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NONLINEAR COMPENSATOR SYNTHESIS VIA SINUSOIDAL-INPUT DESCRIBING FUNCTIONS

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ABSTRACT

We describe a new nonlinear compensator synthesis approach and illustrate it with an application to a position servo design problem from robotics. The synthesis technique is based on a set of amplitude-dependent sinusoidal-input describing function (SIDF) models of the nonlinear plant. An intermediate step is the design of a linear compensator set based on these models; final synthesis of the nonlinear control system is accomplished by SIDF inversion to determine the required compensator nonlinearities. The major extension in comparison with earlier research is that the compensator so obtained is fully nonlinear; i.e., there is a nonlinear operator associated with each term (proportional, integral, derivative) in the compensator. This approach is capable of treating nonlinear plants of a very general type, with no restrictions as to system order, number of nonlinearities, configuration, or nonlinearity type, and can be extended readily to include other compensator types, e.g., lead/lag. The end result is a closed-loop nonlinear control system that is relatively insensitive to reference input amplitude.

1. INTRODUCTION

The primary intent of this paper is to describe a new nonlinear compensator synthesis approach for designing a closed-loop system that is as insensitive to input amplitude as possible. The method will be briefly illustrated with an application to a position servo design problem from robotics; however, the application is much more fully detailed in [1], which we consider to be a companion to this presentation.

This technique uses a set of sinusoidal-input describing function (SIDF) models of the nonlinear plant as the basis for nonlinear compensator synthesis. SIDF models are used because we believe they provide the best characterization of the major nonlinear effect with which we are concerned: the sensitivity of the nonlinear plant's input/output (I/O) behavior to the amplitude of the input signal. This issue has been discussed in detail in [2-4]; here we will only mention a few factors which we believe are important:

a. Standard linearized models are not dependent on input amplitude.
b. Standard random-input DF models fail to capture the fact that the effect of each system nonlinearity is frequency-dependent [3,4].
c. *Sets* of SIDF models covering the range of input amplitudes that will be encountered provide an excellent basis for "robust design," in the sense that the sensitivity of the plant behavior to input amplitude is an important issue in robustness, and the SIDF I/O model is the least conservative model that accurately accounts for this factor.
d. SIDF I/O models of single-input plants are parameterized by a single parameter, i.e., the input amplitude, rather than a parameter corresponding to each nonlinearity (e.g., a parameter that takes on values determined by the conic sector or range of the derivative of each nonlinearity). This point is a major factor related to the practicality and conservatism of any approach: compensator synthesis based on models parameterized on the basis of one independent parameter per nonlinearity are difficult to carry out and tend to provide stability conditions that are very conservative.

Once the required set of SIDF models is available, the synthesis of a nonlinear compensator proceeds as follows: First, a linear compensator *set* is designed based on these models, with the objective of making the forward path of the control system as insensitive to input amplitude as possible; then, final synthesis of the nonlinear control system is accomplished by SIDF inversion to determine the required compensator nonlinearities. The particular approach presented here involves the design of PID compensators, both linear and nonlinear; there is no restriction as to compensator structure except that the linear controller set and final nonlinear design must be of one type, e.g., PID, lead-lag, etc.

The major extension in comparison with our earlier research [4] is that the controller obtained is fully nonlinear, i.e., there is one nonlinear operator associated with each term (proportional, integral, derivative) in the compensator. This approach is capable of treating nonlinear plants of a very general type, with no restrictions as to system order, number of nonlinearities, configuration, or nonlinearity type.

2. NONLINEAR COMPENSATOR DESIGN APPROACH

First, let us re-emphasize the underlying premises of the SIDF design approaches that we have been developing [1-4]:

a. The nonlinear system design problem being addressed is the synthesis of compensators that are effective for plants having frequency-domain input/output models that are sensitive to input amplitude (e.g., for plants that behave very differently for "small," "medium," and "large" input signals).
b. The primary objective of compensator design is to arrive at a closed-loop system that is as insensitive to input amplitude as possible.

This encompasses a limited but important class of problems, for which gain-scheduled compensators cannot be used and for which the desired response is similar to that provided by a linear system (in the sense of b. above).

