SYNTHESIS OF LINEAR PID CONTROLLERS FOR NONLINEAR MULTIVARIABLE SYSTEMS

Amir Nassirharand
Dept. of Mechanical Engng.
University of Kentucky
Lexington, KY 40506-0046

James H. Taylor
GE Corporate R & D
Building KW, Room D209A
Schenectady, NY 12301

Abstract

We report on a new, systematic approach to the synthesis of linear multivariable controllers for use with multiple-input/multiple-output (MIMO) nonlinear systems. The approach is based on describing function models of the system followed by numerical optimization in the frequency domain. The end result is a closed-loop feedback system which is insensitive to the amplitude level of the excitation signal, and which approximates a set of user-defined performance criteria with minimum mean square error. We demonstrate the procedure by solving an example problem.

1. Introduction

The objective in designing a multivariable closed-loop feedback system is to devise a controller that causes the system to behave in a desired way. In some cases, the plant may be described adequately by a linear model. In other cases, a nonlinear model may be required to achieve adequate realism. Frequency-domain methods are well established for the design of controllers for linear multivariable plants (e.g., [1-4]); however, controller design methods for use with nonlinear multivariable systems in the general setting considered herein is at its early stages of development [5].

Controller design techniques for use with *special classes* of nonlinear multi-input/multi-output (MIMO) nonlinear systems have received considerable attention. Global transformation methods [6] require that system differential equations have differentiable right-hand sides, and therefore they may not be applied to systems with discontinuous nonlinearities or nonlinearities with discontinuous derivatives (e.g., backlash and saturation). Control design techniques based on the theory of variable structure systems (VSS) are only applicable to systems wherein the nonlinearities of the plant are within the image space of the input distribution matrix [7-10]. It should also be noted that application of the VSS theory to discontinuous systems becomes difficult, and the existence and uniqueness of the sliding mode solutions may not formally be satisfied [7]. There also exist computer-based methods for the design of compensators for nonlinear feedback systems [11, 12] wherein the user designs a preliminary linear compensator using classical linear frequency-domain compensation procedures which then serves as the basis on which the final compensator is designed to meet additional performance functionals using the method of inequalities. The application of these techniques are limited to nonlinear feedback systems characterized by separable nonlinear elements. Finally, optimal [13] or adaptive [14, 15] control laws are difficult or impossible to design for nonlinear plants, and when they can be obtained, usually require a dedicated digital computer for implementation. Such control systems are thus rarely practical.

In this research, controller design is based on the *finite-signal input/output* behavior of the nonlinear plant. We consider the class of nonlinear multivariable systems whose mathematical model is in the following state-variable differential equation form:

$$\dot{x}(t) = f(x(t), u(t)) \tag{1}$$

$$y(t) = g(x(t), u(t)) \tag{2}$$

where x is the state vector of dimension n, u is the input vector of dimension m, and y is the system output of dimension l. There are no restrictions regarding the type or location of the nonlinearities in the system. The finite-signal input/output behavior of the system is characterized using a non-parametric identification technique which is applicable to any system that can be simulated with sinusoidal inputs. This approach allows one to describe the system motion "in the large"; this is of prime importance when analyzing or designing a controller for amplitude-sensitive nonlinear systems. In addition, the controllers designed using the proposed design technique may either be implemented using simple RLC networks or they may easily be implemented on a microprocessor.

Our primary objective is to present a systematic procedure for synthesis of *multi-range* PID controllers for use with multivariable systems that have amplitude-sensitive plants. The term *multi-range* arises from the fact that the controller design is based on several plant characterizations that correspond to different operating regimes. These regimes are characterized not by differing operating points, but rather by the expected amplitudes or ranges of the input signals. The objective of the controller design is to arrive at a closed-loop system that (i) is insensitive to the amplitude level of the excitation signal, (ii) approximates a set of user-defined performance measures with minimum mean square error, and (iii) is approximately decoupled. By "decoupled" we mean that each control input variable affects only one output.

The controller synthesis procedure is composed of seven steps that are defined in detail in the next section. It identifies a minimum-sensitivity linear multivariable controller that achieves a set of user-defined design objectives as closely as possible. If the simulation results in the last step reveal that the design objectives are not met, then the designer may have to synthesize a nonlinear multivariable controller. The design procedure is based on a modeling approach described in [16] and was obtained while attempting to extend the previous work done by Taylor and Strobel [17, 18] for single-input / single-output systems to the multivariable case.

