Robust Fault Detection and Isolation Using a Parity Equation Implementation of Directional Residuals

Maira Omana and James H. Taylor
Department of Electrical & Computer Engineering
University of New Brunswick
Fredericton, N. B. Canada
maira.omana@unb.ca - jtaylor@unb.ca

Abstract

This paper focuses on solving the failure detection and isolation (FDI) problem by developing a model-based approach using a parity equation implementation of directional residuals. This new approach is an extension of the generalized parity vector (GPV) technique based on the stable factorization approach. The present work has improved the approach in Viswanadham, Taylor and Luce [11] in two important respects: (1) a novel transformation matrix computation is presented that enhances the isolation properties of the FDI algorithm, i.e., increases the maximum number of faults that can be isolated and the number of disturbances that can be decoupled above the number of outputs of the system [2]; and (2) disturbance decoupling is implemented in the stable factorization framework to make the residuals immune to measurable disturbance effects. The efficacy and robustness of this technique is demonstrated by applying this FDI scheme to a jacketed continuous stirred tank reactor (JCSTR).

1. Introduction

The continuous and growing advances in process control have resulted in large and complex plants, increasing the need of high performance fault monitoring systems. As a result, fault detection and isolation has become a critical issue for safe and reliable plant operation and reduction of economic losses [9].

Over the past decades, one area of active research is based on the use of analytical redundancy [1], [3], [5], [12] and the generation of residuals which have directional properties in response to particular faults [4], [7], [8] and [11]. This paper solves the FDI problem using an extension of the generalized parity vector (GPV) technique based on the stable factorization approach [11].

The paper is organized as follows. In section 2 a general overview of stable factorization is first given, followed by its application to implement the generalized parity vector technique. Next, in section 3, FDI using directional residuals for sensor and actuator faults is exposed. In section 4 a novel calculation of the transformation matrix is proposed to improve isolation, and section 5 presents the implementation of disturbance decoupling in the stable factorization framework. In section 6, an example using a jacketed continuous stirred tank reactor (JCSTR) model is used to illustrate the method.

2. Residual generation

The residual generation block is implemented using the generalized parity vector (GPV) technique, which is developed in the stable factorization framework. Before introducing the GPV concept, some of the fundamental mathematics of stable factorization are outlined.

2.1. Stable factorization

The significance of using the stable coprime factorization approach is that the parity relations obtained involve stable, proper and rational transfer functions even for unstable plants. Therefore the realizability and stability of the residual generator is guaranteed [11].

Given any *nxm* proper rational transfer function matrix *P(s),* it can be defined in terms of its right and left coprime factors as follows:

$$P(s) = N(s)\, D(s)^{-1} \tag{2.1.1}$$

$$P(s) = \tilde{D}(s)^{-1}\, \tilde{N}(s) \tag{2.1.2}$$

where $N(s)$ and $D(s)$ are said to be right coprime factors and $\tilde{N}(s)$ and $\tilde{D}(s)$ are called the left coprime factors. All factors belong to the set of stable transfer function matrices.

The GPV technique is based on the stable factorization of the system transfer function matrix in terms of its state-space representation. Let the system be described by the set of equations:

$$\dot{x}(t) = Ax(t) + Bu(t) + Gd(t) \qquad (2.1.3)$$

$$y(t) = Cx(t) + Eu(t) \qquad (2.1.4)$$

where x, u, d, and y represent the state variables, inputs, disturbances and outputs of the system, respectively. Assuming that the pairs of state space matrices (A, B) and (A, C) are stabilizable and detectable, it is possible to select a constant matrix F such as the matrix $\widetilde{A}_o = A - FC$ is stable. Using the definition of the coprime factorization of $P(s)$ in [10], the left coprime factors are given by:

$$\widetilde{N} = C(sI - \widetilde{A}_o)^{-1}(B - FE) + E \qquad (2.1.5)$$

$$\widetilde{D} = I - C(sI - \widetilde{A}_o)^{-1} F \qquad (2.1.6)$$

2.2. Generalized parity vector (GPV) technique

Based on the definition of the transfer function matrix $P(s)$ given in equation (2.1.2) and taking the relationship among the desired control input, u_d, and the actual output of the sensors, y, the following relations are obtained:

