Uncertainty Estimation in Wind Power Forecasts Using Monte Carlo Simulations

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***Abstract*—In our earlier paper [9] we took the histories of wind speed forecasts and actual wind speed data available from Environment Canada and presented that the hourly wind speed forecast error distributions are nearly Gaussian in nature.**

In this paper we used the hourly error distribution to model a representative wind-speed realization as the sum of a deterministic term and a stochastic term. The deterministic term was the forecast provided by Environment Canada, while the stochastic component, the error in the forecast, was modeled as a first-order gaussian markov process.

Wind-speed realizations were then input to a wind generator model developed in MATLAB®/Simulink® to get wind power realizations. The uncertainties in the wind speed-realizations were transferred to the wind power realizations as well. Monte Carlo Simulations were performed to assess likely range of wind power production.

It is shown that how the statistics of wind power prediction obtained by performing Monte Carlo Simulation gave an idea of the risk involved in wind power production.

I. INTRODUCTION

The inherent uncertainty in wind makes it hard for a Wind Energy (WE) utility to predict their generation within a range of $\pm 1.5\%$ limits set by the North American Electric Reliability Council (NAERC) [4]. The WE utilities face a challenge to operate in day-ahead electricity markets and are subject to high financial risk in trading. Therefore, rather than participating in day-ahead markets most WE utilities enter into contracts with local conventional suppliers. However, these contracts offer a low price compared to the electricity markets [10], [2].

Some Independent System Operators (ISOs) have introduced new electricity market rules to improve participation of the wind power utilities; these rules allow wind generation to be sold in hour-ahead markets and receive the hour-ahead market prices without any penalty [8]. But ISOs do not consider wind production as a capacity resource, because they have to provide a backup generation source to compensate for the possibility of unanticipated low- or no-wind conditions causing unexpected shortfalls at wind generation facilities [1], [5]. This issue does not arise with a conventional generator because their production can be known in advance with almost certainty.

In most North American Markets, since installed wind capacity is low, their production can essentially be absorbed into the market without any degrading of the system. But as installed capacity increases, there is a common agreement among researchers [1] that wind capacity should be acknowledged by encouraging wind energy participation in day-ahead markets. The ISO of NordPool, a prominent Nordic electricity market, has successfully implemented this, and the results shown by Morthorst [7] show that approximately 20% of total power consumption in Denmark is supplied by wind power and electricity market prices fell approximately between 7 to 13% in the year 2005; since wind energy has low cost of production its participation in the electricity market decreases over all electricity market prices.

The participation of wind energy in a day-ahead market is currently discouraged due to its high uncertainty. If WE utilities can adequately address the issue of uncertainty and variability in wind power generation then they will be allowed to participate in the market for fair pricing, and that will motivate them to invest in better forecasting methods for maximum profit, and the cost associated with running a backup generation. (If the dispatch of a backup generator is linked to wind power production, it may not be possible to run that unit at optimal cost).

The participation in the day-ahead market requires a day ahead-commitment. Any deviation from the committed power will lead to a regulation-up price or regulation-down price which decreases profits. Since a WE utility's production is intermittent, their profits can suffer due to regulation prices. Therefore, a WE utility needs accurate power forecasts and strategies for bidding. In this paper we are addressing statistics of the wind power forecasts to give an idea of the risk involved in estimating the wind power production.

II. FORECASTING WIND POWER PRODUCTION

A. Markov Processes

Since we already established in [10] that hourly wind-speed forecast error distributions can be assumed to be approximately normal, then the random process can be generated as a Gauss-Markov Process (GMP). A continuous process is a Markov Process (MP) if the probability distribution for the current state (range of values) depends only the most recent past state, and if the restriction is added that distribution of the current state is normal then it is called GMP [3]. For example, a continuous process $e(t)$ is a First Order Markov (FOM) process if for every k and

$$t_1 < t_2 < ... < t_k \tag{1}$$

it is true that,

$$F[e(t_k)|e(t_{k-1}), ..., e(t_1))] = F[e(t_k)|e(t_{k-1})] \tag{2}$$

where in this study, e denotes wind-speed forecast error.

