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A NONLINEAR PID AUTOTUNING ALGORITHM

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ABSTRACT

A **nonlinear autotuning regulator algorithm** is obtained via a direct combination of the Åström-Hägglund algorithm for the linear case [1] with the sinusoidal-input describing function (SIDF) approach to nonlinear compensator synthesis of Taylor and Strobel [2] The basic approach for linear autotuning proceeds as follows:

a install a relay with hysteresis in series with the unknown plant to be controlled; close a unity-gain feedback loop around this combination;

b choose several values of hysteresis so that this system exhibits limit cycles; the frequencies and amplitudes of the oscillation at the output of the plant determine points on the plant Nyquist plot; and

c. given points on the plant Nyquist plot, set the PID controller gains using an appropriate tuning algorithm (e.g., Ziegler-Nichols)

This approach produces good results if the plant is linear or nearly so; however, if the plant behavior is strongly amplitude-dependent, there are likely to be problems with implementing this algorithm.

The nonlinear autotuning regulator algorithm which extends the above approach to handle situations where the plant behavior is strongly amplitude-dependent is based on the SIDF approach. In essence, SIDF input/output (I/O) models of the compensated nonlinear system are exploited to directly synthesize a compensator nonlinearity that eliminates or reduces the amplitude dependence of the open-loop I/O relation The nonlinear synthesis portion of this algorithm is reasonably simple to implement, has been shown to be effective [2], and should be of practical utility An example application to a precision position control system is provided as an illustration.

1. INTRODUCTION

A new procedure for automatic tuning of PID regulators was recently introduced in Åström and Hägglund [1]. It is based on performing system identification via relay-induced oscillations The system is connected in a feedback loop with a relay to produce a limit cycle; frequency-domain information about the system dynamics is derived from its amplitude and frequency With an ideal relay, the oscillation will give the critical point where the Nyquist curve intersects the negative real axis Other points on the Nyquist curve can be explored by adding hysteresis to the relay characteristic Linear design methods based on knowledge of part of the Nyquist curve are discussed in Åström and Hägglund [1] and Hägglund and Åström [3]

Basing identification on relay-induced oscillations has several advantages An input signal which is near optimal for identification is generated automatically, and the experiment is safe in the sense that it is easy to control the amplitude of the oscillation.

In this paper the idea of using relay-induced oscillations is applied to determine frequency domain models that can serve as the basis for the synthesis of nonlinear compensators. Relay-induced oscillations are used to map amplitude-dependent describing function models of the open loop and then to apply the nonlinear design method of [2] to determine a nonlinear compensator

The rest of this paper is organized as follows: Identification by relay-induced oscillation is described in Section 2. Nonlinear compensator design is discussed in Section 3. Section 4 combines these approaches to arrive at a formal statement of the nonlinear autotuning algorithm. Section 5 gives an application of the method to a nonlinear model for a servomotor drive. It is shown that the method gives some improvement over a conventional linear design. Conclusions and future directions are provided in Section 6.

2. LINEAR AUTOTUNING

A schematic diagram of a system with automatic tuning is shown in Fig 1. In addition to the process or plant, the system is composed of four subsystems: an ordinary feedback regulator with adjustable parameters, a signal generator, a model identifier, and a block which performs regulator design calculations.

The system works as follows: The process is excited from the signal generator, relevant process dynamic descriptors are estimated from the response of the process to the excitation, and regulator parameters are calculated based on these descriptors. The signal generator, the estimator and the design calculations are then disconnected and the system operates like an ordinary fixed-gain regulator.

Other schemes of this type have been proposed; see, e.g., Åström [4], Clarke and Gawthrop [5], and Isermann [6]. All of these require considerable *a priori* information. Typically, it is necessary to know the time constants approximately. The scheme discussed in this paper builds upon the ideas in Åström [7,8], Hägglund [9], and Åström and Hägglund [1, 10]; this results in a simple system which requires little prior information.