$$P(s) = \widetilde{D}(s)^{-1}\widetilde{N}(s) = \frac{y(s)}{u_d(s)} \qquad (2.2.1)$$

$$\widetilde{D}(s)y(s) - \widetilde{N}(s)u_d(s) = 0 \qquad (2.2.2)$$

Under ideal conditions, when the plant is linear, noise and fault free, equation (2.2.2) holds. However, when a fault happens, this relation is violated showing the inconsistency between the actuator inputs and sensor outputs with respect to the unfailed model. Using this fact, the generalized parity vector, $p(s)$, is defined as:

$$p(s) = T_r(s)[\widetilde{D}(s)y(s) - \widetilde{N}(s)u_d(s)] \qquad (2.2.3)$$

The GPV is a time varying function of small magnitude under normal operating conditions, and exhibits a significant magnitude change when a fault occurs. Each distinct failure produces a parity vector with different characteristics, allowing the use of the GPV for isolation purposes. A transformation matrix $T_r(s)$ is introduced to make it possible to achieve the desired fault response specifications [11]. This paper is focused on fault diagnosis using the direction of the parity vector under various failure conditions. We assume hereafter that T_r is constant, and that F in equations (2.1.5) and (2.1.6) is chosen such that $\tilde{A}_o = -\sigma I$ (which can always be done if (A, C) is observable); this simplifies our discussion of GPV behaviour.

3. Fault detection and isolation (FDI) using directional residuals

The basic idea of FDI using failure directions is that each failure will result in activity of the parity vector along certain axes or in certain subspaces. These reference axes or subspaces are determined by the state space matrices. Depending on the dynamics of the system, some of these reference directions may be close or identical, making the isolation for some faults difficult or unachievable. To overcome the angle separation problem between the reference directions, the calculation of an optimal transformation matrix T_r is introduced in section 4.

3.1. Actuator faults

Assuming an additive fault $a_j(t)$ in the j^{th} actuator and using the definitions in section 2.2, the GPV becomes:

$$p_{a,j}(s) = -(T_r\tilde{N})^j a_j(t) \triangleq T_r B_n^j \frac{a_j(s)}{(s+\sigma)} \qquad (3.1.1)$$

Equation (3.1.1) shows that $p_{a,j}(s)$ is restricted to exhibit activity along the direction defined by the j^{th} column of $\widetilde{N}$, which we denote $B_n^j/(s+\sigma)$. For a system with $y=x$, or state space matrices $C=I_{nxn}$ and $E=0_{nxm}$, as in the JCSTR model, equation (2.1.5) can be rewritten more simply as:

$$\tilde{N} = (sI - \tilde{A}_o)^{-1}B = \left[diag(s+\sigma)\right]^{-1} B \qquad (3.1.2)$$

From equation (3.1.2), $B_n^j = B^j$ is obtained. The actuator fault isolation is based on the angle Θ_j between the GVP and B_n^j as illustrated in figure 1. If the j^{th} actuator is faulty, this angle should be zero in the ideal case or less than a small threshold value, T_h, to account for model uncertainty.

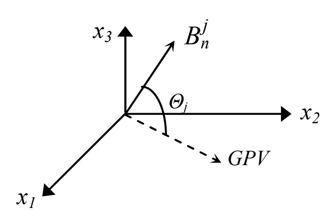

Figure 1. Actuator FDI

3.2. Sensor faults

Similarly, for an additive fault $s_i(t)$ in the i^{th} sensor, the parity vector (section 2.2) reduces to:

$$p_{s,i}(s) = (T_r\tilde{D})^i s_i(t) \triangleq T_r\left[E_d^i + \frac{B_d^i}{(s+\sigma)}\right]s_i(s) \quad (3.2.1)$$

Thus, for the sensor failure case, it is not possible to confine $p_{a,i}(s)$ to lie along a fixed axis. Only for fortuitous cases, depending on the dynamics of the system, can this be achieved. However, for any system, the GPV always lies in a plane SP^i of the generalized parity space, defined by the vectors E_d^i and B_d^i [11]. These vectors are related to $\tilde{D}$ by equation (2.1.6). Assuming the same state space representation used in section 3.1 for the JCSTR, equation (2.1.6) can be simplified as:

$$\tilde{D} = I - (sI - \tilde{A}_o)^{-1}F = I - \left[diag(s+\sigma)^{-1}\right]F \quad (3.2.2)$$

From equation (3.2.2) and the definition of F, B_d and E_d are defined as:

$$B_d = (-A - \sigma I)\ ,\ E_d = I \quad (3.2.3)$$

The sensor fault isolation is based on the angle Θ_i, between the GPV and the i^{th} sensor reference plane, SP^i, as illustrated in figure 2. If the i^{th} sensor is faulty, this angle should be zero or less than T_h.