B. Autocorrelation Function

Given a string of wind-speed forecast error data, e_i, $i = 1, 2, ..., N$, taken at a constant time intervals. The lag τ Autocorrelation Function (ACF) is defined as,

$$r_{ee}(\tau) = \frac{\sum_{i=1}^{N-\tau}(e_i - \mu)(e_{i+\tau} - \mu)}{\sum_{i=1}^{N}(e_i - \mu)^2} \tag{3}$$

where μ is the true mean of the data; if $\hat{\mu}$ or the sample mean is used then this estimate is biased. In this study the MATLAB® function 'autocorr' was used which has a small bias for large N. Typically, a one dimensional ACF for a random variable exhibits exponential behavior [3].

C. Order of a Markov Process

A random process $e(t)$ with an empirical lag τ ACF calculated using equation 3 may be reasonably well approximated by the following equation [3]:

$$r_{ee}(\tau) = exp\left(-\beta|\tau|\right) \tag{4}$$

where β is the correlation time constant of the data. This ACF can be associated with a first-order differential equation [3],

$$\frac{de}{dt} + \beta e = u(t) \tag{5}$$

Alternatively, a random process $e(t)$ with an empirical lag τ ACF calculated using equation 3 may be fit by the following equation [3]:

$$r_{ee}(\tau) = \left[\frac{2\beta_1\beta_2(\beta_1+\beta_2)}{(\beta_2-\beta_1)^2}\right] \left[\frac{e^{-\beta_1|\tau|}}{2\beta_1} + \frac{-\beta_2|\tau|}{2\beta_2} - \left(\frac{e^{-\beta_1|\tau|}+e^{-\beta_2|\tau|}}{\beta_1+\beta_2}\right)\right] \tag{6}$$

This ACF can be associated with second-order differential equation [3],

$$\frac{d^2e}{dt^2} + (\beta_2+\beta_1)\frac{de}{dt} + \beta_1\beta_2 e = u(t) \tag{7}$$

Equation 5 is termed as a First-Order Markov (FOM) process, while equation 7 is called a Second-Order Markov (SOM) process; β_1 and β_2 are the correlation time constants for the SOM. For a GMP the input $u(t)$ is sum of a gaussian white noise process and possibly a deterministic term. The ACF calculated according to the equation 3 for the wind-speed forecast error data is shown in the figure 1. The formula is evaluated only up to 20 lags (τ) or 20 hours; beyond that it is almost zero. From figure 1, wind-speed forecast error autocorrelation value are seen to fall off exponentially. The calculated ACF was then fitted with the theoretical autocorrelation functions, equation 4 for a FOM model, and equation 6 for a SOM model using a nonlinear least-square fit technique (MATLAB® function 'fit') as shown in figure 1. From that figure it is clear that the FOM model is almost identical to the SOM model fit. The correlation time constant found for the FOM was β=0.2982 hour (95% confidence bounds [0.2645, 0.3313]), and the correlation time constants found for the SOM were β_1=30.96 hour (95% confidence bounds [-459.7, 521.6]) and β_2=0.2933 hour (95% confidence bounds [0.2389, 0.3477]) respectively. The confidence bounds of β_1 of the SOM are very high, i.e., the value is essentially meaningless as demonstrated in figure 1, and thus it can be ignored. Therefore the FOM fit was selected over the SOM fit. Once

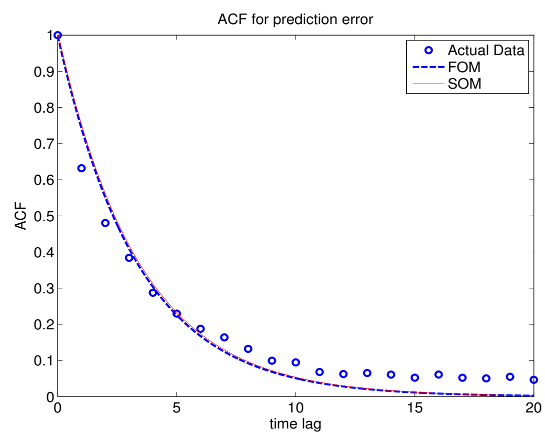

Fig. 1. Fitting an ACF for FOM and SOM processses

the correlation time constant was found the spectral density Q of the white noise that produces $\sigma_e$was calculated [3] as:

$$Q = 2\beta{\sigma_e}^2 \tag{8}$$

After that, solving the differential equation 5 will give wind-speed forecast error dynamics. It should be noted that the statistical properties (mean, variance) of the error data changes on an hourly basis, and therefore this characteristic was incorporated in solving the prediction error dynamics by changing the statistics of the random input $u(t)$ on an hourly basis.

