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Dynamic Optimal Reactive Power Flow

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Abstract: **An efficient method for minimization of energy loss over time is presented. The proposed method uses different loading conditions during a given future time interval instead of one single snapshot of the network. The method finds the optimal conditions during the given interval.**

The given interval is divided into several shorter periods. By increasing the number of periods or load profiles, the dimension of problem will rise substantially. This problem is handled by using the Generalized Bender Decomposition (GBD) technique. With this technique, the loading condition for each period will be solved in a separate NLP subproblem. The results of the NLP subproblems will be coordinated in a master problem. As shown in simulation results, the proposed method not only improves the voltage profile, but it also decrease the total energy loss over the given interval.

Keywords: optimal reactive power flow, energy loss minimization, power loss minimization, GBD algorithm

1 Introduction

Optimal power flow (OPF) problem is one of the major issues in operation of power systems. This problem can be divided into two subproblems, MVar dispatch or optimal reactive power flow (ORPF) and MW dispatch. The main objectives of ORPF address three important aspects: a) keeping the voltage profiles in an acceptable range [1], b) minimizing the total transmission energy loss [2], and c) avoiding excessive adjustment of transformer tap settings and discrete var sources switching [2, 3].

The control variables for this study include the vars/ voltages of generators, the tap ratios of transformers, reactive power generation of var sources, etc. The constraints include the var/voltage limits of generators, the voltage limits of load buses, tap ratio limits, var source limits, power flow balance at buses, security constraints, etc.

ORPF is frequently executed on-line by getting a snapshot from the real-time condition of the network. For tracking on-line load changes, and keeping the network in optimal condition over time, the ORPF should be executed continuously, or at least very often. However, due to application and implementation difficulties, ORPF is run less frequently. Reasons for this include keeping operator workload within acceptable limits and avoiding excessive equipment switching (transformer taps, capacitor banks, etc.). Between runs, the system can move far from the optimal condition. The deviation from optimality depends on the rate and magnitude of load changes.

In most energy management systems (EMS), a static var dispatch problem is solved [4, 5]. However, dynamic dispatch approaches have been applied to optimal active power flow by several researchers [6, 7]. In this paper, a new dynamic ORPF problem is proposed and solved. In this scheme, the total energy loss based on the on-line load conditions and the load forecast during an upcoming interval is minimized. The proposed method keeps the tap ratios and discrete var sources constant during the given interval, at settings that are optimal over the entire time. However, voltage constraint violations are eliminated at the beginning of shorter periods, and power loss is minimized to the extent possible by adjusting continuous controls such as generator vars/voltages.

In Section 2 of this paper, the static and dynamic dispatch methods are compared. In Section 3, the formulation of the problem, and in Section 4, the solution method are given. In Section 5, the application of the new method to the IEEE 30-bus system is addressed. A short summary of results and significant issues are given in Section 6.

2 Static Versus Dynamic OPF

Two methods of static ORPF are selected for discussion. In [4], the power loss is minimized on the basis of predicted load. An on-line program, which runs each 0.5 sec, makes the necessary adjustments for any difference between real-time and forecast load profiles. The method described in [5] divides the control variables into two sets, discrete control variables (capacitors, reactors, and tap ratios) and continuous control variables (vars/voltages of the generators). The ORPF program is run with two different objectives: 1) power loss minimization, and 2) removing voltage constraint violations. The first run has a cycle of half to one hour, and uses all the control variables. The second run has a cycle of 15 minutes; in removing voltage constraint violations only the continuous control variables are employed.

Several approaches for dynamic OPF also exist. In [6], the optimal MW dispatch is simultaneously solved for on-line and twelve other predicted load profiles in the upcoming hour. A parallel processing neural network is used to solve the problem. In [7], a dynamic dispatch for generation scheduling has been used. A time interval consisting of several one-hour subintervals has been selected. The load level during each hour is assumed to be constant, although it differs from one hour to the next. The method comes up to an optimal generation scheduling for the whole time interval. The method which is proposed in this paper is an on-line dynamic var dispatch that has three main advantages:

1. Reduced energy loss- The proposed method minimizes the total energy loss during a given time interval as the main objective.
2. Reduced physical plant changes- This method keeps the tap ratios and discrete var sources constant during the whole interval. This reduces the number of physical plant changes and unnecessary equipment wear and life-cycle costs. This benefit also results in an implicit economic benefit.
3. A decrease in the number of control variables- In this method, the number of controls in the beginning of each period except the first is restricted to the continuous variables.