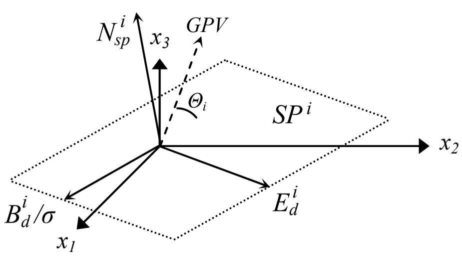

Figure 2. Sensor FDI

3.3. Special case for actuator faults

We consider a special case in terms of the SP^i normal, N_{sp}^i shown in figure 2 and defined by $N_{sp}^i = E_d^i \otimes B_d^i$ as:

$$B_n^j \bullet N_{sp}^i = 0 \quad (3.3.1)$$

If the dot product of B_n^j and the normal to the i^{th} sensor reference plane is zero then the j^{th} actuator axis lies on the i^{th} sensor reference plane as is illustrated in figure 3.

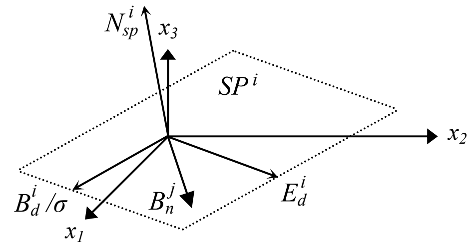

Figure 3. Special case for actuator FDI

This condition would be a result of the system state space structure. For this case it is not possible to calculate a transformation matrix T_r such as the actuator reference direction can be taken out of the sensor reference plane. This can be demonstrated mathematically by proving equation (3.3.2) for arbitrary T_r, which we did by symbolic manipulation in MATLAB®.

$$T_r B_n^j \bullet \left(T_r E_d^i \otimes T_r B_d^i\right) = 0 \quad (3.3.2)$$

Under this circumstance we may still be able to distinguish between these faults by taking a more detailed look at the parity vector relation in equation (3.2.1): Let us assume that $s_i(s) = S_i / s$ (a bias fault); we can apply the initial value theorem to show that the initial GPV activity is in the direction $T_r E_d^i$ and invoke the final value theorem to demonstrate that the steady-state GPV activity is in the direction $T_r\left(E_d^i + \dfrac{B_d^i}{\sigma}\right) \triangleq T_r B_s^i$. Then we can clearly isolate the i^{th} sensor fault from the j^{th} actuator fault unambiguously as long as B_n^j is not in the cone angle (sector) between E_d^i and B_s^i, using the following logic:

$$\left.\begin{array}{l} if\ \angle\left(GPV, SP^i\right) \le T_h\ then\ f_s^i \\ if\ \left\{\angle\left(GPV, SP^i\right)\ \&\ \angle\left(GPV, B_n^j\right)\right\} \le T_h\ then\ f_a^j \end{array}\right\} \quad (3.3.3)$$

where f_s^i and f_a^j denote the i^{th} sensor and j^{th} actuator faults respectively. Furthermore, we can still distinguish between these faults if B_n^j is not aligned with B_s^i by waiting to observe the steady-state GPV direction.

4. Transformation matrix

The transformation matrix T_r is an important issue in FDI using directional residuals. We wish to choose T_r to increase the separation angle between the original set of reference directions as much as possible, to enhance robustness and maximize the number of faults that can be isolated and the number of disturbances that can be decoupled, beyond the number of outputs of the system [2]. This can be formulated as a constrained optimization problem, whose objective is to maximize the angles between the transformed reference directions, to the extent possible.