2. Controller Synthesis Procedure

The controller synthesis procedure requires the following *a priori* information:

a. The mathematical model of the nonlinear plant in state-variable differential equation form (Eqns. (1) and (2)).

b. A number of operating regimes of the nonlinear plant. These operating regimes are defined by: (i) a set of expected amplitudes of the plant excitation signal, and (ii) a set of excitation frequencies of interest.

c. Closed-loop system specifications in the frequency and/or time domain.

Technical details of the seven-step controller synthesis procedure are given below.

2.1. Input/output Characterization. One approach to input/output characterization is to obtain the small-signal linear (SSL) model of the plant by replacing each nonlinear term with a linear term whose gain is the slope of the nonlinearity at the operating point. However, such models are not defined for systems which have discontinuous or multi-valued nonlinearities or nonlinear elements with discontinuous slopes. Furthermore, SSL I/O models fail to capture the amplitude sensitivity of the original nonlinear system. Therefore, we use an alternative approach to I/O characterization which does not have the disadvantages of the small-signal linearization technique.

We characterize the I/O behavior of a nonlinear MIMO plant by obtaining *sinusoidal-input describing function* (SIDF) models of it [16]. The advantages of using SIDF models for nonlinear systems design have been discussed in detail elsewhere (e.g., [17-21]), so we do not elaborate on this issue here. Suffice it to say that representing the nonlinear plant with its sinusoidal-input describing function model has proved to be a powerful tool for analysis and design of nonlinear control systems [17-22].

In order to obtain an SIDF model at the operating point $\underline{u}_0$ with input amplitude $\underline{a} = [a_1, a_2, \ldots a_m]$, we excite all channels at one time with different but nearly equal frequencies, i.e.,

$$u_p(t) = u_{0p} + a_p \cos(\omega_p t), \; p = 1,2,\ldots m \tag{3}$$

where u_{0p} is the "DC component" of the input signal or operating point, a_p is the amplitude of the p^{th} excitation signal, ω_p is the frequency of the p^{th} sinusoidal input, and p is the input channel index. Then, the dynamic equations of motion are numerically integrated to obtain the outputs as a function of time, $y_q(t)$. The matrix Fourier integrals, $I_{m,k}^{p,q}$, for period k, are integrated simultaneously, and accepted as valid when all signals $y_q(t)$, $q = 1,2,\ldots l$ have achieved steady-state:

$$I_{m,k}^{p,q} = \int_{(k-1)T}^{kT} y_p(t)\, e^{-j m \omega_q t}\, dt \tag{4}$$

where $k = 1,2,\ldots$, $m = 0,1,2,\ldots$, $T = 2\pi/|\omega_p - \omega_q|$, and p and q are the input channel index and output channel index, respectively. The constant or DC component of the response is given by $I_{0,k}^{p,q}$, and the pseudo-transfer function at discrete frequency ω_q is given by

$$G^{p,q}(j\omega_q; u_{0p}, a_p) = \frac{|\omega_p - \omega_q|}{\pi a_p} I_{1,k}^{p,q} \tag{5}$$

In order to analyze the importance of the higher harmonic effects, one may also evaluate

$$G_m^{p,q}(j\omega_q; u_{0p}, a_p) = \frac{|\omega_p - \omega_q|}{\pi a_p} I_{m,k}^{p,q}, \; m = 2,3,\ldots \tag{6}$$

where in Eqns. (5, 6) it is assumed that k is sufficiently large that the output signals $y_q(t)$, $q = 1,2,\ldots l$ have achieved steady-state. It should be emphasized that the validity of the Fourier analysis defined above requires that the various input frequencies must be related rationally, e.g., $\omega_1/\omega_2 = 3/4$ for the two-input case. Finally, for each given excitation amplitude vector $\underline{a}_i$, Eqn. (5) is evaluated at the discrete frequencies in the user-defined frequency set to obtain a quasilinear model $\underline{G}(j\underline{\omega}; \underline{u}_0, \underline{a}_i) = [G^{p,q}(j\omega_q; u_{0p}, a_{p,i})]$ of the nonlinear plant. This procedure for various user-defined excitation amplitudes is repeated to obtain a number of quasilinear models of the nonlinear plant.

2.2. Performance Specifications. Once the I/O behavior of the system is characterized for several operating regimes of interest, the designer is in a position to identify a set of closed-loop performance criteria. For example, the desired transient response of the control system may be characterized by specifying an I/O channel pairing and a minimum degree of decoupling in order to approximately "diagonalize" the system, and a desired rise time or 2% settling time and a maximum allowable percent overshoot might be specified for each "diagonal" I/O channel pair. The latter objective may be stated in terms of each output following the corresponding input signal with a response similar to that of a second-order system. This information is used in design of a preliminary PID controller for the nominal case in the next step.

