EVALUATION OF NONLINEAR FILTER ALGORITHMS WITH CADET™*

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Abstract

Extended Kalman filters and related estimation algorithms have been developed for many guidance applications. In general, the filter equations that result are linear in the context of their derivation but often highly nonlinear in their mechanization. In determining filter performance in such situations, linear covariance analysis is not rigorously applicable, and accurate monte carlo analysis may require unacceptably large expenditures of computer time. This paper describes the first application of the Covariance Analysis DEscribing function Technique -- CADET™ -- to such a problem, to obtain results that are both accurate and inexpensive in terms of computer time.

1. INTRODUCTION AND PROBLEM STATEMENT

In this section, the simplified filter design model, the corresponding filter algorithm, and a guidance system simulation are briefly outlined. This material serves as the basis for the discussion of filter evaluation techniques.

1.1 FILTER DESIGN MODEL

Figure 1-1 defines the guidance problem under consideration. The missile-target intercept is assumed to be planar, and an inertial cartesian coordinate frame is established as shown. A digital guidance system is designed to process discrete noisy measurements of instantaneous line-of-sight (LOS) angle, θ, and uncorrupted measurements of missile acceleration, a_m, to provide estimates of missile-target lateral separation, y, lateral separation rate, $\dot{y}$, and target acceleration, a_t. These estimates serve as the basis of an optimal control law(1) which generates a missile acceleration command, a_c, of the form

$$a_c = c_1\hat{y} + c_2\hat{\dot{y}} + c_3\hat{a}_t + c_4 a_m \qquad (1\text{-}1)$$

Other variables shown in Figure 1-1 are

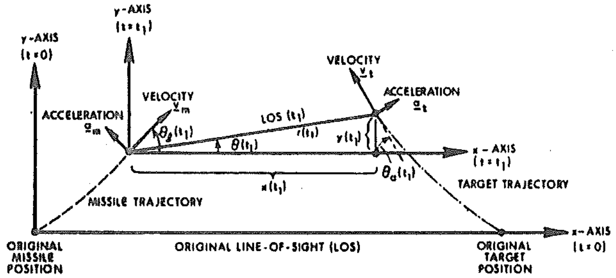


Figure 1-1 Missile-Target Planar Intercept Geometry

$\underline{v}_m$ and $\underline{v}_t$, the missile and target velocity vectors with magnitudes v_m, v_t and orientations θ_ℓ, θ_a respectively; $\underline{a}_m$ and $\underline{a}_t$, the missile and target acceleration vectors with magnitudes a_m, a_t and orientations $(\theta_\ell+\pi/2)$, $(\theta_a+\pi/2)$, respectively; and r, the missile-target range.

The linearized filter design model is portrayed in Figure 1-2. The target acceleration is modeled as a low-pass filtered zero-mean gaussian white noise process, w, with constant spectral density q_0,

$$E\left[w(t)\; w(\tau)\right] = q_o\delta(t-\tau) \qquad (1\text{-}2)$$

and bandwidth ω_t. The kinematic relation, based on assuming that v_m and v_t are constant, is

*Supported by the Office of Naval Research, Contract No. N00014-73-C-0213.

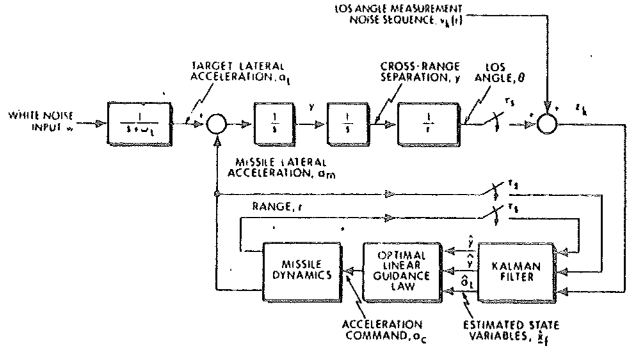


Figure 1-2 Missile-Target Intercept Model for the Derivation of the Digital Guidance System

$$\ddot{y} = v_t\dot{\theta}_a \cos\theta_a - v_m\dot{\theta}_\ell \cos\theta_\ell$$
$$\cong v_t\dot{\theta}_a - v_m\dot{\theta}_\ell = a_t - a_m \tag{1-3}$$

