A Frequency-Domain Model-Order-Deduction Algorithm for Nonlinear Systems

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Abstract

Several model-order deduction algorithms (MODAs) have been developed to coordinate the synthesis of lumped (finite-dimensional), linear system models, of accetable order, that accurately characterize the behavior of a system over a frequency range of interest (FROI) $[\omega_{min}, \omega_{max}]$. The most recent of these techniques considers the frequency response of the model as the "performance metric" and systematically increases model complexity until the frequency response over a FROI has converged to within a user-specifice tolerance.

The linear MODA algorithm based on frequency response is being extended to support the synthesis of models of nonlinear systems. This technique follows a procedure similar to the linear frequency-domain algorithm, but uses a describing-function approach to develop an amplitude-dependent characterization of the nonlinear system frequency response. The extended algorithm synthesizes models that are also of low order; in addition, they include only those nonlinear effects that influence the frequency response significantly over the FROI and for an amplitude range of interest. This significantly extends the class of systems to which model-order deduction can be applied.

1. Introduction

Modelers face a number of decisions when developing a lumped model of some physical device. These decisions include choosing the appropriate model order, the effects to include in the model, and how to treat the nonlinearities inherent in the physical process.

While a general technique for solving this problem is not available, a number of *model-order-deduction algorithms* (MODAs) for determining an acceptable model order do exist for the linear case. The original MODA, described in Wilson and Stein [14, 15], parses a set of components, each of which has one or more possible submodels associated with it, and coordinates the synthesis of a low-order lumped-parameter model that includes all the system eigenvalues within a specified spectral radius, i.e., $|\lambda_i| \in [0, \omega_{max}]$. More recently, an extension of MODA has been proposed in which this eigenvalue-based model performance metric is replaced with a requirement that the *frequency response* must have converged over the frequency range of interest (FROI) [17]. It is argued in [17] that the frequency-domain version of MODA (called FD-MODA) is a better tool for developing models for controls analysis and design.

A limitation of MODA and its extensions is their inability to handle systems with nonlinear behavior. However, describing functions (DFs) provide a means of quasi-linearizing a model, and a MODA can then be applied. The combination of DFs and MODA to synthesize nonlinear models with "eigenvalues" within a spectral radius produced a preliminary algorithm, called *model-order-deduction algorithm for nonlinear systems* (MODANS) [16], which demonstrated the overall approach. Here we focus on the analogous extension of FD-MODA to create FD-MODANS, a DF- and frequency-domain-based technique for nonlinear system modeling. We believe this to be a major advancement over earlier approaches.

1.1. Parameter Lumping

Parameter lumping refers to the process by which continuous elements are modeled with a finite (usually low) number of discrete elements. Cannon [2], Dorny [3] and Huston [6] provide a thorough discussion of the process. As these sources indicate, parameter lumping requires a decision regarding the most appropriate representation (model) of continuous elements, i.e., the proper number of inertial and compliant elements.

1.2. Describing Function Methods

The basic idea of the describing function approach is to replace a nonlinear component with a quasilinear term whose "gain" is a function of "input amplitude", where the form of input signal is assumed in advance and the amplitude-dependence of the gain is based on that assumption plus an approximation-error criterion. In this work, the signals are assumed to be sinusoidal, so we use a sinusoidal-input describing function (SIDF) method. The associated approximation-error criterion is minimization of mean square error. This technique is dealt with thoroughly in [1, 5] for the case of a single nonlinearity; for systems with more than one nonlinearity in arbitrary configurations, the most general extensions of the SIDF approach are described in [9].

Modern SIDF approaches have been used for two purposes: limit-cycle analysis and characterizing the input/output (IO) behavior of a nonlinear plant (e.g., [9, 10]). The latter application serves as the basis for the method presented here. There are two techniques for generating SIDF IO models for nonlinear systems:

1. Develop a state-space model of the system in which every nonlinear element is replaced analytically by the corresponding scalar SIDF, formulate the equations of harmonic balance, select an input amplitude a ($u(t) = a\sin(\omega t)$), solve for the unknown amplitudes of the state variables and scalar SIDF values, and compute the IO model as $G(j\omega, a) = C(a)[j\omega I - A(a)]^{-1}B(a) + D(a)$ (note that all arrays in the quasilinear model may depend on the input amplitude a). This approach was used in [16] in developing MODANS; refer to [10] for further details.
2. Apply a sinusoidal signal to the nonlinear system model, perform direct Fourier integration of the system output in parallel with simulating the model's response to the sinusoidal input, and simulate until steady-state is achieved to obtain the dynamic or frequency-domain SIDF $G(j\omega, a)$ [13].

