An Extended MPC Algorithm for Processes with Variable and Unpredictable Time Delays

James H. Taylor and Khaled M. Akida
Department of Electrical & Computer Engineering
University of New Brunswick
Fredericton, N. B., CANADA
jtaylor@unb.ca – khaled.akida@unb.ca

Abstract

The classical model predictive control (MPC) approach is extended to handle uncertainty in the variable delay time problem by developing a Smart Delay-time Predictor approach. This new approach builds on the variable delay time estimator technique applied to time-variable flow processes. The present work has improved the approach proposed by Sayda and Taylor [1] in one important respect: the time delay prediction method presented here eliminates the adverse transient spikes that occurred due to uncertainty in the variable time delay, i.e., it removes unwanted transients caused by miscalculation of the forced response inside the controller. The efficacy and robustness of this technique is demonstrated by controlling a pulp bleaching process using a model predictive control algorithm with a variable delay-time estimator embedded in that controller.

1 Introduction

Chemical process control systems have to operate over a large envelope and for a variety of recipes. As a result, the controller has to attempt to overcome process nonlinearities, varying time delays and saturation constraints. Time delay is the time it takes from the moment a change is made in the control input until a response is seen in the output variable. Some possible sources of time delays are: (1) transportation of material over long distances, (2) retention of material in large vessels, (3) measurement delays, and (4) actuation signal delays. The presence of time delay may cause the following difficulties in process control: (1) a disturbance entering the process will not be detected until after a significant period of time, (2) the control action will be inadequate since its effects on a current error will affect the process variable only after a long delay, (3) long time delays may cause instability in the system.

A traditional modeling technique for a process is to describe the process as a combination of basic idealized models such as perfect mixers and plug flow vessels. The dynamics of a continuous flow process depend on the mass flow rate. The time constant of the process is determined by the flow rate through the vessel, the liquid volume in it, and the degree of mixing. In traditional design, the process is usually assumed to be at a nominal operating point so that the flow rates and volumes are constant, but this assumption is often not valid. Because of disturbances and intentional changes in the pulp production and utilization rates, the flow rate through the process is continually varying [5], [6].

One may model a plug flow vessel, through which the process material is assumed to flow without mixing, as a pure time delay. The concentration of the constituents at the outlet of the vessel is the same as at the inlet a certain time ago [5]. Under steady flow conditions, the delay time can be calculated by dividing the volume in the vessel by the flow rate. We define the residence time of the feed material as the time between entering at the input of the flow system to exiting at the output. The bleaching tower, which is the main contributing element in the continuous flow system of the bleaching process, can be represented by a plug flow reactor followed by a continuous stirred tank reactor. The division of the residence time into two parts is motivated by a simple model of a flow system. The plug flow of a system is modeled as a transportation part, e^{-sT_d}, and the part where the material is mixed is modeled as a first-order lag $1\,/\,(1+s\tau)$ [4].

The paper is organized in eight sections. Section 2 deals with the dynamics of the pulp bleaching process and its challenges. Sections 3 and 4 are devoted to the calculation of estimates and predictions of time delay in a system. Section 5 presents a model predictive control scheme suitable for time delay processes. Section 6 addresses problems due to the uncertainty in the variable time delay. In Section 7 we present the solution to this problem and results from a typical industrial application. This is followed by conclusions in Section 8.

2 The Pulp Bleaching Process

The main objective of pulp bleaching is to remove the coloring compounds still present in the fibers and thereby increase the brightness of the pulp, and to produce a pulp of satisfactory physical and chemical properties for the manufacture of printing or tissue papers [8]. Modern kraft pulp bleaching is achieved in a multistage plant, using expensive chemicals such as hydrogen peroxide, caustic soda, and alkaline extraction agents. When the unbleached pulp enters the bleach plant, it still includes a significant amount of lignin (bonds that hold fibers together) and chromophores (color constituents of the pulp). Hydrogen peroxide is employed to extract as much of the residual lignin as possible without damaging the pulp. After that, caustic soda extraction is used to remove the alkali-soluble portion of the lignin from the woodpulp. The bleaching of mechanical pulp with hydrogen peroxide is usually carried out by treating the pulp using DPTA or pentasodium diethylenetriaminepentaacetic, which is added to remove transitional metal ions in the pulp; processing conditions include agitation and at least 15 minutes retention time at temperature, ranging from at least 105 to 130^oF (40 to 54^oC). Bleach liquor is generally made up in a cascade mixing system and applied to the pulp. Pulp is held in a tower for at least two hours, though retention in excess of this time is also common. In general, a peroxide residual of 5 to 10 % of the amount applied is desired. Most systems include sulfur dioxide injection to prevent reversion and for pH adjustment. In summary, three stages are generally required in preparing the bleached pulp: (a) washing the pulp, (b) heating to the desired temperature, and (c) retention to complete the reaction. This modification in refiner mechanical pulp process has the name of Thermo-Mechanical Pulping (TMP) [3].

