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Recursive Optimization Procedure for Fuzzy-Logic Controller Synthesis

James H. Taylor & Lan Sheng
Department of Electrical Engineering
University of New Brunswick
Fredericton, NB CANADA E3B 5A3
Internet: *jtaylor@unb.ca*

Abstract

We present a new method for the synthesis of fuzzy-logic controllers (FLCs) for amplitude-sensitive nonlinear plants based on sinusoidal-input describing-function (SIDF) methods plus step-response optimization. This method involves a two-step process wherein an initial controller is obtained via the direct generation of the membership functions and output levels based on the "frequency response" of the nonlinear plant in the describing-function sense ($G(j\omega, a)$ where a is the sinusoidal input amplitude), then the FLC is perfected via recursive optimization of the step responses for a specified set of input amplitudes. By "recursive" we mean that the average step-response obtained from step k is used as the objective for the $k+1$ optimization problem, and the process iterates until convergence, with the objective of achieving closed-loop system performance that is as insensitive to reference-input amplitude as possible for the selected controller configuration. An illustration of the method and its effectiveness is provided, based on a prototypical position control problem where a servo motor plus mechanical load are characterized by torque saturation and nonlinear friction (stiction).

1 Introduction

Previous papers presented preliminary research in developing fuzzy-logic controller system design techniques based on combining a sinusoidal-input describing-function (SIDF) approach with fuzzy logic methodologies to create a direct fuzzy-logic controller synthesis procedure [1, 2]. The background for this ongoing work is presented in those articles; here we focus only on the new extension, recursive time-domain optimization. In particular, those papers describe the steps involved in this new fuzzy-logic controller synthesis approach and its application to a motor + load model with saturation and stiction. Specifically, they outline the approach, including (i) SIDF modeling, (ii) the generation of "amplitude-sensitive proportional, integral and rate-feedback gains" needed to control the nonlinear plant, (iii) the conversion of this model-based information into fuzzy-logic rulebases, and (iv) refinement of this preliminary design by step-response optimization.

2 Optimization of FLC Step Response

The procedure outlined previously in [1, 2] provides the initial FLC design to be optimized. It should be emphasized that the availability of this preliminary design which already closely achieves the objective of reduced sensitivity to input amplitude is critical to the success of the optimization step – without this starting point, there is little hope that optimization by itself could achieve a viable design.

Even with a good first-cut design, it is important to pose the optimization problem correctly to achieve the desired result and a solvable problem. First, the objective function Φ is expressed in terms of the desired insensitive step response $h^*(t)$ as:

$$\Phi = \sum_{i=1}^{I} \sum_{k=1}^{K} w_k (h(t_k; a_i) - h^*(t_k))^2 \qquad (1)$$

where $h(t_k; a_i)$ denotes the step response of the control system to a step of amplitude a_i at the integration times t_k, and w_k are weighting factors to, for example, permit trade-off between features of the transient response (e.g., overshoot) and those of the steady-state solution (e.g., the amount of offset).

The amplitudes a_i, $i = 1, 2, \ldots I$ may be selected to be the same as in the SIDF design stage, or they may differ; it is important, however, that they be roughly consistent. We found in the application presented below that weights had to be higher after the initial transient, to alleviate the effect of "sticking"; in general, the selection of these weights will be application and scenario specific. Most importantly, the desired insensitive step response $h^*(t)$ must be achievable by the controlled plant. We have adopted several strategies for defining it appropriately in the example below: (i) select the most desirable step response achieved by the first-cut FLC (ii) set $h^*(t)$ equal to the average of the set of step responses achieved by the first-cut controller.

Finally, the novel contribution in this presentation is a multi-step recursive optimization process, in which the desired insensitive step response $h^*(t)$ was iterated with each optimization run using the avarage of the set of step responses achieved by the preceeding run.

3 Design Example

A fuzzy-logic controller is designed for the same motor + load model used in [1, 2, 4, 5]. This model uses a substantial gain reduction to represent motor saturation, and a standard stiction nonlinearity. Following the general algorithm presented above, we first select the input amplitudes as before. Then steps 2 through 7 proceed as in [1], namely, performing a SIDF analysis of the motor, synthesizing the nonlinear inner-loop rate-feedback and PI compensators, and using this information to synthesize nonlinear FLC rulebases.