To elaborate on the second method and illustrate its use in the MODA context, we specify a range of input amplitudes $[a_{min}, a_{max}]$ to cover the expected operating range of the system and frequencies $[\omega_{min}, \omega_{max}]$ to span the FROI. Then specific sets of values $\{a_i\} \in [a_{min}, a_{max}]$ and $\{\omega_j\} \in [\omega_{min}, \omega_{max}]$ are selected for generating $G(j\omega_j, a_i)$. The system model is augmented by adding new "states" corresponding to the Fourier integrals

$$\begin{aligned} Re(G_K(j\omega_j, a_i)) &= \frac{\omega_j}{\pi a_i}\int_{KT}^{(K+1)T} y(t)\sin(\omega_j t)dt \\ Im(G_K(j\omega_j, a_i)) &= \frac{\omega_j}{\pi a_i}\int_{KT}^{(K+1)T} y(t)\cos(\omega_j t)dt \end{aligned}$$

where $Re(\cdot)$ and $Im(\cdot)$ are real and imaginary parts of the SIDF $G(j\omega_j, a_i)$, $T = 2\pi/\omega_j$, and $y(t)$ is the output of the nonlinear system. Achieving steady state for a given a_i and ω_j is guaranteed by setting certain tolerances and convergence criteria on the magnitude and phase of G_K where K corresponds to the number of cycles simulated; the integration is interrupted at the end of each cycle and the convergence criteria checked to see if the results are within tolerance so that the simulation could be stopped and $G(j\omega_j, a_i)$ reported. For further detail, refer to [12, 13].

2. Frequency-Domain Method for Nonlinear Model-Order Deduction

2.1. Class of Systems to be Modeled

The work presented here and in the earlier MODA papers has all been done in the context of serially-connected unidimensional electro-mechanical systems. By "serially-connected" we signify simply that the *mechanical* components that comprise the system are connected so that each component follows another. The term "unidimensional" means that each component may undergo one-dimensional rotational *or* translational motion, but not both. (Of course, a system may have some components that are translational and others that are rotational, e.g., a rack-and-pinion assembly.)

The MODA approach in general and the FD-MODANS technique in particular have substantially broader applicability. These algorithms can coordinate the synthesis of *finite-dimensional, time-invarient, and (in the case of* MODANS *and* FD-MODANS*) nonlinear systems.* The algorithms have been applied to serially-connected, unidimensional electro-mechanical systems because (i) these help to demonstrate the algorithms and (ii) the rest of the automated modeling tools available are subject to these restrictions.

2.2. Objective

The required input and the desired output specify the functionality of the algorithm. Inputs include:

- a system, $\mathcal{S}$, of the class defined in Section 2.1, e.g., $\mathcal{S} = \{c_1, c_3, c_2, c_3, c_8 \ldots\}$ where c_i denotes various types of electro-mechanical components,
- an input-amplitude range of interest, $[a_{min}, a_{max}]$,
- a frequency range of interest, $[\omega_{min}, \omega_{max}]$, and
- a frequency-response convergence tolerance, `TOL`.

The output of FD-MODANS is the set of ranks of the components ("rank" specifies the complexity of a component submodel, see [14, 15] for details) of $\mathcal{S}$, i.e., $\mathcal{R} = \{r_1, r_3, r_2, r_3, r_8 \ldots\}$, and the set of nonlinearity indices, $\mathcal{V} = \{v_1, v_3, v_2, v_3, v_8 \ldots\}$, which contain 1 for nonlinearities to be included in the model, 0 for those to be neglected. These sets satisfy two conditions:

1. the frequency response over $[\omega_{min}, \omega_{max}]$ predicted by a model synthesized based on $\mathcal{R}$ and $\mathcal{V}$ has converged within `TOL`,
2. the sum of the ranks, $\sum_i r_i$, is not excessive and only nonlinear effects that influence $G(j\omega,\, a)$ significantly are included.

This provides a (non)linear system model that is of appropriate complexity given the specification that the frequency response should be accurate over the FROI and for input amplitudes in the AROI.