The optimization routine maximizes the minimum of $F_{i,j}(T_r)$, where $F_{i,j}(T_r)$ is the objective function containing the angles between the reference directions that are separable. In this context, separable refers to those directions, which do not satisfy equation (3.3.2). The mathematical formulation is given by:

$$F_{i,j}(T_r) = \angle\left(Z_i, Z_j \right) \qquad (4.1)$$

$$\max_{T_r} \min_{\{F_{i,j}\}} \left\{ F_{i,j}(T_r) \right\} \qquad (4.2)$$

$$\textit{such that} \quad c(T_r) \le 0, \; c_{eq}(T_r) = 0$$

where $c(T_r) \le 0$, $c_{eq}(T_r) = 0$ represent nonlinear inequality and equality constraints, respectively; and Z_i and Z_j are transformed reference directions. These directions are given by transforming B_n^j, B_d^i and E_d^i. In order to refine the optimization problem the following constraints of the form $c(T_r)$ and $c_{eq}(T_r)$ are implemented.

1. A compulsory constraint is imposed on $Cond(T_r)$, the condition number of T_r; $Cond(T_r) \le C_{\max}$ (where C_{max} depends on the application) improves the stability and robustness of the FDI response. Specifically, smoother behaviour of the GPV angles and smaller fault-free $|GPV|_{peak}$ is achieved during transients if $Cond(T_r)$ is constrained.

2. An optional constraint normalizing T_r is applied; $||T_r||=1$ keeps the transformed GVP magnitude in a scale similar to the original one.

3. A final constraint is required only for failure cases where the faulty $|GPV|$ is not large enough in comparison with the transient fault free $|GPV|$. This is solved by constraining the faulty steady state $|GPV|$ to be as large as possible compared to the maximum value of the fault free $|GPV|$ during the transient.

It is necessary to modify the above optimization problem definition when dealing with the special case discussed in section 3.3. As mentioned, when a fault in the i^{th} sensor is applied, the GPV lies on the plane SP^i within the cone defined by vectors B_s^i and E_d^i. Figures 4 and 5 illustrate how these sensor fault sectors are defined for the JCSTR model and their relations with B_n^j.

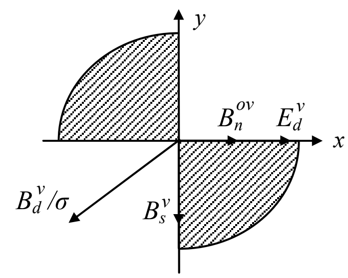

Figure 4. Volume sensor fault sector

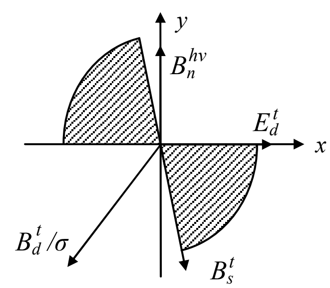

Figure 5. Temperature sensor fault sector

In figure 4 we observe that the outflow valve reference direction B_n^{ov} lies on the boundary of the volume sensor fault sector, making isolation difficult. However, basing isolation on the steady-state GPV activity along B_s^v should still be feasible. Conversely, in figure 5 the heating inflow valve reference direction B_n^{hv} is well outside of the temperature sensor fault sector, which is desirable for the sensor fault isolation.

We are still investigating the best strategy to implement in such cases. For the case shown in figure 5 we may want to maximize the angular separation between B_n^{hv} and the cone, in the situation depicted in figure 4 we can maximize the angle between B_n^{ov} and B_s^v if the steady state strategy is used.

5. Disturbance decoupling

Disturbances are considered to be measurable extra inputs acting on the plant, assuming no particular temporal behaviour [6]. The distinction between a disturbance and certain additive faults is indeed subjective. We model faults as additive inputs at particular sensors and actuators, and may specify temporal behaviour (e.g., bias faults); any other extra inputs we categorize as disturbances. We desire that our FDI approach not be affected by such extra inputs.