Linear autotuning schemes based upon features of the Nyquist curve of the open loop transfer function can be extended most readily to the nonlinear case. Typically, knowledge of the critical point, i.e., the first point where the Nyquist curve intersects the negative real axis, is used. This point can be characterized by the critical gain K_c and the critical frequency ω_c. The Ziegler-Nichols method is a typical example of such design methods; there are many other similar ones (see [8] and [10]).

To use such design methods it is necessary to find an identification or estimation method which determines the critical point. This could be done by supplying a sinusoidal input to the plant and sweeping over frequency until a phase shift of $-\pi$ radians is obtained. Such an approach is, however, time consuming and not easy to implement.

A major contribution of [1] is a new method for automatically determining points on the Nyquist curve. It is based on the observation that a system with a phase lag of at least π radians at high frequencies will usually oscillate when a relay is introduced in the control loop. To determine the point, the system is connected in a feedback loop with a relay in series with the plant, as shown in Fig. 2. The error e is then a periodic signal and K_c and ω_c can be determined approximately from the first harmonic component of the oscillation:

Let D be the relay output amplitude and let a be the amplitude of the first harmonic of the error signal. The describing function of the relay is $N(a) = \frac{4}{\pi}\frac{D}{a}$, and the describing function condition for oscillation at frequency ω_c is $N(a) * G(j\omega_c) = -1$; since a and thus $N(a)$ are known, the critical gain for the plant is simply given by

$$K_c = \frac{-1}{G(j\omega_c)} = N(a) \qquad (2.1)$$

The critical frequency ω_c is also known from observation of the oscillation period T_o, i.e., $\omega_c = 2\pi / T_o$.

The period of the oscillation can easily be determined by measuring the times between zero-crossings. The amplitude may be determined by measuring the peak-to-peak values. These estimation methods are very easy to implement because they are only based on counting and comparisons. More elaborate estimation schemes may also be used to determine the amplitude and the period of the oscillation. The method outlined above has, however, been shown to work well in practice [1].

The estimation method will automatically generate a process input signal with significant frequency content at ω_c, ensuring that the point can be determined accurately. The only *a priori* information required is that the process engineer should select D and thus the input signal amplitude to excite the plant properly without creating dangerous excursions in the plant variables. As mentioned before, other points on the Nyquist curve can be estimated by introducing known dynamics and/or hysteresis (positive or negative) in the relay.

In this paper, we use relay-induced oscillations to determine the frequency-domain characteristics of a nonlinear system. This is done via relay experiments with different relay levels D_i. If the system is linear, the amplitude of the oscillation is proportional to D; if not, the various levels will generally yield different behavior.

3. NONLINEAR COMPENSATOR SYNTHESIS

The basic idea presented in Taylor [11, 12] is that sinusoidal-input describing function (SIDF) models of a nonlinear plant provide a good basis for the design of linear and nonlinear compensators Two methods are suggested for obtaining such models: the derivation of analytic SIDF models of the dynamics plus the solution of equations of harmonic balance, and simulation of the nonlinear plant with sinusoidal inputs plus fourier analysis of the output signal to determine the input/output (I/O) gain and phase relations The SIDF modeling and harmonic balance approach is detailed in [11, 12]; the simulation method (and a computer-aided design package that implements it) is described in Taylor [13].

Using either technique, the designer models the plant by assuming that the input to the plant is sinusoidal with amplitude a and frequency ω and obtains a transfer function representing the I/O relation in the form $G(j\omega, a)$ Several compensator synthesis methods based on SIDF I/O models are suggested in Taylor [12] and refined and applied in Taylor and Strobel [2, 14].

The nonlinear compensator synthesis method of [2] proceeds as follows:

a select a nominal input amplitude a_0, obtain $G(j\omega, a_0)$, and design a nominal linear compensator C_0 based on that model Denote the combined I/O relation CG_0

b. determine a set of SIDF I/O models for a set of error signal amplitudes $\{e_i, i=1, 2. \ldots\}$ for the compensated plant, denoted $\{CG(j\omega, e_i)\}$ or, more simply, $\{CG_i\}$

c. inspect the behavior of $\{CG_i\}$ near the critical point, and if there are large differences for various error amplitudes, synthesize a nonlinearity that minimizes the variation as much as possible A particular implementation of this step is as follows [2]:

  i. determine the M-circle that is just tangent to CG_0,

  ii for each i determine the gain K_i such that $K_i CG_i$ is just tangent to that same M-circle, and

  iii use the information $\{K_i(e_i)\}$ as the basis for synthesizing the nonlinearity $f_c(e)$ that must be placed before the compensator to regularize the overall open loop I/O relation. Synthesis is carried out by SIDF inversion, i.e., by finding the parameters of a piece-wise linear function so that the SIDF of that nonlinearity fits the points $\{K_i(e_i)\}$ as well as possible (we use a minimum-mean-square-error criterion in [2]).