A preliminary FLC design was achieved based on an SIDF approach that produces proportional, integral and rate-feedback gain/amplitude information generated via an optimization method that minimizes the open-loop frequency response (SIDF) sensitivity to input amplitude. This data is used to parameterize the FLC rulebases. This is illustrated for the proportional part of the fuzzy PI controller as follows: First, the rule was taken to have eight membership functions, PZ, NZ ("positive-zero, negative-zero"), PS, NS ("positive-small, negative-small"), PM, NM ("positive-medium, negative-medium") and PB, NB ("positive-big, negative-big"). The membership function "corners" and output levels were taken to be adjustable parameters defining the "fuzzy proportional nonlinearity" $f_{FL,P}$; they were determined by SIDF inversion, i.e., adjusting them to minimize the fitting error between the SIDF of $f_{FL,P}$ and the gain/amplitude data; this resulted in the membership functions and nonlinear behavior depicted in Fig. 1. Note that the five-segment piece-wise-linear function exhibits very high gain for small signals, moderately low gain in the middle range, then a somewhat higher gain; ultimately the function saturates.

The final stage of the design was optimization to improve the time-domain performance of the resulting closed-loop system. First, we show some previous results to serve as a reference for the efficacy of the optimization process. Two cases were studied [1]: The nominal linear PID used in generating the "frequency-domain objective function", and a nonlinear FLC based on frequency-domain (SIDF) methods, used here as the first-cut controller. The resulting time histories, with input amplitudes ranging from $a_1 = 0.20$ to $a_8 = 10.2$ in both cases, are shown in Fig. 2. Comparing these two sets of time histories, it is evident that the linear PID yields rather poor performance – at small and large input amplitudes stiction and integral windup, respectively, tend to spoil the transient response. The preliminary nonlinear FLC, on the other hand, has produced a set of step responses that is remarkably less sensitive to input amplitude. (The performance of the FLC is slightly different from that shown in [5], although the nonlinear controller is functionally very similar, due to the fact that we are using a better numerical integration method [6] which more accurately and effectively manages the discontinuous behavior caused by stiction.)

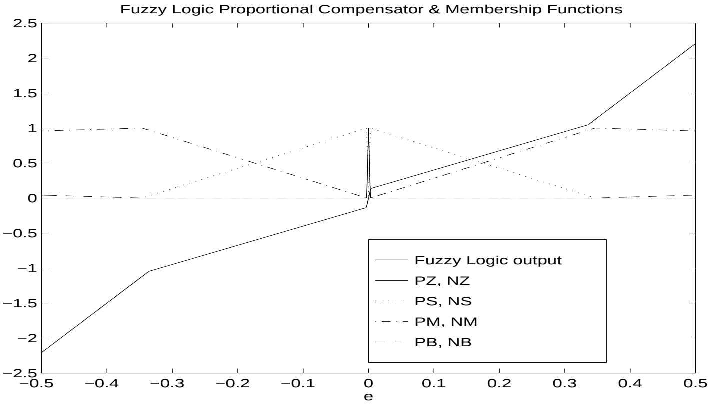

Figure 1: $f_{FL,P}$ and Membership Functions

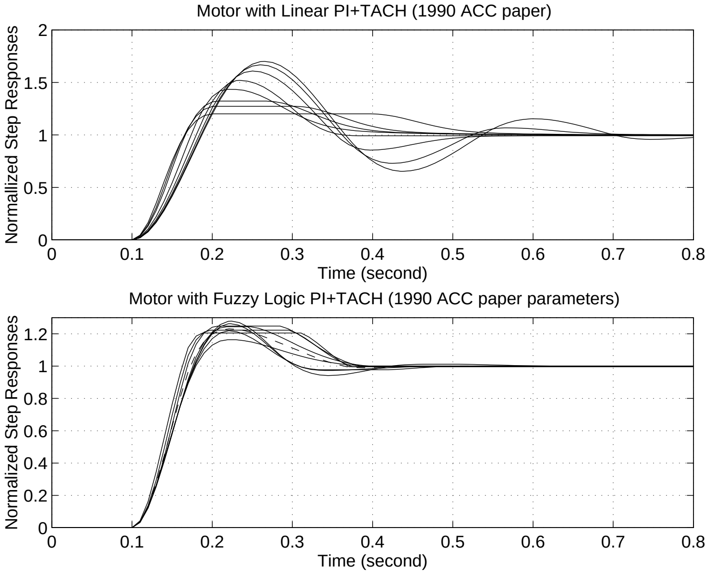

Figure 2: Linear and Nonlinear (SIDF-based) Controller Step Responses

The performance of an optimized FLC is depicted in Fig. 3. A single-run optimization using the average of the normalized step responses of the preliminary FLC for the desired $h^*(t)$ (dashed line, bottom plot of Fig. 2) resulted in the step-response set shown in this figure. We note that optimization resulted in a substantial improvement in the insensitivity of the normalized step responses compared with the first-cut FLC. Finally, the improvement that may be obtained by recursive optimization is demonstrated in Fig. 4. The average of the