We demonstrate that residual directionality can be unaffected by extra inputs whose measurements are available. This can be demonstrated by rewriting equation (2.1.3) as follows:

$$\dot{x}(t) = Ax(t) + \tilde{B}\tilde{u}(t) \tag{5.1}$$

where $\tilde{B}=[B \quad G]$, $\tilde{u} = [u \quad d]^T$ and d represents the disturbance inputs. Using equation (5.1), the coprime factorization definition given by equation (2.1.5) can be rephrased as:

$$\tilde{N} = C(sI - \tilde{A}_o)^{-1}(\tilde{B} - FE) + E \tag{5.2}$$

Using the modified definition of $\tilde{N}$ given by equation (5.2), disturbance decoupling is implemented in the stable factorization framework to make the GPV immune to disturbance effects.

6. JCSTR description and preliminary results

The FDI algorithm has been implemented using MATLAB®, based on a simulated model of the jacketed continuous stirred tank reactor shown in figure 6. In this JCSTR, the tank inlet stream is received from another process unit and there is a heat transfer fluid circulating through the jacket to heat the fluid in the tank. The objective is to control the temperature and the volume inside the tank by varying the jacket inlet valve flow rate (the temperature control or TC loop) and tank outlet valve flow rate (the level control or LC loop) respectively.

In order to test the FDI performance, different sensor and actuators fault were applied, as well as disturbances. Both linear and nonlinear models were used in this study; time histories from both models are shown in all plots, although they coincide in a number of cases.

The effectiveness of sensor FDI is evident in figures 7, 8 and 9 which show the result of applying a -20% bias fault to the volume sensor at *t=3 sec*. It is observed in figure 8 that immediately after the fault is applied, the $|GPV|$ increases significantly, allowing the unambiguous detection of the fault. Isolation is made based on the behaviour shown in figure 9, where only the GPV angle with respect to the volume sensor reference plane goes to zero, while the remaining angles move further away from zero.

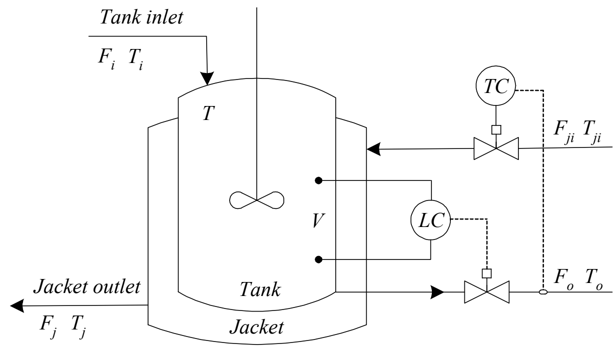

Figure 6. Jacketed continuous stirred tank reactor

For the JCSTR model, both actuators satisfy equation (3.3.1) with respect to one of the sensors, falling into the special case for actuator fault isolation. As expected from the mathematical derivation, the outflow valve fault GPV lies on the volume sensor fault reference plane. This is shown in figure 12, where both angles go to zero, when a -20% bias outflow valve fault is applied. However, the FDI algorithm is capable of isolating this fault successfully by using the logic described in equation (3.3.3). By comparing figures 9 and 12, its effectiveness is confirmed, since only for the actuator fault case are both angles zero, while for the sensor failure case, only the angle corresponding to the faulty sensor is zero. This verifies that even for systems whose reference directions fall in the special case presented in section 3.3, the GPV may provide a different signature for each fault.

Finally, a -20% temperature sensor fault is applied at *t=3 sec* followed by a +20% high mix inflow disturbance at *t=6 sec*. The influence of the disturbance in the temperature measurement is observed in figure 13, for the period of 6 to 9 sec. Disturbance rejection is verified in figures 14 and 15: The $|GPV|$ and the GPV angles are only slightly and briefly affected by the disturbance after t=6 sec. Since none of the thresholds are exceeded, correct detection and isolation is achieved, validating the immunity of the residual to disturbance effects.

7. Conclusion

In this paper we have described an extension of the generalized parity vector approach first introduced by Viswanadham, Taylor and Luce [11]. A systematic approach to calculate an optimal transformation matrix has been effectively developed, enhancing the FDI properties and the scope in terms of the number of faults that can be isolated.