The procedure in step c is illustrated in [2].

4. NONLINEAR AUTOTUNING ALGORITHM

One way in which the linear autotuning scheme (LAS) presented in Section 2 can be extended to the nonlinear case is by combining it with the nonlinear compensator synthesis approach (NCSA) outlined in Section 3. This leads to the following nonlinear autotuning algorithm:

a. install a relay of nominal output level D_0 and hysteresis h_0 in series with the unknown plant to be controlled; note that the square wave has first harmonic amplitude $a_0 = 4D_0/\pi$; close a unity-gain feedback loop around this system;

b. generally the closed-loop system will exhibit limit-cycle oscillations; determine the frequency domain descriptors (critical gain and frequency) and design a nominal linear PID controller C_0 as in the LAS;

c. apply NCSA: obtain $\{K_i(e_i)\}$ for a reasonable number of cases: i = 1, 2, ..., m as follows:

  i install a relay of output level D_i plus hysteresis h_i in series with C_0 and unknown plant; close a unity-gain feedback loop around this combination;

  ii. adjust the hysteresis to obtain limit cycles; the frequencies and amplitudes of the oscillation at the plant output give points P_i on the linear-compensated nonlinear plant Nyquist plot CG_i;

  iii. determine the gain K_i that moves the point P_i to the desired location in the Nyquist plane (e.g., on an M-circle);

d take the gain/amplitude sets $\{K_i(e_i)\}$ and use SIDF inversion to synthesize a nonlinearity that can be placed before the PID compensator to eliminate or reduce the sensitivity of the open loop to nonlinear amplitude dependence.

Less formally, the LAS applied for a nominal amplitude a_0 provides the basis for designing the nominal linear compensator C_0 as in Section 2; if the Ziegler-Nichols algorithm is used, then a PID compensator is obtained Then relay-induced oscillations are used to find points on the Nyquist plot of the linear-compensated plant (C_0 in series with the nonlinear plant) to obtain the $C_0\, G(j\omega,\, e_i)$ information required for step c of the NCSA. The latter step is carried out to synthesize the compensator nonlinearity f_c to complete the autotuning algorithm. Once the nonlinear compensator $f_c(e) * C_0(j\omega)$ is designed, it is implemented in a programmable controller and the autotuning mechanism is deactivated. Note that the nonlinear synthesis portion of this approach can be applied to any compensator type, not merely PID.

5. AN APPLICATION

The nonlinear autotuning algorithm is demonstrated by applying it to a nonlinear position servo control problem. The plant is identical to that described in [14], as portrayed in Fig. 3. The two nonlinear effects are nonlinear friction and torque motor saturation. The servo motor saturation is modeled by a substantial reduction in gain; specifically, the parameters are $m_1 = 5.0$ Nm/v, $\delta = 0.5$ v, $m_2 = 1.0$ Nm/v. "Stiction" is modeled by

$$T_m = \begin{cases} T_e - f_v\dot{\theta} - f_c\,\mathrm{sign}(\dot{\theta}), & |T_e| > f_c \\ T_e - f_v\dot{\theta} - f_c\,\mathrm{sign}(\dot{\theta}), & \dot{\theta} \neq 0.0 \\ 0.0, & |T_e| < f_c \text{ and } \dot{\theta} = 0.0 \end{cases}$$

where $f_v = 0.1$ Nm-s/rad, $f_c = 1.0$ Nm; the moment of inertia is J = 0.01 kg-m^2.