2.3. Preliminary PID Design. In this step, we design a linear multivariable PID controller for one nominal amplitude $\underline{a}^*$ via time-domain optimization. We obtain the parameters of a multivariable linear PID in the following fashion: Based on the performance criteria specified in the previous step, we determine the time response of each output channel for a step change in each input channel and compare that with the specification. For example, for a two-input two-output system, we use the following procedure and notation: We first excite input channel 1 with a step input of amplitude a_1^* while the second input channel is excited with a step of zero amplitude. The corresponding responses of the first and the second output channels are denoted by $y_1(t)$ and $y_2(t)$, respectively. In this case, we would ideally like to see $y_1(t) = y_{d1}(t)$ and $y_2(t) = 0.0$, where $y_{d1}(t)$ is the user-defined desired time response consistent with the performance criteria of the previous step. Similarly, the second input channel is excited with a step input of amplitude a_2^* while input channel 1 is held at zero and the corresponding responses of the first and the second output channels are denoted by $y_3(t)$ and $y_4(t)$, respectively. Again, we would ideally like to see $y_3(t) = 0.0$ and $y_4(t) = y_{d2}(t)$, where $y_{d2}(t)$ is the user-defined desired time response consistent with the performance criteria of the previous step. Therefore, we use time-domain optimization to search for the 12 parameters of the linear multivariable PID by minimizing the following objective function:

$$F = \left[y_{d1}(t) - y_1(t)\right]^2 + \left[0.0 - y_2(t)\right]^2 + \left[0.0 - y_3(t)\right]^2 + \left[y_{d2}(t) - y_4(t)\right]^2 \tag{7}$$

It should be noted that the first and the fourth terms of the objective function attempt to achieve command following, while the second and the third terms of the objective function relate to decoupling.

2.4. Reference Open-Loop System Model. In this step, we connect the preliminary PID controller $\underline{C}^*$ of the previous step to the nonlinear plant. Then the I/O characterization approach of Step 2.1 is applied and the matrix transfer characteristics of the desired open-loop system $\underline{C}^* \cdot \underline{G}^*(j\omega; \underline{e}^*)$ is calculated, where $\underline{e}^*$ is chosen to be consistent with the excitation amplitude at the nominal conditions and the PID gain near the cross-over frequency, denoted $|\underline{C}_{co}^*|$, i.e., $\underline{e}^* = |\underline{C}_{co}^*|^{-1} \cdot \underline{a}^*$. At this stage, the objective of the controller design problem is to desensitize the dynamic behavior of the open-loop system with respect to the variations in amplitude level of the error vector. Hence the need for the reference model, which is used in the next step to design a number of linear multivariable PID controllers at different operating regimes.

2.5. Amplitude-Dependent Controller Parameters. In this step, a *set* of linear multivariable controllers $\{\underline{C}_i(j\omega)\}$ is designed so that the error

$$\underline{E}_i(j\omega) = \underline{C}_i(j\omega) \cdot \underline{G}(j\omega, \underline{a}_i) - \underline{C}^* \cdot \underline{G}^*(j\omega, \underline{e}^*) \tag{8}$$

is minimized over the desired set of frequencies, $\{\omega_k\}$, in the least-squares sense, where $\underline{a}_i = \underline{e}_i\, |\underline{C}_i(j\omega)|$. This yields a set of multivariable linear PID controller parameters for each value of $\underline{e}_i$, denoted $\{K_p^{p,q}(\underline{e}_i)\}$, $\{K_I^{p,q}(\underline{e}_i)\}$, and $\{K_D^{p,q}(\underline{e}_i)\}$.

2.6. Final Linear Multi-Range PID Synthesis. In this step, an average value of the PID parameters of the previous step is calculated to synthesize a multi-range linear PID controller. The simulation results of the next step will reveal if the outlined scheme of controller synthesis is acceptable. If the linear PID is not acceptable, then the controller parameters obtained in the previous step might be interpreted as describing functions for controller nonlinearities; then, controller nonlinear elements might be obtained by SIDF inversion; the details of this approach have not yet been worked out. As an alternative approach, nonlinear gains may be expressed as a nonlinear function of the two excitation amplitudes; then the coefficients of this nonlinear function may be identified via optimization.