3 Zenger's Delay-time Estimation Method

In chemical reaction engineering, the concept of residence time distribution (RTD) is fundamental to reactor design. The RTD is the exit age distribution of material leaving a reactor. The classical residence time distribution covers only the case of stationary operating conditions, i.e., the flow rate through the system and the liquid volume in the system are constant. However, there is a strong practical need to consider processes under unsteady operating conditions also, because of disturbances and intentional changes in the process operation. Consideration of systems with time-varying behavior is beyond the scope of the classical RTD theory, so extensions to the theory are needed.

The volume and the flow through vessels and tanks are time varying, and this causes difficulties in identification and control procedures. For example, the flow in an industrial process may change randomly (e.g., due to disturbances) or intentionally (when the production and/or utilization is increased or decreased). Similarly, the volume changes in buffer vessel, whose purpose is to control the flow variations [6].

Zenger introduced the concept of variable delay time and presented an approach which can be used to estimate the delay time even though the flows and volume are varying [2], [7]. Models with varying liquid volumes are more complex than those with varying flow rates only, and it is often impossible to find a transformation that will change the representation into one with constant coefficients.

Consider the case of a plug flow vessel, in which both the input and output flow rates and the liquid volume change. The model equations are:

$$\dot{V}(t) = Q_i(t) - Q_o(t) \quad (1)$$

$$V(t) = \int_{t-T_d(t)}^{t} Q_i(\tau)d\tau \quad (2)$$

where $Q_i(t)$ and $Q_o(t)$ are the in- and out-flow rates in liters per minute, $V(t)$ is the liquid volume in the plug flow vessel, and T_d in equation (2) can be understood as the past time when the material exiting the vessel entered, i.e., the present delay time. We justify equation (2) by observing that the present volume $V(t)$ of "new liquid" must have entered in the time interval since the exiting material entered. An illustration of this explanation is shown in figure 1 where t_o stands for the initial time the particle entered the vessel, whereas t_f is the time the particle leaves the vessel.

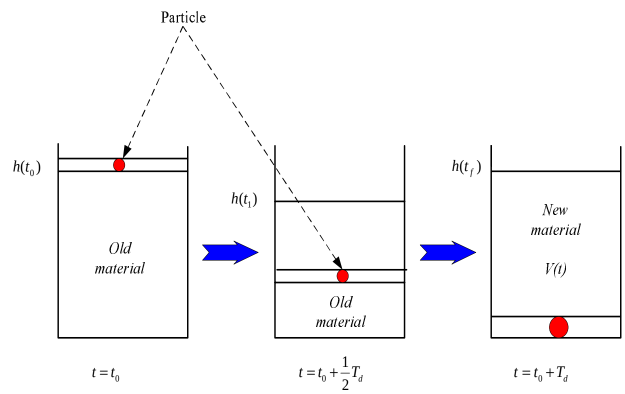

Fig. 1. Estimation of the delay time based on the inflow

By differentiating equation (2) we derive:

$$\frac{d}{dt}\left[\ \int_{t-T_d(t)}^{t} Q_i(\tau)d\tau\ \right] = \frac{d}{dt}V(t)$$

or

$$Q_i(t) - Q_i(t-T_d(t)) * (1-\dot{T}_d(t)) = \dot{V}(t)$$

Substituting for $\dot{V}(t)$ using equation (1) we obtain

$$\dot{T}_d(t) = 1 - \frac{Q_o(t)}{Q_i(t-T_d(t))} \quad (3)$$

Zenger [2] proposed solving equation (3) to determine the delay time numerically. This approach has one deficiency: we cannot predict the delay time initial condition exactly to solve this differential equation. That yields inaccurate results in the calculation of the delay time.