A linear PID compensator was designed for a nominal input amplitude for this plant in [14]; we used that compensator as C_0 above. Then we tuned the nonlinear compensator (synthesized the nonlinearity f_c to be cascaded with the PID regulator), as specified in steps c and d of Section 4. The results of the set of identification experiments is depicted in Fig. 4, and the use of NCSA (via the synthesis software described in [13]) is shown in Figs. 5, 6.

The nonlinear compensator obtained using this autotuning algorithm was validated by performing a series of step response tests for a variety of reference input levels. The results of these tests (normalized step response plots, wherein the output of the system is divided by the input step amplitude) are shown in Fig. 7. This behavior may be compared with a similar set of tests performed for the linear case (C_0 in series with the plant), which is portrayed in Fig. 8. The nonlinear compensator synthesized by this autotuning procedure has reduced the amount of "sticking" and the large variation of peak overshot shown in the linear compensated case; the latter is at the expense of a slower response for large step inputs (as dictated by the torque motor saturation which should not be overdriven as it was in the linear case). Note that this synthesis approach generated a characteristic that shows some accommodation for anti-wind-up, with no prior knowledge of the saturation.

6. CONCLUSIONS AND FUTURE WORK

The two approaches for regulator design in [1] and [2] fulfill two completely different requirements: handling *unknown* plants and *nonlinear* plants. The two are based on the behavior of the plant in the frequency domain, so they can be combined in the way suggested in this paper to address both problems simultaneously.

We believe that there is a need to develop more sophisticated nonlinearity synthesis methods than that presented here. In particular, it would be desirable to have an algorithm that requires fewer experiments and less computation. Still, this algorithm is not at all beyond the capabilities of many microprocessors that are being used for control applications at this time.

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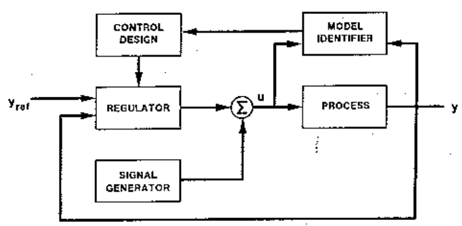