2.3. Model Order Deduction Procedure

1. **Initialize**

   (a) Set all ranks to zero ($r_i = 0$) and all nonlinearity indices to zero ($v_i = \underline{0}$)

   (b) Specify: a tolerance `TOL`, a frequency range of interest $[\omega_{min}, \omega_{max}]$, an input-amplitude range of interest $[a_{min}, a_{max}]$

   (c) Create a grid of K frequency points, $\omega_k \in [\omega_{min}, \omega_{max}]$, pick a set of L input-amplitude values, $a_l \in [a_{min}, a_{max}]$

   (d) Synthesize a system model $\mathcal{S}$, based on current ranks and nonlinearity index vectors

   (e) Set $G^*(j\omega) = G(j\omega)$ for $\omega \in \{\omega_k\}$,

2. **Determine most rank-sensitive component**

   (a) $dG_{max} = 0$

   (b) Cycle over the components c_i:

      i. Increase rank of c_i

      ii. Synthesize system model, based on current ranks

      iii. Compute $G(j\omega)$ for all $\omega_k \rightarrow G(j\omega_k)$

      iv. $\delta G_k = (G^*(j\omega_k) - G(j\omega_k))/G^*(j\omega_k)$

      v. If $(\| \delta G_k \|_\infty = \max_k | \delta G_k |) > dG_{max}$ then: $dG_{max} = \| \delta G_k \|_\infty$, $i^* = i$

      vi. Decrease rank of component c_i

3. **Evaluate need to increase rank**

   (a) If $dG_{max} >$ `TOL` then:

      i. Augment model order by increasing rank of the i^*th component

      ii. Synthesize system model, based on current ranks

      iii. Compute $G(j\omega)$ for all ω_i

      iv. $G^*(j\omega) = G(j\omega)$

      v. go to 2

   (b) else set $G^*(j\omega_k, a_l) = G^*(j\omega)$ for all a_l, and continue below

4. **Determine most sensitive nonlinearity**

   (a) $dG_{max} = 0$

   (b) Cycle over the '0' places in each component's nonlinearity index vector v_i:

      i. Set the m^{th} place in v_i to 1

      ii. Synthesize system model, based on established ranks and current nonlinearity index vectors

      iii. Compute $G(j\omega_k, a_l)$ for all ω_k and a_l

      iv. Compute $\| \delta G_k \|_\infty = \max_k | \max_l (1 - G(j\omega_k, a_l)/G^*(j\omega_k, a_l)) |$

      v. If $\| \delta G_k \|_\infty > dG_{max}$ then: $dG_{max} = \| \delta G_k \|_\infty$, $i^* = i$, $m^* = m$

      vi. Reset m^{th} place in v_i to 0

5. **Evaluate need to add nonlinear effect**

   (a) If $dG_{max} >$ `TOL` then:

      i. Increase model complexity by setting the m^*th place in v_{i^*} to 1.

      ii. Synthesize system model, based on established ranks and current nonlinearity index vectors

      iii. Compute $G(j\omega, a)$ for all ω_k and a_l

      iv. $G^*(j\omega_k, a_l) = G(j\omega, a)$

      v. go to 4

   (b) else, continue below

6. **Output results**

   (a) Output component ranks and nonlinearity index vectors

   (b) Output (non)linear system model

3. Illustration of the Algorithm

To demonstrate FD-MODANS, we modeled and obtained frequency-domain "transfer functions" $G(j\omega, a)$ for the ATB1000, a "benchmark" problem that has received considerable study [7]. This electro-mechanical pointing system (a surrogate gun-turret testbed) consists of two subsystems: a drive subsystem, including a DC motor (with coulomb friction), a gear train (with backlash), and a compliant shaft; and a wheel/barrel subsystem, including an inertial wheel (also with nonlinear friction) and a flexible gun barrel. The drive subsystem dynamics are governed by two sets of differential equations, depending on whether the gears are engaged or not. When the gears are not engaged, we have two decoupled second-order differential equations:

$$J_m \ddot{\theta}_m = T_m - T_{mf} \quad (1)$$

$$J_b \ddot{\theta}_b = -T_s \quad (2)$$

where θ_m and θ_b are the angles of the driving gear and driven gear, respectively. J_m and J_b are the inertias of the motor and compliant shaft assemblies; T_m is the mechanical torque produced by the motor, $T_m = T_{m0} \sin(\omega t)$; T_{mf} is the coulomb friction torque on the motor, $T_{mf} = b_m \operatorname{sgn}(\dot{\theta}_m)$; and T_s is the reactive torque of the compliant shaft,

$$T_s = k_s(\theta_b - \theta_i) + b_s(\dot{\theta}_b - \dot{\theta}_i) \tag{3}$$

where k_s and b_s are spring and viscous friction constants respectively and θ_i is the inertial wheel yaw angle. When the two gears are engaged, the differential equation governing the dynamics of θ_b is

$$(J_m + J_b)\ddot{\theta}_b = T_m - T_s - T_{mf} \tag{4}$$

We note that there is a "jump" in the states $\dot{\theta}_m$ and $\dot{\theta}_b$ at the moment the two gears become engaged; if we neglect the compliance of the gear material, then by conservation of angular momentum we have

$$\dot{\theta}_m(t_c^+) = \dot{\theta}_b(t_c^+) = \frac{J_m\dot{\theta}_m(t_c^-) \ + \ J_b\dot{\theta}_b(t_c^-)}{J_m + J_b} \tag{5}$$

The conditions for engagement and disengagement are (i) contact and (ii) individual component angular accelerations achieving values that permit separation, respectively. This is modeled and simulated rigorously, using a new integration algorithm that exactly captures these "state events" [11].