The main steps of the proposed method are shown in Fig. 1, and explained below:

- **Step 1: selection of interval duration-** The on-line load profile and the load forecast for the upcoming hours are inspected. Depending on the size of load variations and the experience of the operator, an interval varying from half an hour up to several hours will be selected. By observing the same data, the interval will be divided into "N" periods. The number and duration of periods depend on the anticipated load profile changes (see Fig. 2).
- **Step 2: dynamic var dispatch-** At the beginning of each interval, a dynamic var dispatch to minimize total energy loss for the interval will be executed. The on-line load condition and load forecast for the N periods are included. In this stage, all the continuous and discrete control variables are adjusted at the beginning of interval.
- **Step 3: static var dispatch-** At the beginning of each period, a static ORPF will be executed. At each run, the constraint violations for the on-line load conditions are removed. If no violations exist, the power loss is minimized. In these runs, only continuous control variables are allowed to vary.

3 Problem Formulation

The formulation of this problem is explained in two stages. In the first step, the minimization of power loss (or constraint violations) is addressed. In the second stage the formulation for minimization of total energy loss is given.

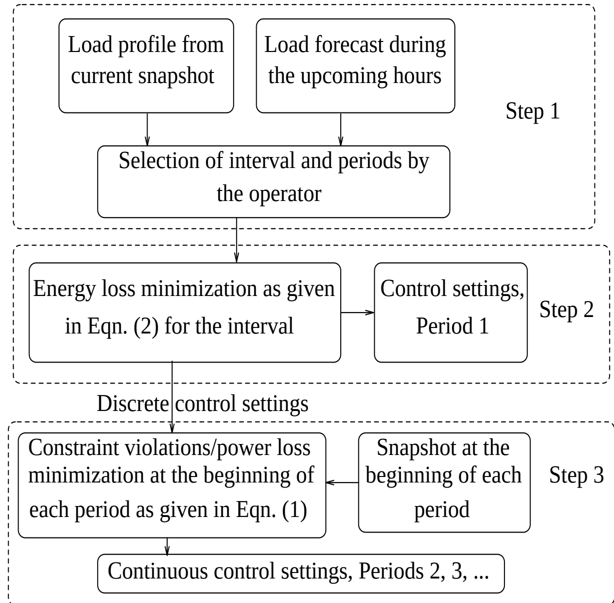

Figure 1: Flow chart of the energy loss minimization method

3.1 Formulation for Constraint Violation and Power Loss Minimization

It is assumed that the optimal MW dispatch is already executed, and the active power generation of all the generators except at the slack bus are constant. With this assumption, thd problem can be formulated as:

$$\begin{array}{ll} & \min_{x} \ f(x, y*) \\ \text{subject to:} & \\ & h(x, y*) = 0, \\ & g(x, y*) \leq 0 \end{array} \tag{1}$$

where f is the transmission power loss or the amount of constraint violations; x is the vector of continuous variables; y^* are the discrete variables which are constant during the period; the equality constraints, h(x), are related

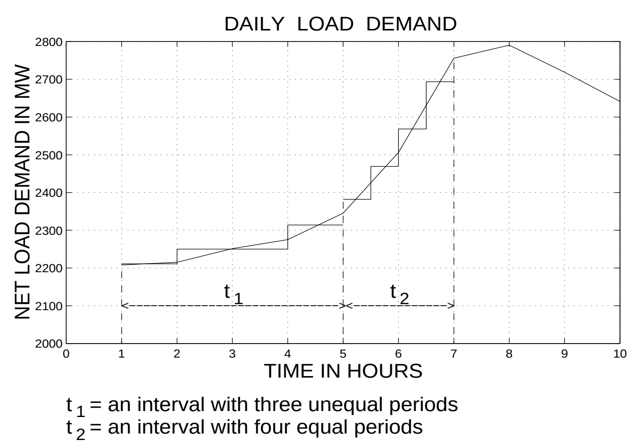

Figure 2: The selection of intervals and periods

to power flow balance equations, and the inequality constraints, g(x), include functional and simple constraints on continuous variables. This formulation will be used in the static var dispatch at the beginning of each period.