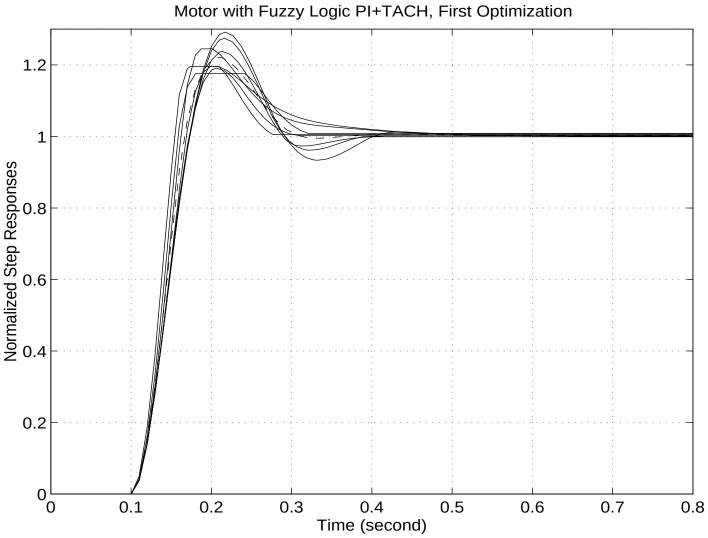

Figure 3: Nonlinear FLC Step Responses

normalized step responses of the single-run optimized FLC was used as the new target $h^*(t)$ (dashed line, Fig. 3), the same 8 step inputs are applied to the system, and the output levels of the three FLC nonlinearities were adjusted to minimize the weighted error between the 8 normalized step responses and the target (Eqn. 1). The most noticeable improvement is in the significant reduction in the peak overshoot and subsequent undershoot – this can be attributed to the fact that the target $h^*(t)$ used in this step is more nearly achievable than that used in the first optimization run.

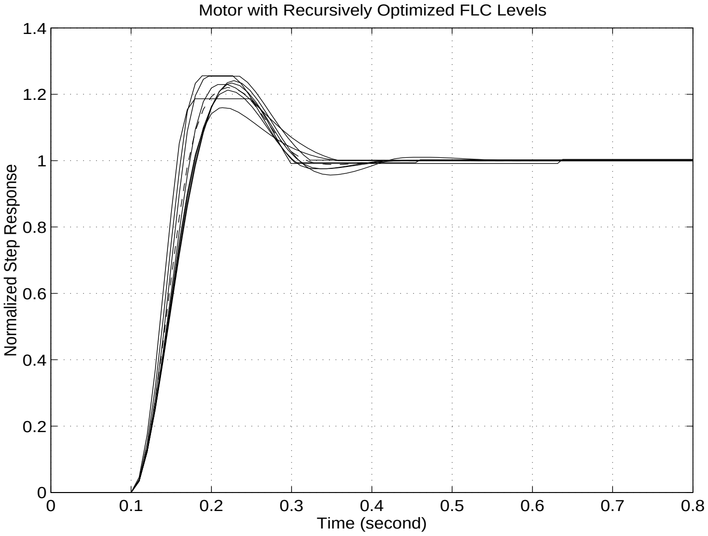

Figure 4: Recursively Optimized FLC Step Responses

4 Summary and Conclusions

The method outlined in [1, 2] and extended and applied in Sections 2 and 3 is a second-generation realization of the basic concept of using SIDF I/O models as the basis for nonlinear compensator design proposed in [3] and developed for the synthesis of several specific controller configurations [4] – [5]. One contribution of this paper is to extend these ideas to fill an important gap in fuzzy control, namely, the generation of FLCs for plants that are too fast and/or too complicated for heuristic methods to be effective. The novel feature in this paper's approach is the addition of recursive optimization to further improve the FLC performance insensitivity. We emphasize our conviction that one must have a preliminary FLC that gives nearly the desired behavior before optimization is feasible – that important role is played by use of the SIDF method. Based on the example shown in Section 3, we feel that this technique shows good promise in dealing with one of the more difficult problems in nonlinear systems design: the design of controllers to accommodate the amplitude-dependence of nonlinear plants.

This approach is capable of treating general nonlinear plants, with no restrictions as to system order, number or type of nonlinearities, or configuration. These results make the use of SIDF-based nonlinear controller design methods substantially more effective. It is also believed that this design approach will provide a framework for further developments in the realm of fuzzy-logic controller design for nonlinear systems.

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