D. Solving a First Order Markov Process

The FOM process was solved using the Euler technique. The FOM model equation 5 can be rewritten as:

$$\frac{de}{dt} = u(t) - \beta e \tag{9}$$

The first-order approximation of the Taylor series solution given as,

$$e(t_0 + h) \simeq e(t_0) + h\ \frac{de}{dt}|_{t=t_0} \tag{10}$$

implies that starting at point $t = t_0$, the value after small time step h, $e(t_0 + h)$ can then be approximated by the value $e(t_0)$ plus the time step multiplied by the slope of the function, i.e., the derivative of function $e(t)$ at $t = t_0$. With this background the Euler technique method for solving an FOM is as follows:

1) Choose an initial condition $e(t_0)$, a time step h, and a terminal time t_f; set $t = t_0$.
2) Select a random value of input $u(t)$, add this to the deterministic component of $u(t)$.
3) Substitute $e(t)$ into equation 9 to determine de/dt.
4) Substitute that value into equation 10 for an approximate value of $e(t + h)$.
5) Let $t = t + h$, $e(t) = e(t + h)$.
6) Repeat steps 2 to 5 until t equals the termination time t_f.

The FOM process describes the dynamics of the prediction error over time (24 hour); solving the FOM using Euler's technique for random initial conditions and random inputs, chosen according to the statistics of the prediction error data, produced realizations of prediction error, and adding those to the forecasted wind speeds gave wind-speed realizations The wind-speed realizations were then passed through a wind power generator model to assess the power production. A rated 9 MW fixed-speed, grid-connected wind farm consisting of three units located in one area was used in this work, it is a customized form of the wind farm model 'power_wind_ig' given in the MATLAB®/Simulink® [6].

III. Uncertainty estimation in power forecasts

The power predictions were obtained by passing wind speed realizations to the wind power generator. Since there are uncertainties in the realized wind speeds, the uncertainties get transferred to the power predications as well. Therefore, Monte Carlo Simulations (MCS) were performed to quantify the risk in the power predictions. The quantification of uncertainty will permit assessing the risk of relying on the power prediction forecasts.

A. Monte Carlo Simulations

MCS converts uncertainty in the input variables of a system into an approximate probability distribution of the outputs. It provides an approach to the statistical analysis of the performance of a system with random inputs by direct simulation. It entails determining system response to a finite number of initial conditions and random input functions generated according to their specified statistics. Thus information required for MCS are the system model, initial condition statistics and random input statistics [11]. Generally, the initial conditions are specified by the mean and the variance of the response. The statistical properties of the random input determines the response after the initial condition. The state space formulation of the FOM model is given as,

$$\dot{e} = u(t) - \beta e \tag{11}$$

where e is the wind prediction error and $u(t)$ is the sum if a white noise process and a deterministic term. The input $u(t)$ is determined by requiring that the statistics (the mean and the standard deviation) of e match empirical values of the hourly forecast-error data. This process is simplified by separating e into its random component and deterministic part:

$$e = e_r + \hat{\mu}_r(t) \tag{12}$$

where, for $(r-1) \leq t < r$, $r = 1, 2, ..., 24$, the deterministic component $\hat{\mu}$ is given by equation 13:

$$\hat{\mu}_{(r)} = \frac{1}{N} \sum_{j=17}^{40} \sum_{i=1}^{N} (v_f(i,j) - v_a(i,j)) \tag{13}$$

where, $v_f(i,j)$ represent the value of the wind speed in Forecast i at the j^{th} hour while $v_a(i,j)$ represent the corresponding actual wind speed. Also $\hat{\mu}_{(r)}$ represent the sample mean wind-speed forecast error of the r^{th} distribution.