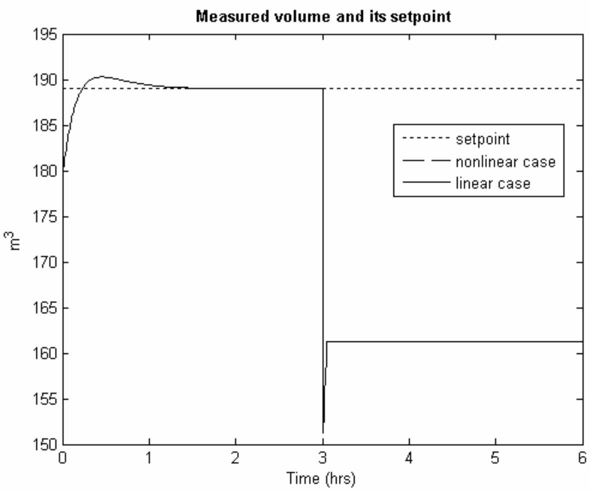

Figure 7. Volume sensor time-histories (-20% bias fault)

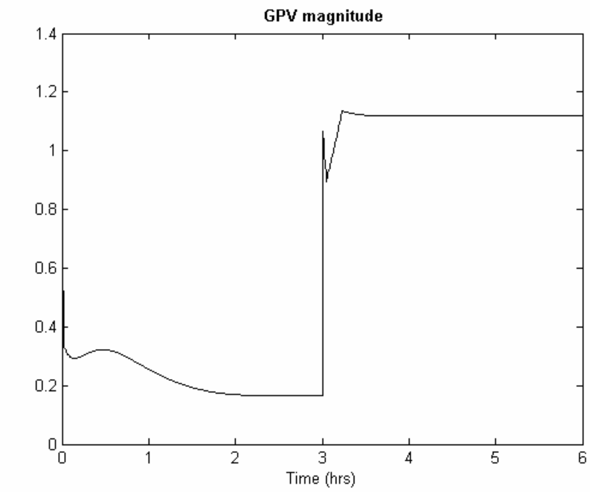

Figure 8. |GVP| for -20% bias volume sensor fault

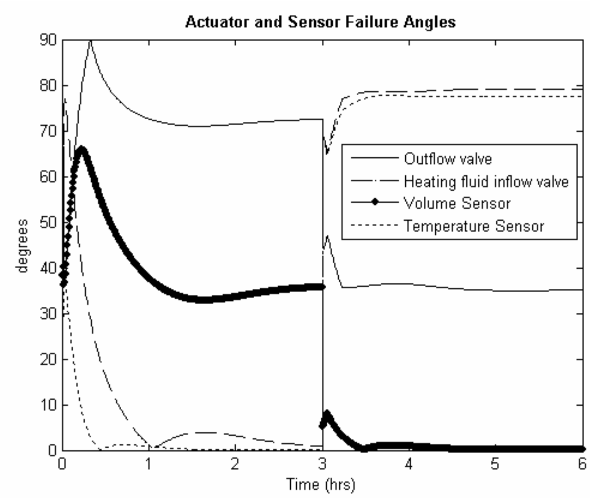

Figure 9. GPV angles for -20% bias volume sensor fault

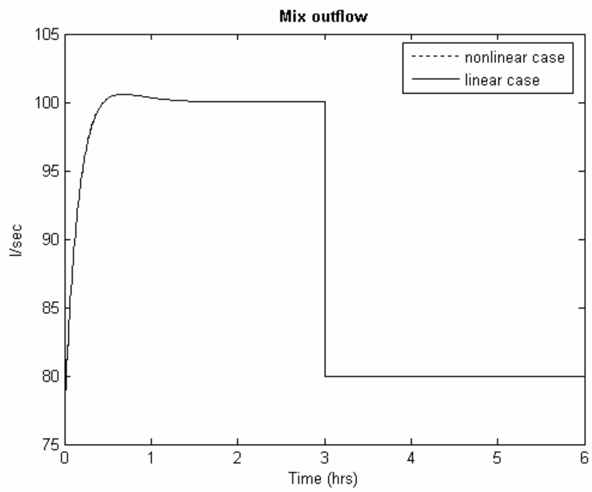

Figure 10. Outflow valve time-histories (-20% bias fault)