As mentioned above, SIDF I/O models are central to this approach. The generation of such models has been dealt with in detail in [4,5]. There are two basic approaches: solving the nonlinear algebraic equations derived from the principle of harmonic balance and simulation coupled with fourier analysis.

The harmonic balance approach is not easy to apply, especially if it is desired to develop a general package that substitutes the appropriate SIDFs into the corresponding nonlinear algebraic equations and solves them. Also, the assumption is made that the input to each nonlinearity is approximately sinusoidal (refer to [6,7]), which may leave the analysis open to question. However, there is an advantage to this approach: the SIDF model is obtained in a form that lends itself to further analysis such as finding the roots of the quasilinear characteristic equation.

The second technique is easier to implement, given a good package for integrating nonlinear differential equations, and avoids the need to justify the assumption that the inputs of every nonlinearity are nearly sinusoidal — there is no such assumption made using simulation. We have recently extended the nonlinear simulation package SIMNON [8] to perform SIDF I/O model generation for nonlinear system models. The basic idea is to drive the nonlinear plant with a sinusoid of the desired amplitude for the set of frequencies of interest, and evaluate Fourier integrals as the simulation proceeds. The simulation for a given frequency is stopped when the Fourier integrals have converged, and the I/O model is evaluated; for details, refer to [5]. Henceforth, we will call this extended simulation package FRSIMN, and use the notation $G(j\omega,a)$ to designate the SIDF I/O model generated by driving a nonlinear system with the input $u(t) = a \cos(\omega t)$.

The method we have developed to synthesize a nonlinear compensator for a nonlinear plant proceeds as follows:

a. select sets of input amplitudes $\{a_i\}$ and frequencies $\{\omega_k\}$ that cover the range of plant input amplitudes and frequencies of interest,
b. use FRSIMN to generate the corresponding set of I/O models $G_i(j\omega_k, a_i)$, or more succinctly $G_i(j\omega, a_i)$,
c. select one of the I/O models $G^*(j\omega, a^*)$ to be the nominal case for which a preliminary linear compensation will be designed ($a^* \epsilon\ a_i$),
d. design a PID compensator $C^*(j\omega)$ for the nominal model,
e. add a model of the PID compensator $C^*(j\omega)$ to the simulation in series with the nonlinear plant model,
f. use FRSIMN with the compensator input signal $e = e^* \cos(\omega t)$, for a value of e^* chosen to be consistent with a^* and the PID gain near the crossover frequency, denoted $|C_{co}|$, i.e., $e^* = a^*/|C_{co}|$, and generate the SIDF I/O relation for the PID in series with the plant, denoted $CG^*(j\omega, e^*)$,
g. select a set of compensator input amplitudes $\{e_i\}$ that corresponds to the set $\{a_i\}$ in roughly the same way, i.e., $e_i = a_i/|C_{co}|$,
h. design a set of linear compensators $\{C_i(j\omega)\}$ so that the error

$$E(j\omega) = 1 - C_i(j\omega G(j\omega, e_i|C_i(j\omega)|)/CG^*(j\omega, e^*) \qquad (1)$$

over the frequency set $\{\omega_k\}$ is minimized in the mean square sense; this yields a set of PID parameters for each value of e_i, denoted $\{K_P(e_i)\}$, $\{K_I(e_i)\}$, $\{K_D(e_i)\}$,
i. use the sets of PID parameters $\{K_P(e_i)\}$, $\{K_I(e_i)\}$, $\{K_D(e_i)\}$ with SIDF inversion to synthesize the nonlinearities $f_P(e)$, $f_I(e)$, $f_D(e)$,
j. construct a nonlinear PID compensator model that incorporates these nonlinear functions, and
k. simulate the closed-loop system comprised of the nonlinear compensator and plant to validate the design.

The first three steps require only a knowledge of the expected operating conditions of the nonlinear plant and a software package such as FRSIMN for generating frequency-domain I/O models. Since this part of the procedure is described in detail in [5], we will not dwell on it here. The selection of e^* and the design of a PID compensator based on G^* is a straightforward application of classical control system design methods. Steps e and f involve adding the compensator to the simulation model and using the same package to obtain the frequency-domain I/O model of the compensated plant.