The small-angle approximations have been used, since it is assumed that the intercept is nearly head-on. A second linearization occurs in the measurement model,

$$z_k = \theta(t_k) + v_k$$
$$\cong y(t_k)/r(t_k) + v_k \tag{1-4}$$

where the range r is considered to be a deterministic parameter. The noise sequence v_k is zero mean, with three components having range dependent variances,

$$E\left[v_k^2\right] = \left(\frac{\sigma_1}{r(t_k)}\right)^2 + (\sigma_2 r(t_k))^2 + \sigma_3^2 \triangleq \sigma_{v_k}^2 \tag{1-5}$$

where σ_1, σ_2 and σ_3 are constant and may represent receiver noise and distant stand-off jamming, target amplitude scintillation, and angular scintillation, respectively.

The above comments provide the basis for the filter design model

$$\dot{\underline{x}}_f = F_f\underline{x}_f(t) + \underline{g}_f w(t) + \underline{d}_f a_m(t),$$

$$\underline{x}_f = \begin{bmatrix} y \\ \dot{y} \\ a_t \end{bmatrix},\ F_f = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\omega_t \end{bmatrix},\ \underline{g}_f = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ \underline{d}_f = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$z_k = \underline{h}_k^T\underline{x}_f(t_k) + v_k,\quad \underline{h}_k^T = \begin{bmatrix} \frac{1}{r(t_k)} & 0 & 0 \end{bmatrix} \tag{1-6}$$

1.2 FILTER ALGORITHM

The principles of optimal estimation(2) lead to the following algorithm to provide estimates, $\hat{\underline{x}}_f$, of $\underline{x}_f$:

$$\dot{\hat{\underline{x}}}_f(t) = F_f\hat{\underline{x}}_f(t) + \underline{d}_f a_m(t),\qquad t_{k-1} \le t < t_k$$
$$\hat{\underline{x}}_f(t_k^+) = \hat{\underline{x}}_f(t_k) + \underline{k}_k(z_k - \underline{h}_k^T\hat{\underline{x}}_f),\quad t = t_k \tag{1-7}$$

where $\hat{\underline{x}}_f(t_k^+)$ represents the estimate after the measurement and update take place. The gain vector $\underline{k}_k$ is generated according to the following well-known Kalman filter relations: At time $t_k = k\tau$,

$$P_k = \Phi P_{k-1}^+\Phi^T + Q,\quad \Phi \triangleq \exp(F_f\tau) \tag{1-8}$$

$$\underline{k}_k = P_k\underline{h}_k(\underline{h}_k^T P_k \underline{h}_k + \sigma_{v_k}^2)^{-1} \tag{1-9}$$

$$P_k^+ = P_k - \underline{k}_k(\underline{h}_k^T P_k \underline{h}_k + \sigma_{v_k}^2)\underline{k}_k^T \tag{1-10}$$

where Φ is the transition matrix, and

$$Q \triangleq q_0 \int_0^\tau e^{F_f(\tau-t)}\underline{g}_f\underline{g}_f^T e^{F_f^T(\tau-t)}dt \tag{1-11}$$

The estimate update term of Eq. (1-7) is expressed in terms of the elements of P as

$$\underline{k}_k(z_k - \underline{h}_k^T\underline{x}_f) = \frac{(r(t_k)z_k - \hat{y})}{(p_{11}+\sigma_1^2)+\sigma_3^2 r^2(t_k)+\sigma_2^2 r^4(t_k)}\begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \end{bmatrix} \tag{1-12}$$

This relation often requires a "secondary measurement", r, to be available to the filter, in addition to the primary measurements θ and a_m.