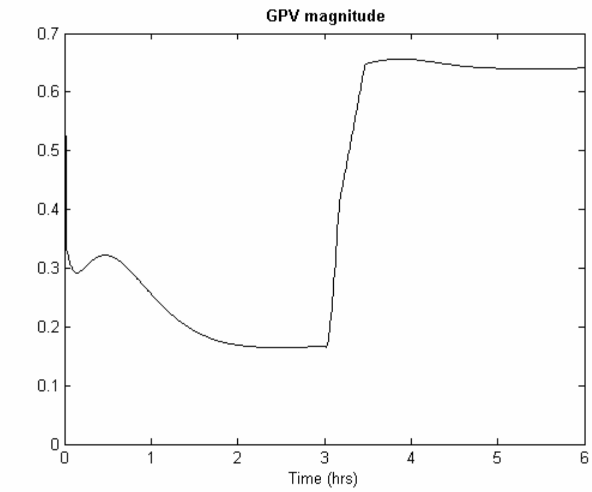

Figure 11. |GPV| for – 20% bias outflow valve fault

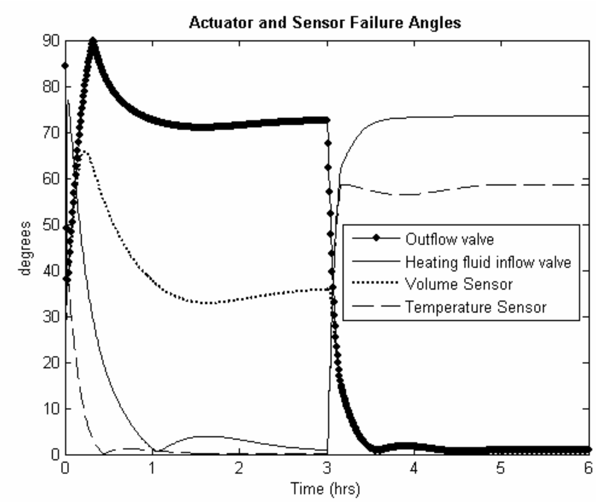

Figure 12. GPV angles for – 20% bias outflow valve fault

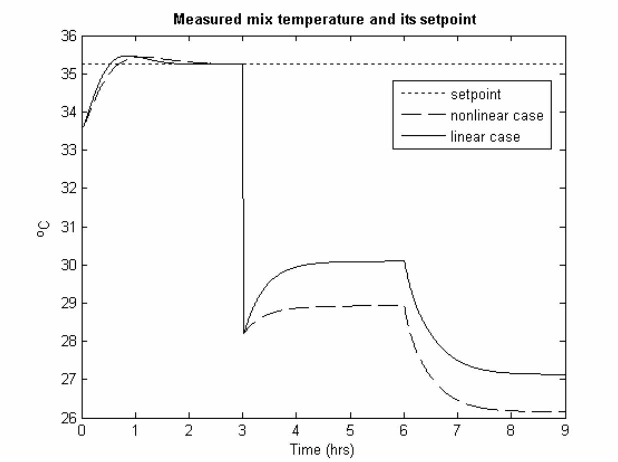

Figure 13. Temperature sensor time-histories (-20% sensor bias fault at *t=3 sec* + 20% high mix inflow disturbance at *t=6 sec*)

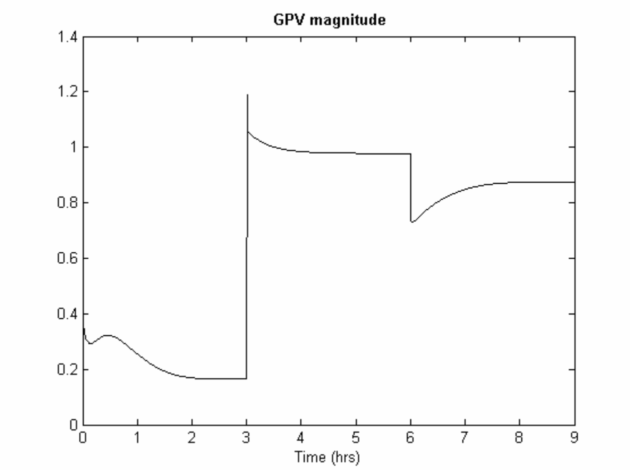

Figure 14. |GPV| for -20% bias temperature sensor fault at *t=3sec* + 20% high mix inflow disturbance at *t=6sec*