It is important to note that steps d through f may require iteration, in the sense that CG^* is not the same as $C^*(j\omega)G^*(j\omega, a^*(\omega))$, because the plant input amplitude is not constant; rather, it is $a(\omega) = e^*|C^*(j\omega)|$, so the initial design objectives (e.g., gain and phase margins) may not be met. Once the designer has obtained a satisfactory CG^*, steps g through i result in the direct synthesis of a nonlinear compensator that provides the best fit possible to CG^*, for the appropriate set of compensator input signal amplitudes $\{e_i\}$.

The most difficult step to visualize in nonlinear compensator synthesis is h. The original set of I/O models $\{G(j\omega,a_i)\}$ defines a complex-valued "surface" above the (ω, a) plane, which we call the G-surface; an illustration of the real part of a G-surface (actually, $-Re\ G$) is depicted in Figure 1. The synthesis of the set $\{C_i\}$ requires evaluating $C_i(j\omega)G(j\omega, a(\omega))$ where $a(\omega) = e_i|C_i(j\omega)|$, which implies that the values of $G(j\omega,a(\omega))$ are not immediately available from the original SIDF I/O model data. It would be possible, in principle, to obtain the necessary data by further use of FRSIMN; however, since these calculations take place inside a mean square error minimization algorithm, this would be very costly. Instead, we obtain the required data from the G-surface by interpolation. This is also shown in Figure 1, where a curve $a(\omega)$ for some e_i is shown on the (a,ω) plane and the interpolated $G(j\omega, a(\omega)$ trajectory is plotted on the G-surface. For each value of e_i, the mean square error minimization algorithm determines the parameter set $\{K_P(e_i),\ K_I(e_i)$ that minimizes the error (Equation (1)) in that sense; this defines one linear PID compensator

$$C_i(s) = K_{P,i} + K_{I,i}/s + K_{D,i} \tag{2}$$

is in the set $\{C_i\}$; the error minimization is obtained using MINPACK [9]. The designer should choose the frequency set $\{\omega_k\}$ with some care, so that the curves $C_i(j\omega)\ G(j\omega, a(\omega))$ are close to the objective $CG^*(j\omega, e^*)$ near the crossover frequency; if $\{\omega_k\}$ over emphasizes low or high frequencies, this condition may not be met. This could also be accomplished by introducing a weighting function in $E(j\omega)$, but we have not found that to be necessary.

The final synthesis step — SIDF inversion — amounts to determining the three nonlinearities whose SIDFs fit the amplitude-dependent gain data generated in step h. This we did with a routine which adjusts the parameters of a rather general piecewise linear function until the corresponding SIDF has minimum mean square error with respect to the e-dependent gains obtained in h; again, we use MINPACK for this process. The nonlinearity is portrayed in Figure 2; the four parameters adjusted in the fitting procedure are the two slopes S_1, S_2, breakpoint δ, and discontinuity D. Note that the nonlinearity has considerable generality. For example, the catalog of SIDF formulae in [7] lists 8 nonlinearities that are special cases of f_{pwl} in Figure 2, and the qualitative behavior of the SIDF versus a can exhibit the following basic trends: monotonically increasing, monotonically decreasing, concave up (having a minimum), and concave down (having a maximum). One could use a more general nonlinearity than this; however, given the nature of the K versus e data determined as above, it is not advisable to go too far in "fine-tuning" the SIDF fitting process. For the example discussed in [1], the results of SIDF inversion to determine the nonlinearities in the PID compensator paths are portrayed in Figure 3. Many of the issues discussed here are illustrated in these plots showing the $K(e)$ data and the SIDF fits that were obtained by our algorithm. This routine is also described in greater detail in [5].

It is also worth noting that SIDF inversion may require insight on the part of the designer: SIDFs cannot fit arbitrary gain/amplitude data, so selection of the set $\{e_i\}$ must be made so that the values are "well spread out." This is because SIDFs tend to be rather smooth and thus cannot fit substantial variations of gain over a closely-spaced set of amplitudes meaningfully. This factor is best seen in Figure 3a., where it is not reasonable to expect a good fit for small values of e. Also, the designer must be aware that rather different nonlinearities may have very similar SIDFs over a substantial range of input amplitudes (cf., the SIDFs for a saturation and for a relay with dead-zone, Figure 4); this may be confusing, but in fact gives the designer some freedom in the final selection of the appropriate nonlinearity to implement.