1.3 GUIDANCE SYSTEM SIMULATION

The filter sketched above is incorporated in the higher-order system model depicted in Figure 1-3. In general terms, the seeker or target tracking device has a second-order linear representation, with LOS angle, missile body angle, θ_m, and three seeker noise sources, w_1 to w_3, as inputs, the latter representing measurement noise components indicated in Eq. (1-5). The guidance system is composed of the filter, defined above, followed by the linear optimal control law, Eq. (1-1) and an acceleration command limiter, or ideal saturation

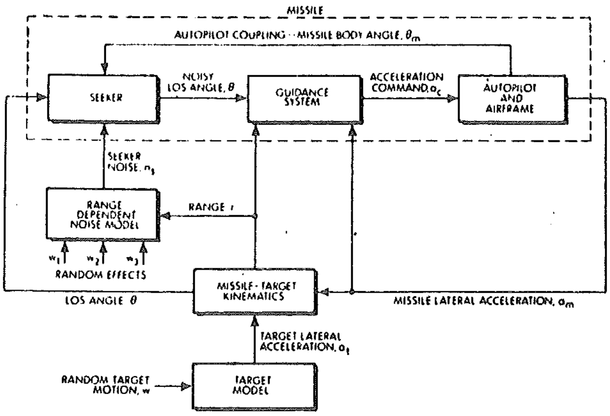


Figure 1-3 Basic Simulation Block Diagram

which maintains $|a_c| \leq a_{max}$. The autopilot and airframe model is a linear third-order subsystem. The missile-target kinematics block is nonlinear, given by Eq. (1-3) without the small angle approximation and $\theta = \tan^{-1}(y/x)$. Finally, the target model generates the target acceleration amplitude a_t as in Figure 1-2. Some of the important parameters quantifying this model are given in Table 1; see (4) for details.

TABLE 1 SIMULATION SYSTEM MODEL PARAMETERS

Parameters	Values
Seeker Poles	s = -10, -50
Seeker rms Noise Levels	σ_1 = 0.707 rad-ft σ_2 = 0.707×10^{-8} rad/ft σ_3 = 0.707×10^{-4} rad
Acceleration Command Limit	a_{max} = 750 ft/sec^2
Autopilot/Airframe Poles	s = -3.16, -7.56 ± 13.0j
Missile, Target Velocities	v_m = 3000 ft/sec v_t = 1000 ft/sec
Target rms Acceleration Level	σ_{a_t} = 150 ft/sec^2

2. GUIDANCE FILTER PERFORMANCE EVALUATION

Given the above missile-target intercept model, evaluating the performance of the guidance filter in a "typical" engagement is a primary concern. In addition to the system parameters given in Table 1, an ensemble of engagements must be specified in terms of initial condition statistics. In the present study, the only variables with non-zero initial conditions are

$$x(0) = E[x(0)] = 24{,}000 \text{ ft}$$

$$E\left[\theta_\ell(0)\right] = 0 \text{ rad},\ E\left[\theta_\ell^2(0)\right] = (0.01745 \text{ rad})^2 \quad (2\text{-}1)$$

The initial value of x, in combination with the closing velocity given by $v_m + v_t$ or 4000 ft/sec (Table 1) gives a nominal engagement time* (time to intercept) of 6.0 sec.

Since the filter is suboptimal, its P matrix may not reliably describe the estimation error. One way to evaluate filter performance is monte carlo analysis: From a large ensemble of simulations with suitable random initial conditions and random inputs, one can approximately determine the statistics of the estimation error. The problem is that many trials are required for an assurance that the analysis is accurate; this may take a prohibitive amount of computer time. A second approach is to assume that the impact of random effects on range is negligible. The filter is then linear time-varying, with gains deterministically parameterized according to a "nominal" r(t), and covariance analysis(2) is used to obtain the estimation error covariance. In the present case,

$$r(t) \cong r_n(t) = 4000(6.0-t) \text{ ft} \quad (2\text{-}2)$$

is substituted into the filter equations. Although such an analysis can be performed using a small fraction of the comptuer time needed for an accurate monte carlo analysis (typically, 10 percent or less), its validity is questionable.