2.7. Multi-Range PID Controller Validation. In this step, time-history simulations are performed in much the same manner as in Step 3, except all of the desired input amplitudes $\underline{a}_i$ are used instead of one nominal case $\underline{a}^*$. Again, for a two-input two-output system, we excite input channel 1 with step inputs of amplitude $a_{i,1}$ while the second input channel has zero excitation, and the resulting responses of the first and the second output channels are compared with $y_{d1}(t)$ and 0.0, respectively to see if the first I/O channel has a suitably "diagonalized" response at every input amplitude. The same simulation procedure is carried out for the second input channel; again, we would ideally like to see that the output of channel 1 is 0.0 and the output of channel 2 is a good approximation to $y_{d2}(t)$ where $y_{d2}(t)$ is the user-defined desired time response consistent with the performance criteria of Step 2.

3. Demonstration Problem

Consider the schematic block diagram of the nonlinear system shown in Fig. 1. The mathematical model of the system is given as follows:

$$\dot{x}_1 = x_2 + T_1 \tag{9}$$

$$\dot{x}_2 = -x_1^3 - 2.0\,x_2 + T_2 \tag{10}$$

$$T_1 = \begin{cases} m_1 u_1 & \text{if } |u_1| < \delta_1 \\ Sign\,(u_1)\cdot(m_1\delta_1 + m_2(|u_1| - \delta_1)) & \text{if } |u_1| > \delta_1 \end{cases} \tag{11}$$

$$T_2 = \begin{cases} \tilde{m}_1 u_2 & \text{if } |u_2| < \delta_2 \\ Sign\,(u_2)\cdot(\tilde{m}_1\delta_2 + \tilde{m}_2(|u_2| - \delta_2)) & \text{if } |u_2| > \delta_2 \end{cases} \tag{12}$$

It is desired to synthesize a PID controller for this system that would form a closed-loop feedback system that is insensitive to the amplitude level of the excitation signal and approximates a set of user-defined performance criteria with minimum mean square error. We proceeded as in Section 2, and synthesized the following multi-range controller:

$$\underline{K_P} = \begin{bmatrix} 431.14 & 0.0696 \\ 0.0426 & 713.10 \end{bmatrix}, \tag{13}$$

$$\underline{K_I} = \begin{bmatrix} 187.86 & 3.05 \\ 0.0376 & 1441.66 \end{bmatrix}, \tag{14}$$

$$\underline{K_D} = \begin{bmatrix} 0.0748 & 0.391 \\ 0.0567 & 0.143 \end{bmatrix} \tag{15}$$

We validate the design as indicated in Section 2.7. We excite channel one with step inputs of various magnitudes while the second channel is excited by a zero input signal. We then observe the first channel output, $y_1(t)$, and the second channel output, $y_2(t)$. This set of simulation results are shown in Figs. 2 and 3. We then excite the second channel with step inputs of various magnitudes while the first channel is unexcited, and observe the first output, $y_3(t)$, and the second output, $y_4(t)$. This second simulation results are shown in Figs. 4 and 5. From examination of Figs. 2-5, it is apparent that we have achieved a closed-loop feedback system which is *fairly* decoupled, is relatively insensitive to the amplitude level of the excitation signal, and approximately satisfies the specified performance criteria. Ideally we would like to see $y_2(t)$ and $y_3(t)$ to be exactly zero for all time; this would indicate perfect decoupling (see figures 3 and 4). We would also like to see that $y_1(t)$ and $y_4(t)$ be completely amplitude insensitive and satisfy the specified performance criteria; this clearly cannot be achieved with linear compensation (see Figs. 2 and 5).

4. Summary and Conclusions

The goal of this study was to develop a systematic controller synthesis procedure for the design of feedback systems with nonlinear, multi-input / multi-output, time invariant, and continuous-time plants. The premise of this research was that SIDF models of a nonlinear plant can be used to design effective control systems for nonlinear multivariable plants. The work presented herein supports the conclusion that such a control design procedure does indeed produce closed-loop systems that are exhibit reduced amplitude sensitivity. Therefore, this goal has been met.

Three types of information are required for this controller synthesis procedure: (a) a mathematical model of the nonlinear multivariable plant in standard state-variable differential equation form, (b) a definition of the desired operating regimes (amplitudes and frequencies), and (c) a set of performance criteria. The mathematical model of the plant is used to generate the SIDF models of the plant at the specified operating regimes of interest. A minimum-sensitivity linear PID controller is then identified using the controller synthesis procedure given in Section 2. This procedure is systematic, and it may be employed without a high level of subjectivity.

The method and associated software were applied to an example problem, and satisfactory results were obtained. There are several areas of future work that may prove to be fruitful:

a. extension of the method to permit the solution of the disturbance rejection problem, and

b. extension of the method to allow synthesis of nonlinear multivariable PID controllers.