The random component of e then satisfies,

$$\dot{e}_r = w_n(t) - \beta e_r \tag{14}$$

where w_n is a gaussian white noise process. The initial condition statistics are given as,

$$E[e_r(0)] = 0 \tag{15}$$

$$E[(e_r(0))e_r^T(0)] = S_0 \tag{16}$$

where S_0 is the variance of the prediction error distribution at time t=00:00. The random component of e_r often characterized by its standard deviation:

$$\sigma_0 = \sqrt{S_0} \tag{17}$$

As mentioned earlier, the statistics of the random input change on an hourly basis; therefore the statistics of the $w_n(t)$ are given as,

$$E[w_n(t)] = 0 \tag{18}$$

$$E[w_n(t)w_n^T(\tau)] = Q_r(t)\delta(t - \tau) \tag{19}$$

where, for $(r-1) \leq t < r$, $r = 1, 2, ..., 24$, the spectral density of the white noise $Q_r(t)$ is given by the equation 20 [11], [3].

$$Q_r(t) = 2\beta\hat{S}(r) \tag{20}$$

Equation 19 indicates that the input random component has zero autocorrelation for $t \neq \tau$, i.e., the quantity $w_n(t)$ is white noise as mentioned above [11]. The sample variance $\hat{S}_{(r)}$ of the wind-speed forecast error of the r^{th} distribution is give by equation 21,

$$\hat{S}_{(r)} = \frac{1}{N-1} \sum_{j=17}^{40} \sum_{i=1}^{N} (v_f(i,j) - v_a(i,j) - \mu^{(r)})^2 \tag{21}$$

B. Prediction Error Realizations

Given the system model, initial condition statistics and random input statistic, the MCS technique generates an ensemble or large number n of the system responses to wind speed realizations. The ensemble of system responses is generated by performing the following procedure n times: choose a random initial condition, i.e., a value $e_r(0)$, according to the statistics provided by equations 15 and 16. Then select a random input vector $w_n(mh)$; the value of white noise with spectral density $Q_r(t)$ was simulated by using the MATLAB® random number generator 'randn' to obtain a sequence of random values $w_n(mh), m = 1, 2, ..., t_f/h$ satisfying [11]

$$E[w_n(mh)] = 0 \quad (22)$$

and

$$E[(w_n(mh))w_n^T(mh)] = \frac{Q(mh)}{h} \quad (23)$$

where h is the simulation time-step, then the random input $w_n(t)$ is defined as,

$$w_n(t) = w_n(mh), \; mh \leq t < (m+1)h \quad (24)$$

where h is small time increment [11]. The input vector is then passed to the Euler integration technique, as mentioned in section 2.4, to propagate the solution from $t = 0$ to $t = h$, and so on until the final time $t_f = 23 : 59 : 59$ hours is reached [11] (only the deterministic term in step 2 is zero). Performing n independent trials yields an ensemble of n prediction trajectories or realizations of e_r, each denoted $e_r^{(i)}(t; x^i(0), w^i(t))$ to show the dependence of trajectory on the random initial condition and random input value [11]:

$$\begin{pmatrix} e_r^{(1)}(t; x^1(0), w^1(t)) \\ e_r^{(2)}(t; x^2(0)w^2(t)) \\ ... \\ e_r^{(n)}(t; x^n(0), w^n(t)) \end{pmatrix}$$

Adding the above realizations of e_r to the deterministic components of the prediction error (equation 12) gives n realizations of the prediction error, each denoted $e^{(i)}(t; e_r^i(t), \hat{\mu}(t))$ to show the dependence of trajectory on e_r and the deterministic component $\hat{\mu}$.

$$\begin{pmatrix} e^{(1)}(t; e_r^1(t), \hat{\mu}(t)) \\ e^{(2)}(t; e_r^2(t), \hat{\mu}(t)) \\ ... \\ e^{(n)}(t; e_r^n(t), \hat{\mu}(t)) \end{pmatrix}$$

C. Power Predictions

Adding the above error realizations to the forecasted wind speeds v_f, gave the wind speed realizations $v_r(t)$, and power predictions were then obtained by passing wind speed realizations to the wind power generator. The block diagram shown in figure 2, shows the process of getting wind power predictions. The limiter was placed after the addition of prediction error realization and forecasted wind speed to avoid negative values of the wind speed realization $v_r(t)$, i.e., for each trial $v_r^i(t)$ is given as,

$$v_r^i(t) = max\left[(v_f + e^i(t)), 0\right] \quad (25)$$

Each realized power prediction is determined by inputting each realized wind speed $v_r^i(t)$ to the wind power generation model. Generating n realizations of forecast error gives n statistically meaningful realizations of wind speed, which yields an ensemble of n realized power predictions:

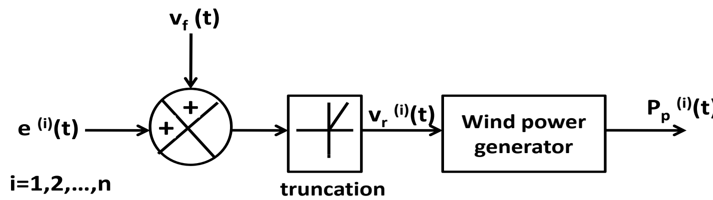

Fig. 2. Wind power realization generation

$$\begin{pmatrix} P_p^{(1)}(t; v_f(t), e^1(t)) \\ P_p^{(2)}(t; v_f(t), e^2(t)) \\ ... \\ P_p^{(n)}(t; v_f(t), e^n(t)) \end{pmatrix}$$

The sample mean $\hat{m}_p(t)$ and variance $\hat{S}_p(t)$ of the power predictions was calculated by the following equations [11],

$$\hat{m}_p(t) = \frac{1}{n} \sum_{i=1}^{n} (P_p^i(t)) \quad (26)$$

$$\hat{S}_p(t) = \frac{1}{n-1} \sum_{i=1}^{n} (P_p^i(t) - \hat{m}_p(t))(P_p^i(t) - \hat{m}(t)_p)^T \quad (27)$$

D. Confidence Intervals

It was desirable to determine the confidence intervals to ensure that true mean is guaranteed to lie within these intervals with a specified probability or confidence. If n is sufficiently large then the confidence interval $[\underline{m}, \overline{m}]$ in which the true mean $\mu(t)$ will lie is centered on the sample mean $\hat{m}(t)$ and the range is governed by the sample variance $\hat{S}(t)$ (or standard deviation $\hat{\sigma}$) and the level of confidence β, is given as,

$$\underline{m} = \hat{m} - \frac{\psi\hat{\sigma}(t)}{\sqrt{n}} < \mu < \frac{\psi\hat{\sigma}(t)}{\sqrt{n}} + \hat{m} = \overline{m} \quad (28)$$

The true value of the mean $\mu(t)$ lies between the values of lower and the upper bound of the inequality, as indicated in equation 28. The lower bound ($\underline{m}$) and upper bound ($\overline{m}$) quantities are referred to as the lower and upper confidence band limits. The values of ψ for various values of confidence β are given in table I, with whatever level of confidence is chosen, e.g., $\beta = 0.90$ or 90% confidence, for ψ=1.645.

ψ	β
1	0.6827
1.645	0.90
1.960	0.95
2.576	0.99

TABLE I
VALUES OF ψ FOR VARIOUS VALUES OF β

The lower and upper confidence limits [$\underline{\sigma}$, $\overline{\sigma}$] of a sample $\hat{\sigma}$ is expressed as [11]:

$$\underline{\sigma} = \underline{\rho}\hat{\sigma}\overline{\sigma} = \overline{\rho}\hat{\sigma} \quad (29)$$

where $\underline{\rho}$ and $\overline{\rho}$ are determined by the level of confidence, β, the kurtosis of the random variable λ and the number of trails performed n, given as [11],

$$\underline{\rho} = \frac{1}{\left[1 + \psi\sqrt{\frac{\lambda-1}{n}}\right]^{\frac{1}{2}}} \overline{\rho} = \frac{1}{\left[1 - \psi\sqrt{\frac{\lambda-1}{n}}\right]^{\frac{1}{2}}} \quad (30)$$

The values of ψ for various values of confidence β are given in the table I. The reasonable choice of λ must be determined before the confidence intervals are calculated. One option is to determine it by the following relation [11]:

$$\lambda \simeq \frac{\hat{\mu_4}}{\hat{S}^2} \equiv \hat{\lambda} \quad (31)$$

where $\hat{\mu_4}$ is the sample fourth central moment and $\hat{S}$ is the sample variance.