3. APPLICATION TO A POSITION SERVO DESIGN PROBLEM

The nonlinear plant for which we designed a nonlinear compensator is depicted in Figure 5. The servo motor saturation is modeled by a substantial reduction in gain; specifically, the parameters are $m_1 = 5.0$ Nm/v, $\delta = 0.5$ v, $m_2 = 1.0$ Nm/v. "Stiction" is modeled by the relation

$$T_m = \begin{cases} T_e - f_v\dot{\theta} - f_c\ \mathrm{sign}(\dot{\theta}), |T_e| > f_c \\ T_e - f_v\dot{\theta} - f_c\ \mathrm{sign}(\dot{\theta}),\ \dot{\theta} \neq 0.0 \\ 0.0, |T_e| < f_c \text{ and } \dot{\theta} = 0.0 \end{cases} \tag{3}$$

where $f_v = 0.1$ Nm-s/rad, $f_c = 1.0$ Nm; the moment of inertia is $J = 0.01$ kg-m^2.

The nonlinear compensator synthesis procedure procëeded exactly as outlined in Section 2; the highlights are shown in Figures 3 through 7 of [1]. For this presentation, we will restrict ourselves to reporting on the final design: the nonlinear control system is depicted in Figure 6, and the nonlinearities obtained via SIDF inversion are shown in Figure 7. As noted in [1], the different break-point values and slope ratios show that the compensator is not merely a saturation-inverter.

The final validation of this design is obtained by simulating the performance of this system for step inputs. For the sake of comparison, the same test is applied to the closed-loop system having the linear PID compensator C^* in the forward path. We see in Figure 8 (a and b) that the effect of saturation and stiction are quite striking in the case with a linear compensator, while nonlinear compensation substantially reduces the effect of input-amplitude dependence. For example, the percent overshoot ranges from 12 to 57% for the linear PID, while the nonlinear PID keeps the percent overshoot in the range 32 to 38%. Also, there is severe stiction evident in the linear case; the nonlinear PID almost completely eliminates it.

4. SUMMARY AND CONCLUSIONS

The method outlined in Section 2 is a specific realization of the basic concept of using SIDF I/O models as the basis for nonlinear compensator design proposed in [2 and 3]. Based on the example presented in [1] and outlined in Section 3, we believe that this approach shows considerable promise in dealing with one of the more difficult problems in nonlinear systems design — the design of compensators to correct for the amplitude-dependence of nonlinear plants.

REFERENCES

1. Taylor, J H. and Strobel, K.L., "Nonlinear Control System Design Based on Quasilinear System Models," *Control 85*, Cambridge Univ., Cambridge, UK, July 1985.
2. Taylor, J.H., "Robust Computer-Aided Control System Design for Nonlinear Plants," *Proc. Applic. Multivariable Systems Theory*, Manadon, Plymouth, UK, October 1982.
3. Taylor, J.H., "A Systematic Nonlinear Controller Design Method Based on Quasilinear System Models," *Proc. American Control Conf.*, San Francisco, CA, pp. 141-145, 1983.
4. Taylor, J.H., and Strobel, K.L., "Applications of a Nonlinear Controller Design Approach Based on Quasilinear System Models," *Proc. American Control Conf.*, San Diego, CA, June 1984.
5. Taylor, J.H., "Computer-Aided Control Engineering Environment for Nonlinear Systems," accepted by the Third IFAC Symposium CAD in Control and Engineering Systems, Lyngby, Denmark, August 1985.
6. Atherton, D.P., *Nonlinear Control Engineering*, Van Nostrand Reinhold, London, 1975.
7. Gelb, A. and VanderVelde, W.E., *Multiple-Input Describing Functions and Nonlinear System Design*, McGraw-Hill, New York, 1968.
8. Elmqvist, H., "SIMNON — An Interactive Simulation Program for Nonlinear Systems," *Proc. Simulation 77*, Montreux.
9. More, J.J., Garbow, B.S., and Hillstrom, K.E., *User Guide for MINPACK-1*, Argonne National Laboratory Report No. ANL-80-74, August 1980.