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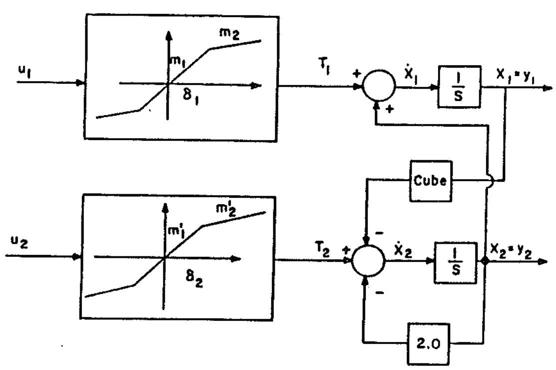

Figure 1. Schematic block diagram of the example problem

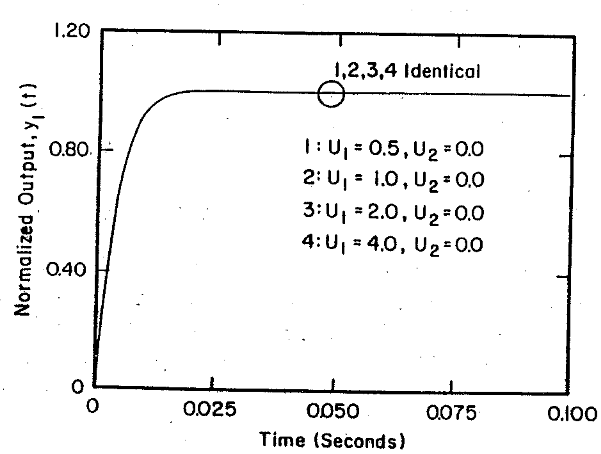

Figure 2. Normalized step response of the first output channel validating the command following features

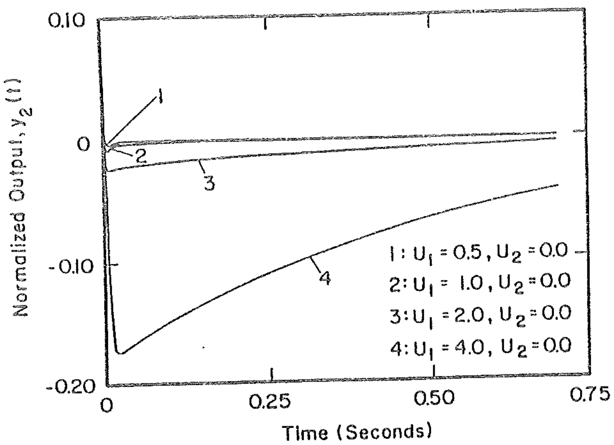

Figure 3. Normalized step response of the second output channel validating decoupling features

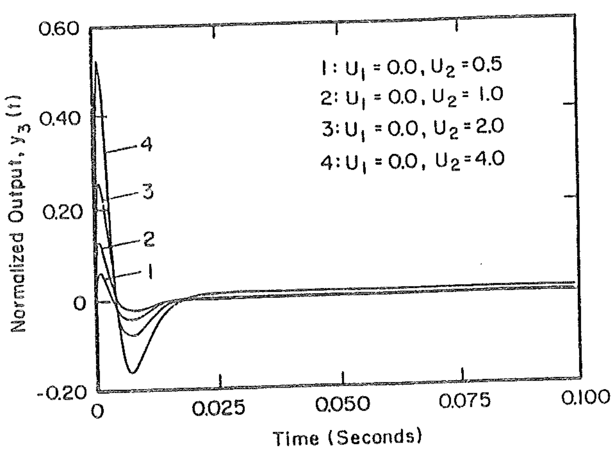

Figure 4. Normalized step response of the first output channel validating decoupling features

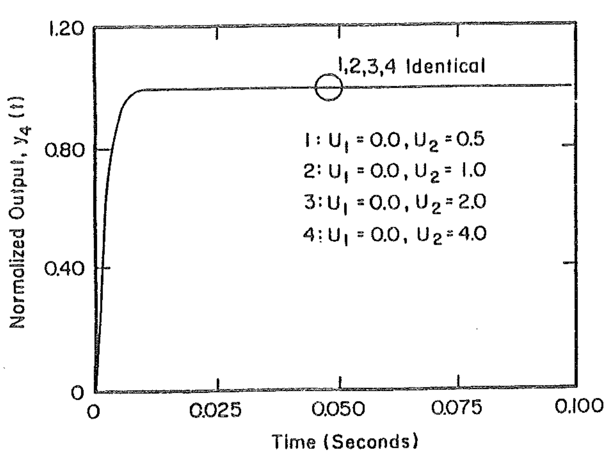

Figure 5. Normalized step response of the second output channel validating command following features