The gun barrel is a distributed-parameter system that can be approximated by a lumped-parameter model obtained using a modal expansion approach. The wheel/barrel subsystem is then described by a state-space model of the following form:

$$\ddot{x} + D\dot{x} + Kx = B(T_s - T_{if}) \tag{6}$$
$$y = Cx \tag{7}$$

where $x \in \mathcal{R}^n$ is the subsystem state vector (vector of modal coordinates) and $y^T = [\theta_i \ \theta_{tip}]$ is the output vector; θ_i is the inertial-wheel angle and θ_{tip} is the gun-barrel tip-angle; T_{if} combines viscous and coulomb effects; and matrices D, K, C and B are of appropriate dimensions. For parameter values, see [7].

We chose this problem because it is a benchmark, and has exactly the desired characteristics. It is too complex to explore exhaustively in this paper, so we will show results that demonstrate the procedures and issues discussed in the previous sections. First, in Fig. 1 we illustrate the process of determining one value of $G(j\omega,\, a)$, i.e., for $a = T_{m0} = 2$ volts, $\omega = 10$ radians/sec. The effects of backlash are evident for this small amplitude excitation; one sees the separation and re-engagement of the gears and the associated velocity reset at each point of re-engagement. The routine for checking the convergence of $G(j\omega,\, a)$ after each cycle stopped after the second cycle, because the tolerance is quite relaxed (on magnitude it's 1 dB, and on phase 5 degrees; simulation continues until the difference from one cycle to the next is within tolerance).

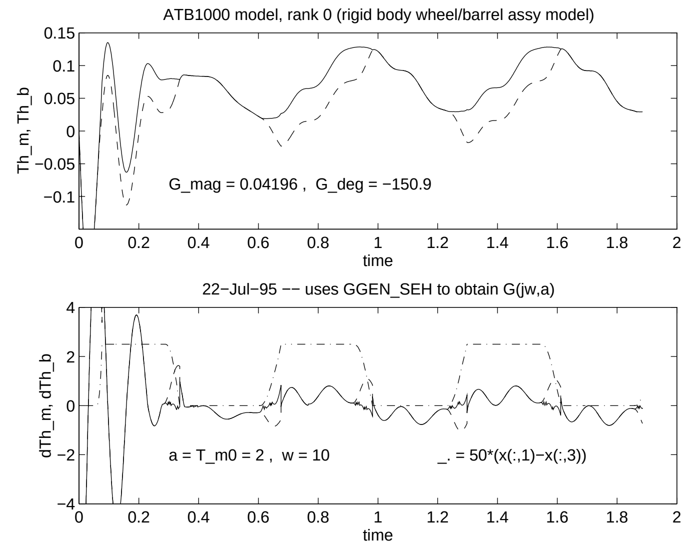

Figure 1: Obtaining $G(j\omega,\, a)$ by Simulation

An executive based on this routine was used to generate frequency response plots for various amplitudes and conditions. In Fig. 2 we see the standard Bode representation for $G(j\omega,\, a)$ determined for two amplitudes ($a = 2,\, 10$) and for rank $= 0,\, 1,\, 2$. We note simply that the nonlinear effects are definitely important over the amplitude range of interest $a \in [2,\, 10]$, as is the inclusion of the first mode of the flexible member. The second mode is at $\omega_n = 138$ rad/sec., so it can be safely neglected over the frequency range of interest $\omega \in [2,\, 50]$ rad/sec, as you may barely discern the difference.