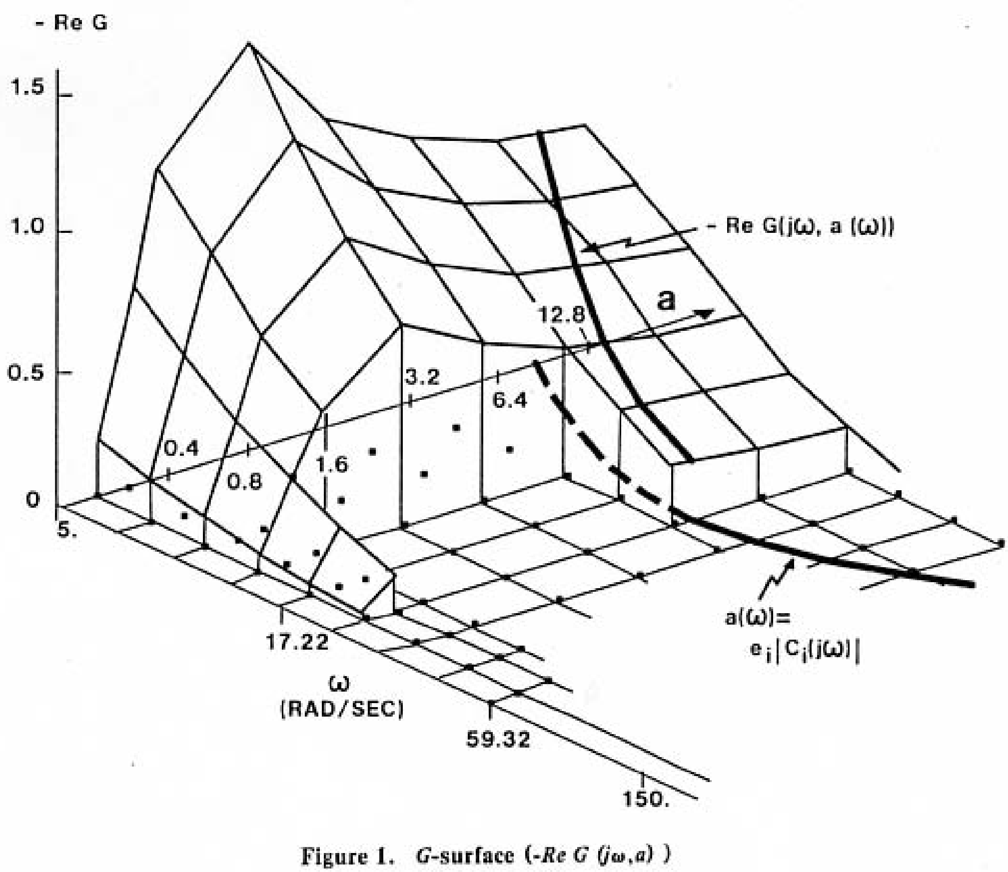


Figure 1. *G-surface* (*-Re G (jω,a)*)