Figure 1. Block Diagram of a Linear Autotuner

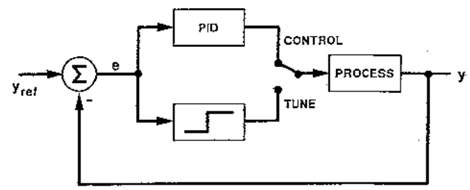

Figure 2. Relay Control of the Process

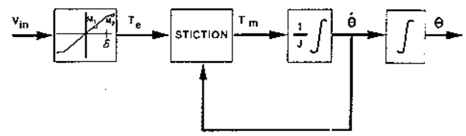

Figure 3. Nonlinear Position Servo Model

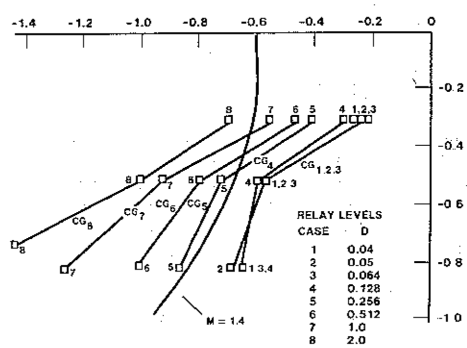

Figure 4. Identification of $CG(j\omega, e_i)$

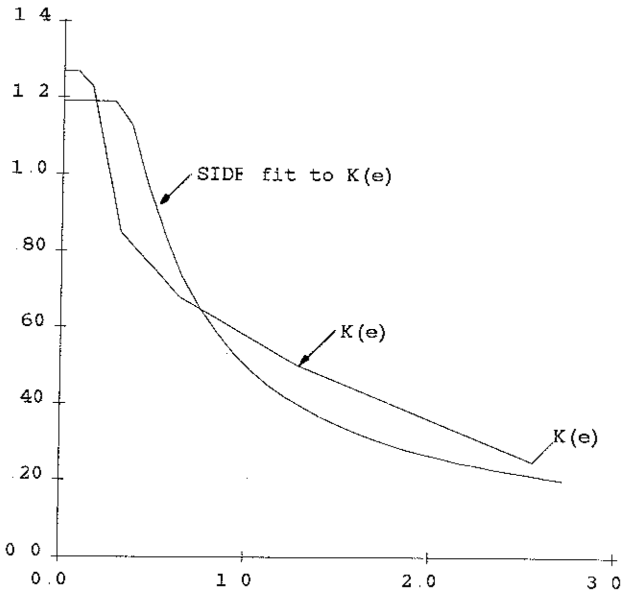

Figure 5. Gain / Amplitude Fit by SIDF Inversion

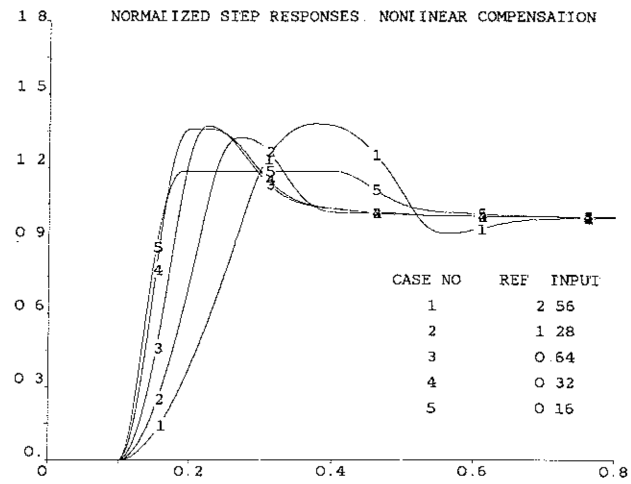

Figure 7. Step Response Tests, Nonlinear Compensation

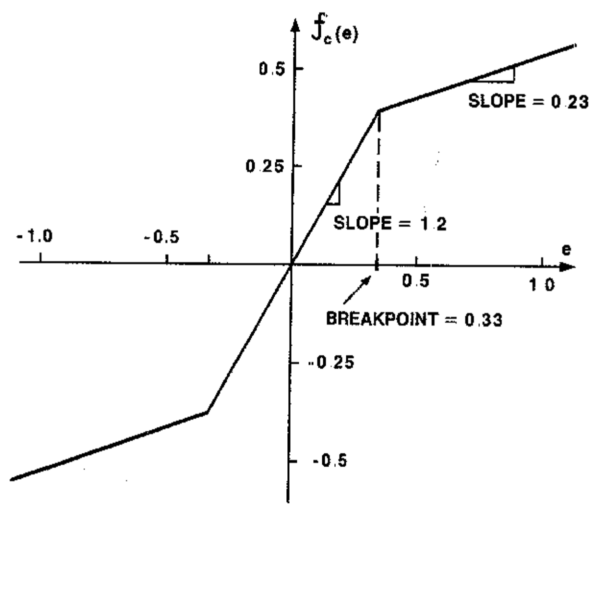

Figure 6. Nonlinearity from SIDF Inversion

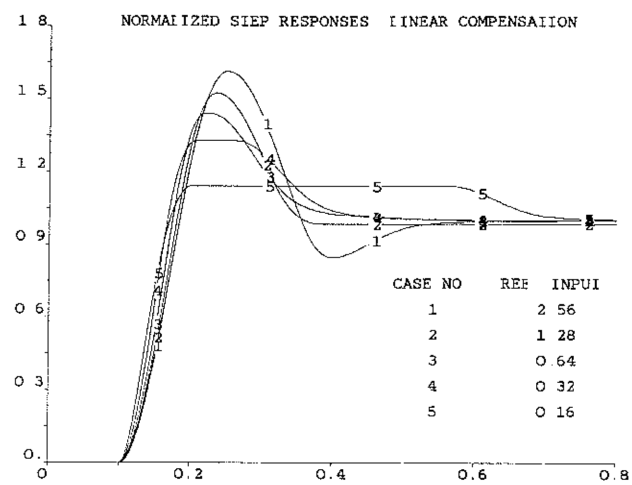

Figure 8. Step Response Tests, Linear Compensation