4 A New Delay-time Prediction Method

As shown in equation (2), we can *estimate* $T_d(t)$ by integrating the pulp inflow backward in time until that integral equals the present volume. Alternatively, we could *predict* the variable delay time $T_{dp}(t)$ if we know the future outflow by integrating it forward in time until the integral equals the present volume. The latter is expressed as follows:

$$\int_{t}^{t+T_{dp}(t)} Q_o(\tau)d\tau = V(t) \quad (4)$$

where $V(t)$ and $t + T_{dp}(t)$ are respectively the present volume and the predicted time delay at the instant t. Equation (4) can be understood as the definition of the "predicted delay time" as follows: If a *new particle* enters at time t and $V(t)$ is the corresponding volume in the vessel, then $T_{dp}(t)$ corresponds to that future time when that volume of liquid $V(t)$ has exited the vessel. However, predicting the pulp outflow $Q_o(t)$ is generally not practical because the operators would have to specify their future need for pulp, which is usually not feasible.

An algorithm can be realized to determine the delay time either backward as in equation (2) or forward as in equation (4) as follows:

1) Store the pulp inflow and bleaching tower level measurements over a time interval equal to the maximum retention time of the tower, with a suitable sampling time h.
2) Calculate the pulp volume in the tower.

Backward delay time estimation

3) Measure the volume at time t and set a counter $k = t - h$.
4) Integrate the inflow from k to t; if the integral equals the volume at time t then stop and $T_d(t) = t - k$, else set $k = k - h$ and repeat this step.

Forward delay time prediction

5) Measure the volume at time t and set a counter $k = t + h$.
6) Assume the future outflow and integrate it from t to k; if the integral equals the volume at time t then stop and $T_{dp}(t) = k - t$, else set $k = k + h$ and repeat this step.

Either algorithm can be used to identify the bleaching process delay time using offline data, but the forward method is problematical for the real-time control, since $T_{dp}(t)$ and thus the future outflow is required but difficult to predict. Those problems will be discussed in section 6. Figures 2 and 3 exhibit the volume and the pulp inflow for one data set, and the pulp inflow and both the estimated and the predicted delay time, respectively.

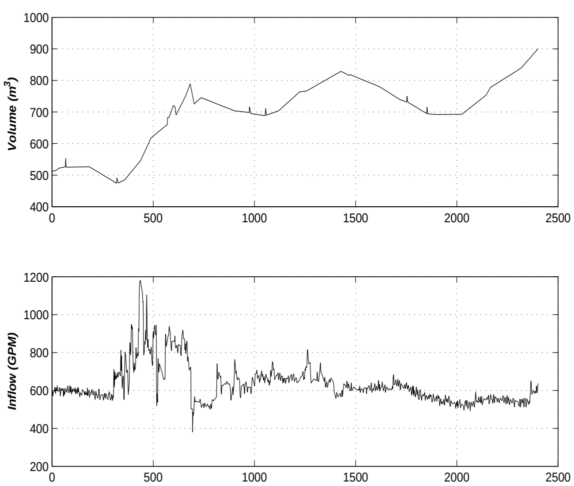

Fig. 2. Volume and inflow of the first data set

5 Model Predictive Control

Model predictive control (MPC) is the class of advanced control techniques most widely applied in the process industries. A primary advantage is its explicit handling of constraints. In addition, the formulation for multivariable systems with time-delays is straightforward. MPC was developed in the process industries in the 1960's and 70's, based primarily on heuristic ideas and input-output step and impulse response models [9].

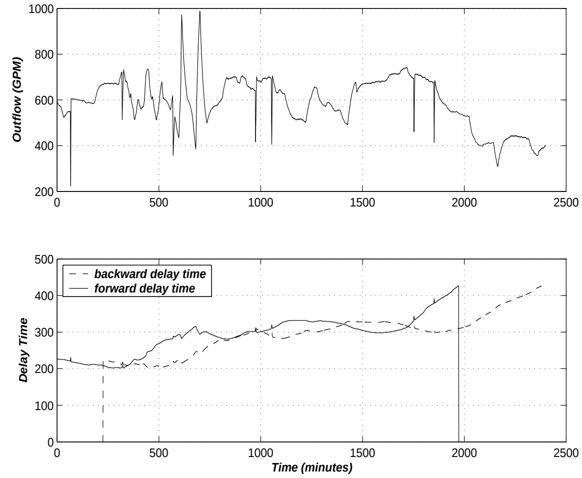

Fig. 3. Outflow, estimated and predicted delay time, first data set

The basic idea is to solve an open-loop optimal control problem at each time step. The decision variables are a set of future manipulated variable moves and the objective is to minimize deviation from a desired trajectory; constraints on manipulated, state and output variables are naturally handled in this formulation. Feedback is handled by providing a model update at each time step (often called "additive disturbance correction"), and performing the optimization again. Dynamic matrix control (DMC) is the most popular MPC algorithm used in the chemical process industry today due to its simplicity and efficiency [10]. The basic strategy may be developed as follows [11]:

The process model utilized in DMC is the step response of the plant, while disturbances are regarded as constant over the prediction horizon. The discrete-time response of the plant model is thus:

$$y(t) = \sum_{i=1}^{\infty} g_i \Delta u(t-i) \tag{5}$$

where g_i are the sampled output values for the step response and $\Delta u(t) = u(t) - u(t-1)$. The predicted values over the horizon are:

$$\hat{y}(t+k|t) = \sum_{i=1}^{\infty} g_i \Delta u(t+k-i) + \hat{n}(t+k|t) \tag{6}$$

where disturbances are assumed to be constant over the horizon, i.e., to be equal to the measured value of the output (y_m) minus that estimated by the model ($\hat{y}(t|t)$), as follows:

$$\hat{n}(t+k|t) = \hat{n}(t|t) = y_m(t) - \hat{y}(t|t) \tag{7}$$

Then equation (6) can be rewritten as:

$$\hat{y}(t+k|t) = \sum_{i=1}^{k} g_i \Delta u(t+k-i) + f(t+k)$$

where $f(t+k)$ is the free response of the system, that is, the part of the response that does not depend on the future actions:

$$f(t+k) = \sum_{i=k+1}^{\infty} g_i \Delta u(t+k-i) + y_m(t) - \hat{y}(t|t) \quad (8)$$

For a stable process, the coefficients g_i of the step response tend to a constant value after N_p sampling periods, which yields:

$$f(t+k) = y_m(t) + \sum_{i=1}^{N_p} (g_{k+i} - g_i) \Delta u(t-i) \quad (9)$$

Computing $\hat{y}$ over the horizon $(k = 1, \ldots, N_p)$, with N_u control actions, yields:

$$\hat{y}(t+N_p|t) = \sum_{i=1}^{N_u} g_i \Delta u(t+N_p-i) + f(t+N_p) \triangleq GU + F \quad (10)$$

where G as defined to be the system's *dynamic matrix*, which is made up N_u columns of the system's step response compatibly shifted down in order, U denotes the N_u-dimensional vector of control increments, and F is the free response vector.

The set of future control values is obtained by optimizing a performance criterion J in order to keep the process as close as possible to the reference trajectory $\omega(t+k)$ which for step set-point changes is approximated by means of the following first order system:

$$\omega(t+k) = \alpha\, \omega(t+k-1) + (1-\alpha)\, r(t+k) \quad k = 1, \ldots N_p \quad (11)$$

where α is a parameter between 0 and 1 (the closer to 1 the smoother the approximation) that constitutes an adjustable value that will influence the dynamic response of the system, and $r(t+k)$ is the constant future reference. The specific criterion usually takes the form of a quadratic function (cost function) of errors between the prediction output signal and the prediction reference trajectory plus a weighted quadratic input term as follows:

$$\begin{aligned} J &= \sum_{i=1}^{N_p} [\hat{y}(t+j|t) - \omega(t+j)]^2 + \sum_{i=1}^{N_u} \lambda [\Delta u(t+j-1)]^2 \\ &\triangleq e^T e + \lambda U^T U \end{aligned} \quad (12)$$

where $e = GU + F - \omega$ is the vector of future errors over the prediction horizon and U is the vector composed of the future control increments $\Delta u, \ldots, \Delta u(t+N_u)$. The parameter λ is a positive constant that can be used to tune the DMC controller to achieve the required performance. If there are no constraints, the solution to the minimization of the cost function J can be obtained analytically by setting the derivative of J equal to 0, which provides the general result:

$$U = (G^T G + \lambda I)^{-1} G^T (\omega - F) \quad (13)$$

The control move $u(t|t)$ is sent to the process, while the subsequent control moves calculated are ignored, because at the next sampling instant $y(t+1)$ is already known and the output prediction is repeated with this new value and all sequences are updated.