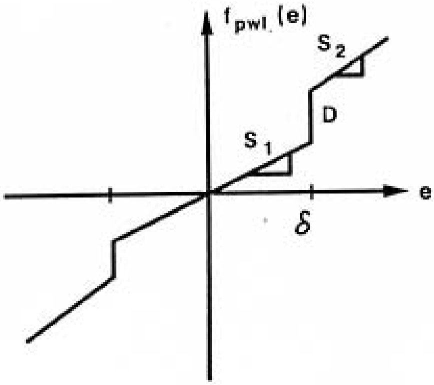


Figure 2. Nonlinearity Class for SIDF Inversion

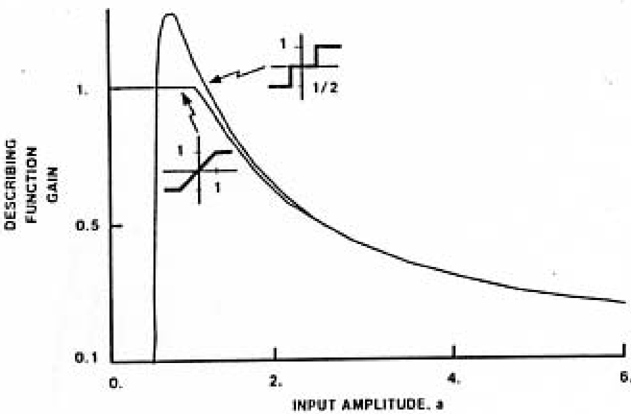


Figure 4. SIDFs for a Limiter and Relay with Deadzone

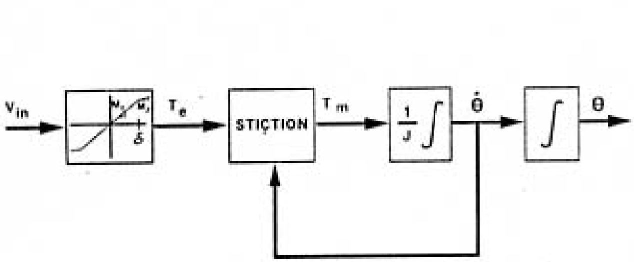
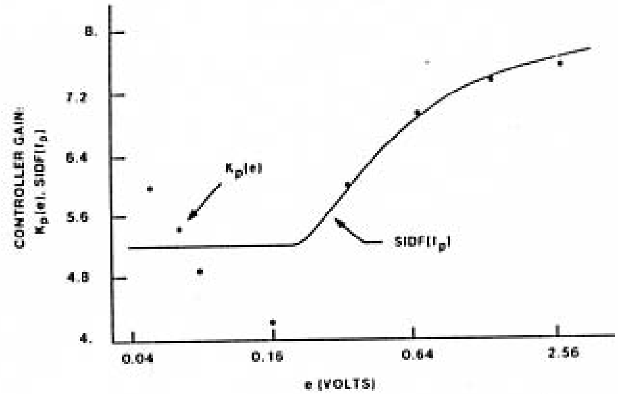
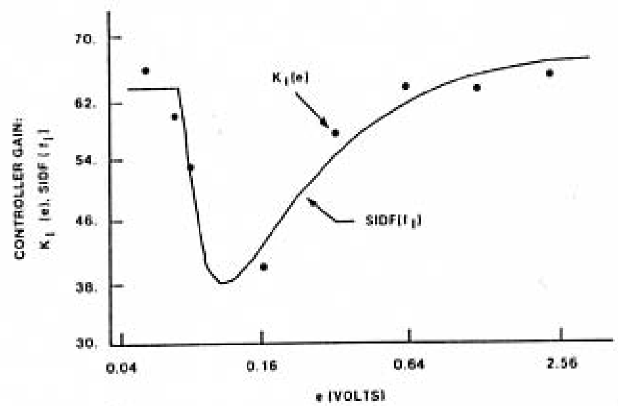


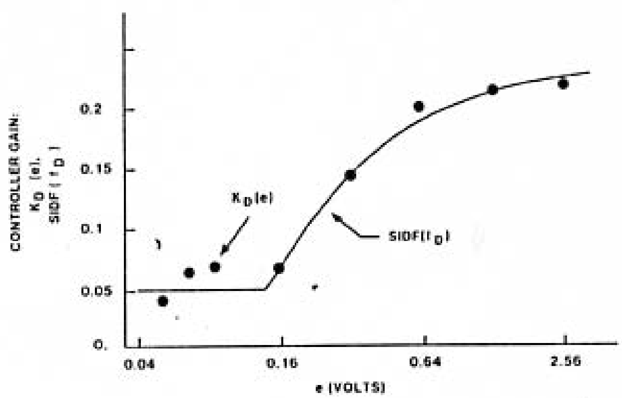
Figure 5. Position Servo Model Schematic



(a) Proportional Path



(b) Integral Path



(c) Derivative Path

Figure 3. Nonlinearity Synthesis (SIDF Inversion)