E. Selection of Time Step (h)

The selection of the time step h is challenging and it is done heuristically. Since the Euler technique (section II), relies on the derivative of the error function to approximate its trajectories, the smaller the step size, the smaller the error. A small time step of h=.01 hour was chosen which is very small compared to the time constant of the FOM β=0.2982 hour; for $h << \beta$, $w_n(mh)$ is a good approximation to a white noise process.

IV. Test Results

MCS were performed to get a large ensemble (n) of power predictions for the given forecasted wind speeds: a wind speed error realization was generated and then the realization was added to the forecasted wind speeds. The addition of error realization with the wind speed forecast gave the wind speed realization. The realized wind speed was then passed into the wind power generator model to get a wind power prediction. Doing it in the same way for n times gave an ensemble of n power forecasts. The equation 26 and 27 were then used to calculate the statistics of wind power production.

75 MCS were performed, which ensures that the difference between true mean μ and the estimated mean $\hat{m}(t)$ will be less than roughly 11% with 90% certainty [11]. It was not possible to perform more trials since MATLAB® ran out of memory due to large amount of data being stored for post processing. Figure 3 shows the mean and standard deviation of the wind power forecasts, obtained by performing 75 MCS trials.

A. Effects of n on the confidence limits

As mentioned above, confidence limits ensures that true mean lies between the lower confidence limit

$$\underline{m} = \hat{m} - \frac{\psi\hat{\sigma}(t)}{\sqrt{n}} \quad (32)$$

and the upper confidence limit

$$\overline{m} = \frac{\psi\hat{\sigma}(t)}{\sqrt{n}} + \hat{m} \quad (33)$$

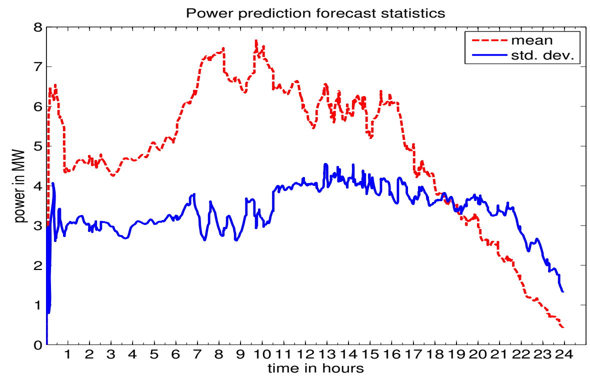

Fig. 3. Wind power statistics, 75 MCS trials

with a specified level of confidence β; The values of n corresponding to the different values of ψ are given in [11]. It must be carefully distinguished between uncertainty due to the number of trials and uncertainty due to wind-speed forecast error. We do not have any control on the uncertainty of wind-speed forecast error, but equation 32 and equation 33 demonstrates the effect of n to achieve a desired degree of a confidence limits, i.e., we can make uncertainty due to the number of trials arbitrarily small by increasing the number of MCS trials performed.

Deciding how many trials to perform requires comparing the wind power production forecast using the EC forecast, P_f, versus that produced with the wind speed forecast error model and MCS, P_p; it is desirable to obtain a clear separation between these two results, i.e., P_f should lie outside the MCS confidence bands of P_p for it to be helpful. The mean of P_p may help a WE utility operator to decide how much power to bid in an electricity market:

1) If $E[P_p] = \hat{m}_p$ lies above P_f then the WE utility operator can be more aggressive in terms of how much power to bid.
2) If $\hat{m}_p$ lies below P_f then the operator should be careful.
3) If $\hat{m}_p$ is approximately equal to P_f then the operator should be neutral.

To illustrate these concepts, the statistics as shown in figure 3 were then substituted into equation 28 to provide the $\hat{m}_p$ confidence intervals, with level of confidence 90% as shown in figure 4 along with the P_f plot.

From figure 4, the $\hat{m}_p$ confidence band lies well above P_f for $02:00 \leq t \leq 08:00$, so bidding could be aggressive over that interval. Clearly, 75 trials of MCS was sufficient for this assessment.

The above strategies will be more effective if $\hat{\sigma}_p$, the sample standard deviation of P_p, is also taken into account. For example, if $\hat{m}_p$ lies above P_f but $\hat{\sigma}_p$ is large, then it may not be wise to be aggressive in bidding.