6 Problems Due to Variable Delay Time

Figure 4 from [1] portrays the peroxide dosage (top plot) and the pulp brightness output (bottom plot) due to controlling the pulp bleaching process using a DMC controller with ± 5 % errors in delay time (nominally 550 minutes). The brightness response exhibits "*blips*" at approximately 500 minute intervals. To explain those blips, let us consider the -5% case, in which the brightness response occurs "earlier than expected". This early response of the brightness will cause an error in the estimation of the free response in the DMC control algorithm as given in equation (9). Consequently, the future error between the predicted free response and the set point profile is not zero, which causes an immediate downward blip in the control action. This in turn causes a blip in the brightness response after the delay time which will result in another error, and the story is repeated every delay time, resulting in the series of blips portrayed in figure 4. In other words,

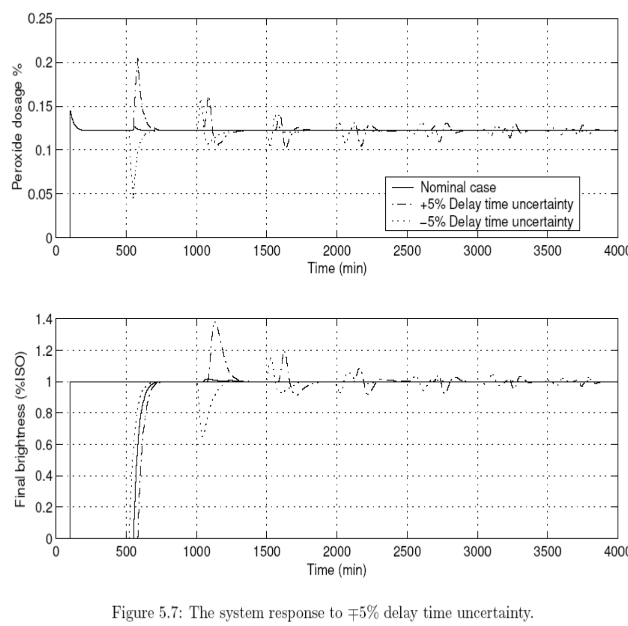

Fig. 4. The system response for ∓ 5 delay time uncertainty [1]

the early brightness response is considered to be a positive disturbance which means that DMC controller will order the peroxide dosage to decrease starting from the current time until the time the final estimated brightness is reached. This explains the downward blip about 500 minutes later.

7 MPC with a Smart Delay-time Predictor

We propose a new approach to solve the problem discussed in the previous section, namely a *Smart Delay-time Predictor* which corrects the delay time uncertainty in the final pulp brightness as follows: Assume a brightness setpoint change is required at time t_0 and a recommended control action (hydrogen peroxide dosage) is then activated to achieve that required pulp brightness. At this time the backward delay time is known to be $\hat{T}_d^0$, i.e., the DMC algorithm "expects" that the brightness will respond at t_0 + $\hat{T}_d^0$. As time advances we continue to back-estimate the exact delay time after which the output brightness should begin to track the new set point, based on the variable inlet pulp flow and level data, and we advance the delay time inside the MPC controller, step by step from $\hat{T}_d^1$ to $\hat{T}_d^2$ to $\ldots\hat{T}_d^k$, where k is the number of steps after the set point change, and we continue to integrate the inlet flow backwards to a point where $\hat{T}_d^k$ equals the time back to the application of the step change. The difference between this algorithm and the one employed in [1] is that the delay time used by Sayda and Taylor was calculated outside the DMC controller, then stored and used by the controller without updating. In our method, we calculate the delay time continually and update the controller inside the loop; once the backward integration of the variable time delay reaches the past setpoint change, the delay time calculation stops until the next time a set-point change is requested. In this way the DMC controller has a more accurate idea of the time to anticipate the brightness to respond, thus eliminating the "blips".

Figure 5 shows the result of applying this indirect adaptive / predictive controller using the embedded smart delay-time predictor to the pulp bleaching process. For more details about this controller and these results, the reader can refer to [12].