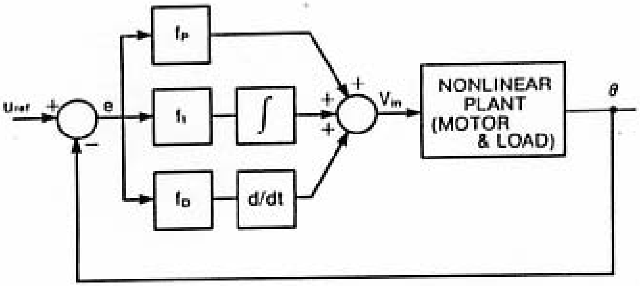
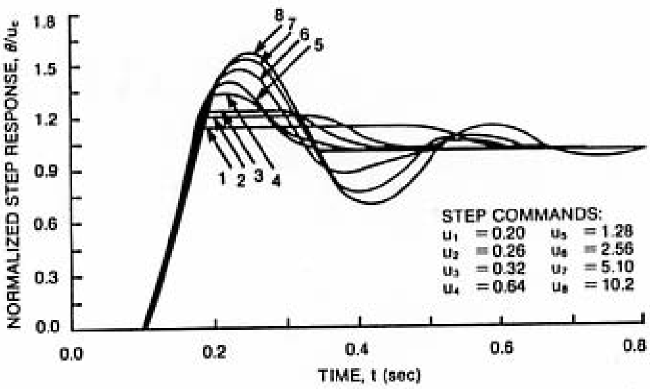
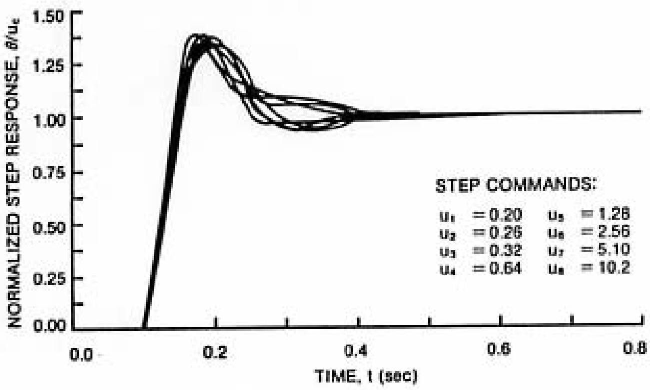


Figure 6. Final Nonlinear Control System

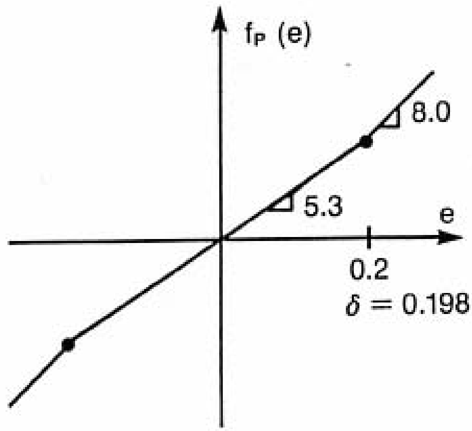


(a) Linear PID and Nonlinear Plant

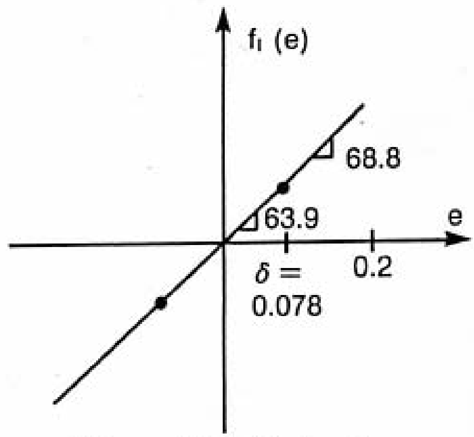


(b) Nonlinear PID and Plant

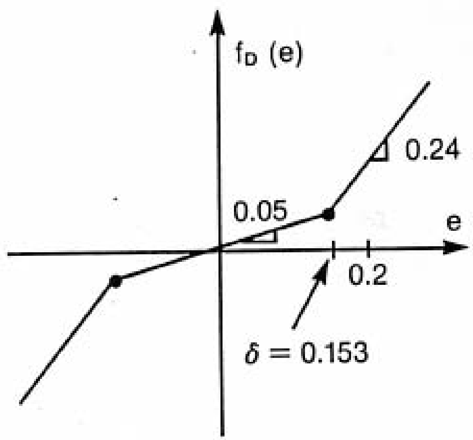
Figure 8. Step Response Plots



(a) Proportional Path Nonlinearity



(b) Integral Path Nonlinearity



(c) Derivative Path Nonlinearity

Figure 7. Controller Nonlinearities Synthesized by SIDF Inversion