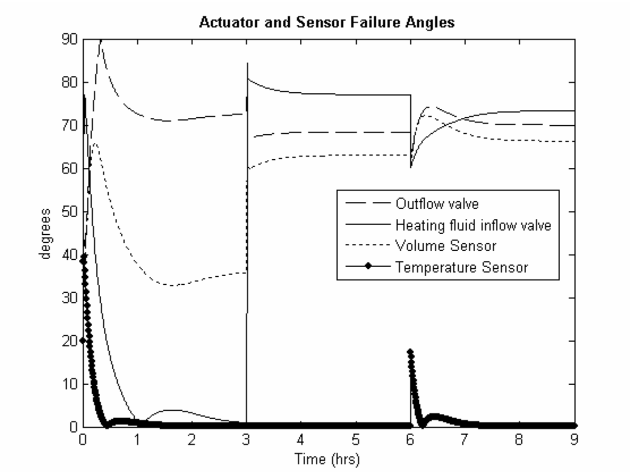

Figure 15. GPV angles for -20% bias temperature sensor fault at *t=3sec* + 20% high mix inflow disturbance at *t=6sec*

The special case when an actuator fault GPV direction lies on a sensor fault plane has been clarified, and additional logic has been added to deal with the special case defined for actuator fault detection. Finally, robustness has been improved by incorporating disturbance decoupling in the stable factorization framework. These results demonstrate the capability of a powerful model-based approach for FDI, even if there are significant nonlinearities involved, as there are in the JCSTR problem (but not discussed in detail here).

Acknowledgment: This research is supported by Atlantic Canada Opportunities Agency (ACOA) under the Atlantic Innovation Fund (AIF) program. The authors gratefully acknowledge that support and the collaboration of the University College of Cape Breton (UCCB), the National Research Council (NRC) of Canada, and the College of the North Atlantic (CNA).

References

[1] Chow, E. Y. & Willsky, A. S., (1984). Analytical Redundancy and the Design of Robust Failure Detection Systems. *IEEE Transactions on automatic control, Vol. AC-29, No. 7, pp. 603 - 614.*

[2] Frank, P. M. (1995). Residual evaluation for fault diagnosis based on adaptive fuzzy thresholds. *IEEE Colloquium on Qualitative and Quantitative Modelling Methods for Fault Diagnosis, No. 79, pp. 4/1 – 4/11.*

[3] Gertler, J. J. (1988). Survey of Model-Based Failure Detection and Isolation in Complex Plants. *IEEE Control Systems Magazine, Vol. 8, pp. 3 – 11.*

[4] Gertler, J. J. & Monajemy, R. (1995). Generating Directional Residuals with Dynamic Parity Relations. *Automatica, Vol. 31, No. 4, pp. 627-635.*

[5] Gertler, J. J. (1997). Fault detection and isolation using parity relations. *Control Eng. Practice, Vol. 5, No. 5, pp. 653-661.*

[6] Gertler, J. J. (1998). *Fault Detection and Diagnosis in Engineering Systems.* Marcel Dekker, Inc.

[7] Hamelin, F., Sauter, D & Aubrun, M. (1994). Fault Diagnosis in Systems using Directional Residuals. *Proceedings on the 33rd IEEE Conference on Decision and Control.*

[8] Pouliezos, A. D. & Stavrakakis, G.S (1994). *Real Time Fault Monitoring of Industrial Processes.* Kluwer Academic publishers.

[9] Venkatasubramanian V., Rengaswamy R., Yin, K. & Kavuri S. N. (2002). A Review of Process Fault Detection and Diagnosis. Part I: Quantitative Model-Based Methods. *Computers and chemical engineering 27, pp. 293 – 311.* Elsevier Science Ltd.

[10] Vidyasagar, M. (1985). *Control System synthesis: A Factorization Approach.* The MIT press.

[11] Viswanadham, N., Taylor, J. H. & Luce E.C. (1987). A Frequency Domain Approach to Failure Detection and Isolation with Application to GE-21 Turbine Engine Control Systems*.* *Control-Theory and advanced technology, Vol. 3, No.1, pp.45-72,* MITA press.

[12] Willsky A. S. , (1976). "A survey of design methods for failure detection in dynamic systems". *Automatica J. IFAC 12, No. 6, pp. 60 1– 611.*