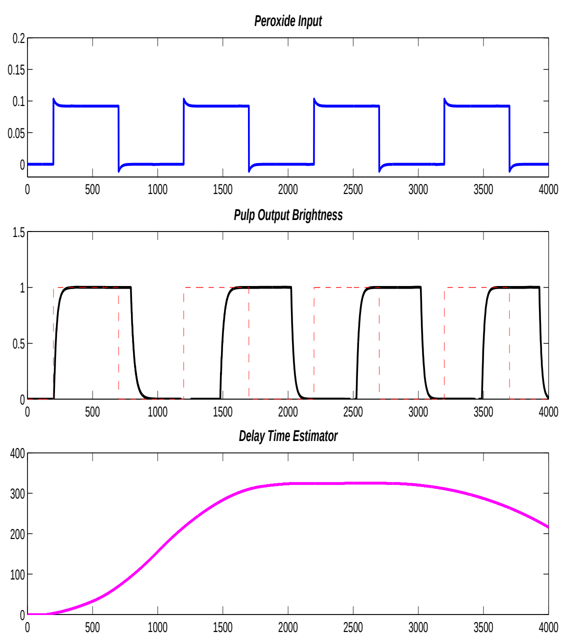

Fig. 5. SISO process with variable delay-time predictor

8 Conclusion

The pulp bleaching process exhibits long time constants and time delays, which tend to degrade controller performance. Variable delay time has been thoroughly defined and illustrated, and its importance in chemical process control applications explained. A model predictive controller designed to cope with such challenging systems has been presented. The performance of the controller achieved good tracking of the final pulp brightness which yields, as a consequence, a significant suppression of undesirable transients in the hydrogen peroxide dosage and immediate reductions in cost. The contributions of the method presented here include a delay time predictor that:

- is straightforward to implement and use,
- corrects uncertainty in delay time estimation, and
- is reliable for a broad class of chemical process applications.

References

[1] A. Sayda and J. H. Taylor, "Model Predictive Control of a Mechanical Pulp Bleaching Process", *Proc. IFAC Workshop on Time-Delay Systems (TDS'03)*, INRIA, Rocquencourt, France, 8-10 September 2003.

[2] K. Zenger, *Modeling, analysis and controller design of a time-variable flow processes*, Helsinki University of Technology, Helsinki, Finland, 2003.

[3] D. W. Reeve, "Introduction to the Principles and Practice of Pulp Bleaching", from *Pulp Bleaching Principles and Practice*, ed. by C. W. Dence and D. W. Reeve, pp. 3-23, Atlanta, Ga, TAPPI Press, 1996.

[4] X. Qian and P. Tessier, "Dynamic modeling and control of a hydrogen peroxide bleaching process", *Pulp and Paper Canada*, vol 89, no. 9, pp. 81-85, 1997.

[5] T. Andersson and P. Pucar, "Estimation of residence time in continuous flow system with dynamics", *Journal of process control*, vol. 5, no. 1, pp. 9-17,1995.

[6] R. Ylinen and K. Zenger, "Computer aided analysis and design of time varying systems", *Proc. international workshop on computer aided systems theory* (EUROCAST) R. Pichler and R. M. Doas, eds., (Kerms, Austria), pp. 73-95, Springer Verlag, 1991.

[7] K. Zenger, "Time-variable models for mixing processes under unsteady flow and volume", *Proc. Fourth IFAC Symposium on dynamics of chemical reactors, distillation columns, and batch reactors*, J. B. Rawlings, ed., (Oxford, UK), pp. 57-63, Pergamon Press, 1995.

[8] G. A. Dumont, C. C. Zervos and G. L. Pageau, "Laguerre-based adaptive control of pH in an industrial bleach plant extraction stage", *Automatica*, vol. 26, no. 4, pp. 781-787, 1990.

[9] J. Richalet, A. Rault, J. L. Testud, J. Papon."Model predictive heuristic control: Applications to industrial processes", *Automatica*, vol. 14, no. 2, pp. 413-428, 1978.

[10] S. J. Qin, and T. A. Badgwell, "An Overview of Industrial Model Predictive Control Technology", *Fifth International Conference on Chemical Process Control*, AIChE Symposium Series 316, 93, J. C. Kantor, C. E. Garcia, and B. Carnahan, eds., pp. 232-256, 1997.

[11] Camacho, E.F. and Bordons C., *Model Predictive Control*, Springer-Verlag, London, 1999.

[12] K. Akida and J. Taylor, "Indirect Adaptive Predictive Control for a Multivariable Pulp Bleaching Process".