To use data $\hat{\sigma}_p$ with assurance we should also check its confidence limits. First we must use equation 31 to calculate estimates for the values of kurtosis $\hat{\lambda}$ each one-hourly wind power forecast distribution obtained by performing 75 MCS

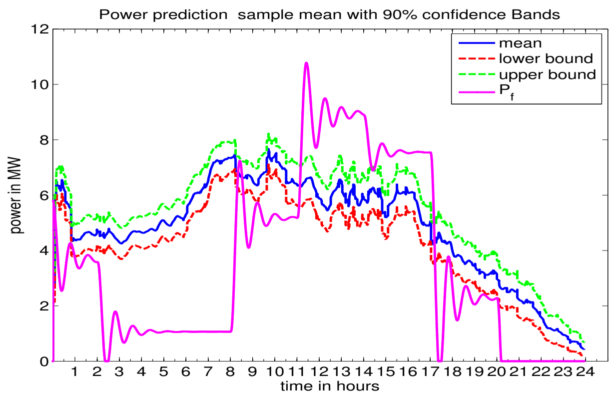

Fig. 4. Wind power sample mean with 90% confidence bands

trials; the results are shown in figure 5.

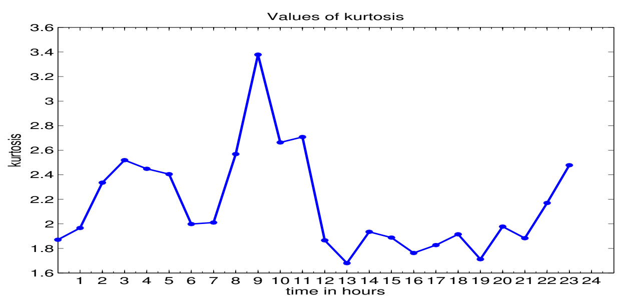

Fig. 5. Estimated kurtosis

The values of kurtosis were then substituted into equation 30 to provide the power prediction confidence intervals with level of confidence 90%, as shown in figure 6.

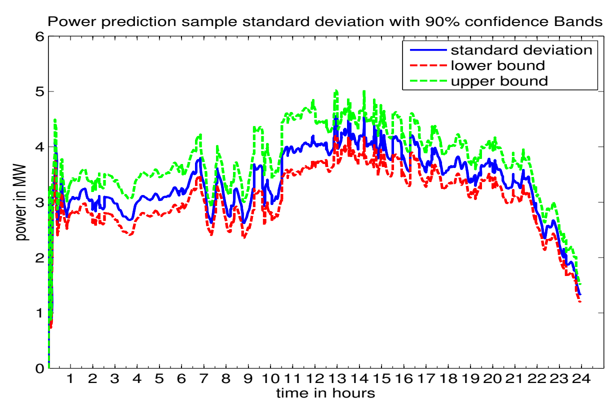

Fig. 6. Wind power sample standard deviation with 90% confidence bands

From figures 4 and 6, the WE utility operator might moderate the three strategies (aggressive, careful and neutral) as $\hat{m}_p$ lies above, below and approximately equal to P_f depending upon the time of the day, noting that the standard deviation $\hat{\sigma}_p$ and its confidence band are quite high (between about 2.4 to 5 MW most of the day) compared to $\hat{m}_p$.

V. Conclusion

The intermittent nature of wind poses operarional and planning chalanges to the electricity market. These challeneges can be adressed with accurate wind power forecasting methods. This paper presents a wind power forecast Using monte carlo-markov Process. It was shown through statistical analysis that the hourly wind-speed prediction error distributions were quite nearly gaussian in nature. After that, the autocorrelation function of the prediction error distribution was fitted with autocorrelation functions, for first- and second-order markov processes. It was shown that a FOM is a more appropriate fit compared to a SOM.

It was shown that solving a FOM using euler's technique for independent initial conditions gave the realizations of the prediction error, and adding those to the forecasted wind speed provided by EC gave the wind speed realizations. The wind speed realizations were then input to an off-the-shelf wind farm model developed in Simulink®. The variability present in the wind speed realizations gets transferred to the wind power realizations by Monte Carlo Simulation. Therefore, it was important to assess the uncertainty in the power production; it was done through MCS.

The MCS essentially gave the variability in the wind power production in terms of statistics and histograms which can used for assessing the risk invloved in power production forecast.

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