A new technique for filter evaluation is to quasi-linearize the nonlinear algorithm, and then use the Covariance Analysis DEscribing Function Technique -- CADET(3). Since the effectiveness of CADET in capturing random

*The presence of a random initial lead angle (Eq. (2-1)) and random target maneuvers gives rise to a mean terminal time of 6.25 sec for the present scenario.

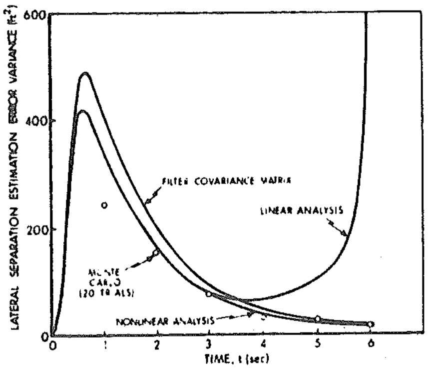
effects in nonlinear systems has been well established,[1,3,4], a filter evaluation using CADET should often be both accurate and efficient.

The above filter performance evaluation techniques have been applied to the guidance filter described in Section 1. Due to computer budget limitations, the monte carlo study was only qualitative (20 trials), to determine which of the covariance analysis techniques is more accurate. To make the comparison as exact as possible, the other system nonlinearities (acceleration command limiter and kinematic nonlinearities) were quasi-linearized in both covariance analyses.

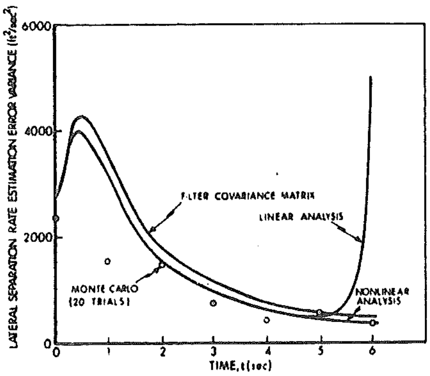
Figure 2-1 depicts the estimation error variances given by the filter covariance matrix, by CADET analysis of the approximate linear time-varying guidance system model, by a CADET analysis of the nonlinear guidance model, and by the 20-trial monte carlo study (encircled data points). In all cases, the linear analysis shows a marked divergence from the true estimation error variance (as given by CADET for the nonlinear model and verified by the monte carlo results) at the end of the engagement. Also, the filter covariance matrix seems to be unrealistically pessimistic in the first two cases, Figure 2-1a and b; the monte carlo data in Figure 2-1c is too scattered to assess which is more accurate in that case.

3. CONCLUSION

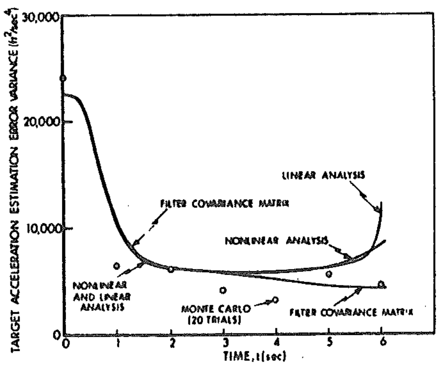
Clearly, analyzing the nonlinear filter algorithm via CADET leads to performance evaluations that are in better agreement with monte carlo results than either the diagonal elements of the filter P matrix, or the approximate covariance analysis based on a linear filter model. It is felt that this study is of quite general significance, since the accurate and efficient assessment of suboptimal nonlinear filter algorithms is of broad interest.



(a) Lateral Separation



(b) Lateral Separation Rate



(c) Target Lateral Acceleration

Figure 2-1 Analysis of Filter Estimation Error Variance

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