3.2 Formulation of Energy Loss Minimization Method

The energy loss minimization problem will be executed at the beginning of each interval. Each interval consists of N periods. The continuous variables have different values for each period, while the discrete variables have the same adjustment during the whole interval. This two-tier strategy is imposed to reduce the number of physical plant changes and thereby avoid unnecessary equipment wear and life-cycle costs.

The ELM method can be formulated as:

$$\begin{array}{l} \min\limits_{x,y} E_L = \sum\limits_{n=1}^{N} P_L^n * t^n \\ \text{subject to:} \\ \quad \left.\begin{array}{l} h^n(x^n, y) = 0 \\ g^n(x^n, y) \leq 0 \end{array}\right\} \text{ for } n = 1, 2, \cdots, N \\ \quad y_{\min} \leq y \leq y_{\max} \end{array} \quad (2)$$

where E_L is the total energy loss of interval; P_L^n is the power loss of period n; t^n is the duration of period n; and x^n (subvector of x) is the vector of continuous variables related to period n; h^n and g^n are the equality and inequality constraints for period n, respectively. To emphasize that the discrete control variables, y, do not have any n index, their inequality constraints are shown separately. The formulation for P_L is given in [2].

4 Solution Method

The ELM problem as formulated in (2) can be solved by using different decomposition techniques [8, 9]. In this paper, the application of the GBD algorithm to the ELM problem is proposed.

In GBD, the set of variables is divided into two subsets, x and y. The y variables are termed as complicating variables. By fixing the y variables the solution of the problem becomes much simpler. This algorithm is recommended for three types of problems [9]. In the problem type discussed in this paper, by fixing the y variables, the problem will be transformed into N independent subproblems.

In GBD, the optimization of x and y variables is decomposed into two separate subproblems, primal and master. In the primal subproblem, the y variables are fixed at their initial (first iteration) or optimal values found in the previous master problem. The optimization is performed by using the x variables. In the master subproblem, the problem is optimized over the y variables. The master and primal problems are solved alternatively until the convergence criteria are met.

In the formulation given in (2), the set of variables has already been divided into two subsets. In GBD, the discrete var sources and transformer tap ratios are recognized as the complicating variables, y. The continuous variables (such as var/voltage of generators) are denoted as x variables. The formulation of master and primal subproblems are explained in the following sections.

4.1 Primal Subproblem Formulation

The formulation of the primal subproblem is similar to (2) except that the values of y should be substituted by the initial or optimal values found in the last run of the master problem. By fixing the y variables, the primal problem will be decomposed into N independent subproblems each involving a different subvector of x as:

$$\left.\begin{array}{l} \min\limits_{x^n} E_L^n = P_L^n * t^n \\ \text{subject to:} \\ \quad h^n(x^n, y^*) = 0 \\ \quad g^n(x^n, y^*) \leq 0, \end{array}\right\} \text{ for } \quad n = 1, 2, \cdots, N, \quad (3)$$

where y^* is fixed y at initial values (first iteration) or optimal values found in the previous master problem.

In the primal problem, the N subproblems (3) are solved independently (in series or parallel). In subproblem n the energy loss during that period is minimized by using x^n. Each subproblem may have a feasible or infeasible solution. In infeasible case, a feasibility problem is solved [9]. If all the subproblems come to feasible solutions, then the value of total energy loss E_L will be an upper bound of original objective function given in (2), UB. In each iteration, UB is updated. In either case, the optimal values of continuous variables, x^*, and Lagrangian multipliers are passed to the master subproblem.

4.2 Master Subproblem Formulation

In the master problem, the x variables are fixed at their optimal values found in the previous primal problems, x^*. In this subproblem, the total energy loss for the interval is optimized over the y variables. This problem can be formulated as:

$$\begin{array}{l} \min\limits_{y, L_M} L_M \\ \text{where:} \\ \quad L(x_i^*, y, \lambda_i, \mu_i) \leq L_M \qquad \text{for } i = 1, 2, \cdots, I \\ \text{subject to:} \\ \quad \hat{L}(x_j^*, y, \hat{\lambda}_j, \hat{\mu}_j) \leq 0 \qquad \text{for } j = 1, 2, \cdots, J \\ \quad y_{\min} \leq y \leq y_{\max}, \end{array} \quad (4)$$

where I and J are the iteration counts for feasible and infeasible primal problems, respectively; and:

$$L(x_i^*, y, \lambda_i, \mu_i) = E_L(x_i^*, y) + \sum_{n=1}^{N} \lambda_i^n \, h^n(x_i^{n^*}, y) + \sum_{n=1}^{N} \mu_i^n \, g^n(x_i^{n^*}, y)$$

$$\hat{L}(x_j^*, y, \hat{\lambda}_j, \hat{\mu}_j) = \sum_{n=1}^{N} \hat{\lambda}_j^n \, h^n(x_j^{n^*}, y) + \sum_{n=1}^{N} \hat{\mu}_j^n \, g^n(x_j^{n^*}, y)$$

where λ_i and μ_i are the equality and inequality Lagrangian multiplier vectors respectively obtained from the feasible primal problem; and $\hat{\lambda}_j$ and $\hat{\mu}_j$ are the equality and inequality Lagrangian multiplier vectors respectively obtained from the infeasible primal problem; in all cases superscript n stands for period number.

The optimal value of L_M is a lower bound of the original objective function given in (2), LB. By solving the master problem, the optimal values of y variables, y^*, will be obtained. At each iteration, LB is updated and y^* is passed to the primal problem for the next iteration.

5 System Studies

For comparing the power and energy loss minimization methods, several small and large size networks have been studied. Due to limitations, only the results of the IEEE 30-bus system (Fig. 1) are given in this section. The Ward and Hale 6-bus system is studied in [2]. The line data and the bus data of the modified IEEE 30-bus system are same as those given in [10]. The load profiles for a one-hour interval with four equal periods are tested. The bus loads are uniformly reduced from 100% to 50% of full load during the hour.

The PLM and ELM methods are applied to the IEEE 30-bus system. In both methods, all the control variables are set at the beginning of interval, and the continuous variables are adjusted at the beginning of each subsequent period. In PLM method the power loss at the beginning of interval and in ELM method the energy loss for the whole interval are minimized.

5.1 The PLM Method

The power loss of the modified IEEE 30-bus system under full load conditions is minimized. The power loss found from this method for the four Periods are given in Table 1. The optimal values of bus voltages and discrete control variables are given in Tables 2 and 3, respectively. As mentioned before, PLM does not consider forecast load variations so the discrete control variable values in "Period 1" are assumed to pertain to the entire interval.

As mentioned in section 3.2, the only control variables that can be varied during the interval are Q_{g1}, Q_{g2}, Q_{g5}, Q_{g8}, Q_{g11}, Q_{g13}. Therefore, at the beginning of Periods 2-4, the ORPF is run with only these control variables used to remove any voltage constraint violation or to minimize the power loss. Due to the load variation at each period, the power loss as shown in Table 1 also changes. The total energy loss achieved by PLM can be determined as:

$$E_L = (P_L^1 + P_L^2 + P_L^3 + P_L^4)/4 = 24.32\ MWH.$$

Table 1: Power loss for the four Periods from the PLM method in MW

Time Period	1	2	3	4
Power loss	39.42	28.40	18.45	11.64

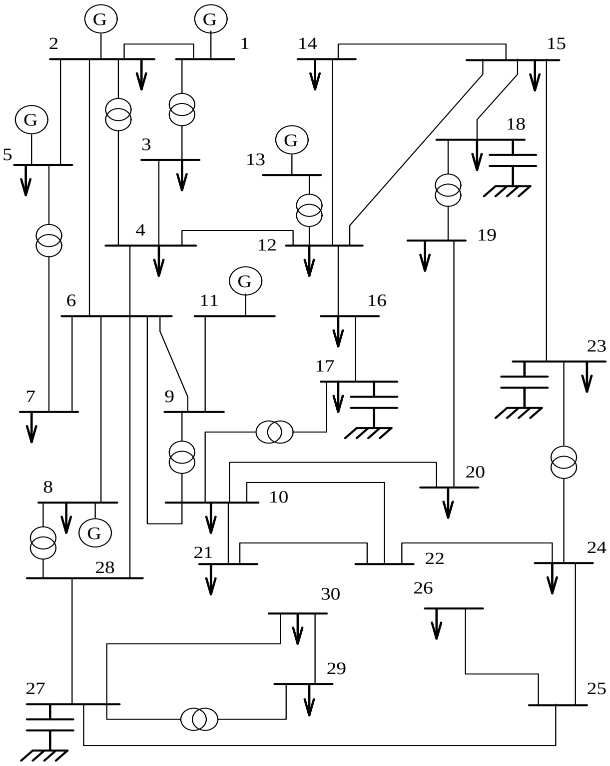

Figure 3: Modified IEEE 30-bus System

5.2 Energy Loss Minimization

The IEEE 30-bus system with the same load profiles is used for the minimization of energy loss. The energy loss is minimized by employing (2). The total energy loss found in this method is equal to 23.89 MWH. This value is less than the energy loss found in the PLM method (24.32 MW). Some of the bus voltages and the discrete control variables computed by this method for all the four Periods are given in Tables 2 and 3, respectively. By comparing the results of these Tables and other simulation studies, the following observations can be made:

1. The voltage profiles from energy loss minimization are smoother than those from the PLM method.
2. The energy loss in the ELM method for the above example is 1.8% less than that from the PLM method.
3. The advantages of the ELM method are more apparent when the load changes significantly. In cases where the load profiles are almost flat during the given time interval, ELM gives slightly better results.
4. In cases where the load changes during the next time interval are large, coming to a feasible solution by the PLM method is not always possible. In these cases the ELM method is more likely to find a feasible solution. The reason is that the load conditions for all periods have been considered in the load flow equations which are enforced as constraints in the ELM formulation. Therefore, the optimal values of the discrete control variables obtained using the ELM method can

Table 2: Several bus voltages from the ELM (PLM) method

Variable	Periods			
	1	2	3	4
V_{g1}	1.05 (1.05)	1.05 (1.05)	1.05 (1.05)	1.04 (0.99)
V_{g2}	1.04 (1.04)	1.04 (1.03)	1.03 (1.02)	1.02 (0.97)
V_{g5}	0.99 (0.99)	1.00 (1.00)	1.00 (0.97)	0.98 (0.95)
V_{g8}	1.01 (1.01)	1.01 (0.99)	1.00 (0.97)	0.99 (0.96)
V_{l12}	1.05 (1.05)	1.05 (1.04)	1.05 (1.01)	1.02 (1.01)
V_{l14}	1.03 (1.03)	1.03 (1.02)	1.03 (1.00)	1.01 (1.00)
V_{l15}	1.02 (1.03)	1.03 (1.02)	1.03 (1.00)	1.02 (1.00)
V_{l28}	1.00 (1.01)	1.01 (1.00)	1.00 (0.99)	0.99 (0.97)

usually handle the load changes predicted by the load forecast.

Table 3: Discrete control settings for the interval from the PLM and ELM methods

Variable	PLM	ELM	Variable	PLM	ELM
Q_{C17}	54	57	$T_{8,28}$	1.01	1.04
Q_{C18}	12	9	$T_{9,10}$	1.02	1.00
Q_{C23}	27	20	$T_{10,17}$	1.04	1.04
Q_{C27}	26	21	$T_{12,13}$	1.05	1.02
$T_{1,3}$	0.90	0.90	$T_{18,19}$	0.99	0.99
$T_{2,4}$	0.98	1.00	$T_{23,24}$	0.94	0.95
$T_{5,7}$	0.98	0.98	$T_{27,29}$	0.98	1.00

6 Conclusion

A new strategy for on-line optimal reactive power dispatch is proposed. The method minimizes the total energy loss during the upcoming interval, while keeping the voltage profiles within an acceptable range. For actual energy loss minimization the duration of periods should be very short. By increasing the number of periods, the dimension of problem will increase substantially. By using the GBD method the dimensionality problem has been solved. With this technique, the loading condition of each period will be solved in a separate NLP subproblem. The results of NLP subproblems will be coordinated in the master problem.

By comparing simulation results, it is found that ELM gives a better voltage profiles than that from the PLM method; in the simulation results, the method produced a nearly constant voltage profiles during the given interval. In addition, the energy loss was reduced at the same time, with the same number of discrete control variable changes as used by PLM.

The ELM method is based on the recognition that certain control variables should not be adjusted too often, as this may cause wear and shorten the life of the corresponding equipment. In this study, there are two categories of control variables established; discrete and continuous control variables are adjusted at the beginning of each time interval, while during the interval, only the continuous control variables are adjusted. The number of categories could be increased, and the frequency of adjustment modified, to fit differing circumstances.

The probability of finding an infeasible solution with the ELM method is much lower than the PLM method. This advantage of the ELM method is obtained by considering the load forecast and making sure that anticipated load changes during the upcoming interval can be accommodated.

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