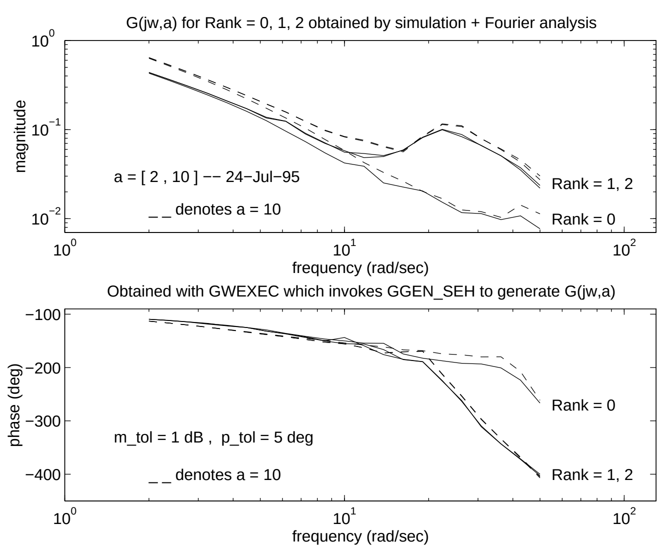

Figure 2: $G(j\omega,\, a)$ for $a \in [2, 10]$, Rank $= 0,\, 1,\, 2$

A more formal exploration of the nonlinear effects is summarized in Table 1. The following cases were considered for $\omega = 5$ rad/sec, a low frequency where the amplitude-dependency is quite evident in Fig. 2: Case A is nominal (fully nonlinear), case B has no nonlinearities, case C has backlash only, case D has motor coulomb friction only, and case E has wheel coulomb friction only. We note significantly less variation for $a = 10$, as would be expected by the nature of the effects (backlash and coulomb friction dominate at small amplitudes). Focussing on $a = 2$, the motor coulomb friction is clearly the most substantive, then wheel coulomb friction, and finally backlash seems to be the least important.

Case	$a = 2$		$a = 10$	
	Magn.	Phase	Magn.	Phase
A	0.1359	-128.13	0.1875	-135.88
B	0.1988	-138.84	0.1988	-138.84
C	0.1993	-138.87	0.1989	-138.85
D	0.1509	-125.27	0.1892	-136.41
E	0.1902	-136.18	0.1971	-138.30

Table 1: Exploration of Nonlinear Effects

Whether or not one may safely ignore the amplitude dependencies shown above depends on the usage of the resulting model. The nonlinear effects are significant at small amplitude, as one might expect – the difference in magnitude at lower frequencies is about 3 dB. One may conclude that a large-amplitude model (which will approach the linear model) is "safe" for designing controllers, for example; however, it will not accurately predict the IO behavior of the ATB1000 for small amplitudes.

4. Discussion

This paper presents an approach that coordinates the synthesis of a nonlinear, state-space system model based on a high-level physical description of a system belonging to the class outlined in Section 2.1. This algorithm, FD-MODANS, employs a two-staged process. The first stage determines the appropriate order of the system model by augmenting the rank of component models that have the most significant affect on $G(j\omega, a)$; this continues until any subsequent increase results in a change to the frequency response that is less than a user-specified tolerance. The second stage of the algorithm works in a similar fashion to determine which component nonlinearity produces the largest change in frequency response when it is added to the system model; this too proceeds until adding nonlinear effects leads only to a negligibly small change in frequency response.

There are several major improvements embodied in FD-MODANS compared with MODANS [16]. FD-MODANS has been extended to determine not only an acceptable low order for the model, but the nonlinearities that should be included to ensure that the specified accuracy is obtained. The new model performance metric, i.e., the frequency response of the nonlinear system over a FROI, is much more suitable than the spectral radius metric used in MODANS. Finally, the change from the analytic DF-based technique for determining $G(j\omega, a)$ used in MODANS to the simulation-based method outlined in Section 1.2 removes some of the barriers to treating arbitrarily complicated systems in terms of the number and types of nonlinearities. In essence, if an electromechanical system with sinusoidal inputs can be simulated, then this approach may be applied.

4.1. Effect of Amplitude on Frequency Response

Both the frequency and the amplitude of the input signal affect the response characteristics of the quasilinear model $G(j\omega, a)$. Depending on the input signal and the particular nonlinearities present in the model, response characteristics may asymptotically approach those of the equivalent linear model as amplitude becomes small (e.g., this would be true for effects such as saturation), or this may happen as input amplitudes become large (as is the case for coulomb friction and backlash, as shown in the previous section). The FD-MODANS approach systematically sorts these issues out, for the conditions specified by the user's FROI and AROI.

4.2. Side-Effects

FD-MODANS provides a means of systematically selecting the required complexity of component submodels within a larger system model. FD-MODANS is intended to be used in an automated-modeling program, with which a user is expected to provide a system configuration description, a desired FROI, a specific input amplitude range AROI, and a tolerance defining the required model accuracy. The program would then use FD-MODANS to build the system model.

The intermediate models obtained during this synthesis process contain useful design information. When FD-MODANS identifies a rank-sensitive component, this is often an indication that this part causes the next mode (generally a structural resonance) in a system model. If the modal frequency caused by this component is too low, an engineer can make the appropriate changes *to this component*, e.g., alter the material or a physical dimension so that the modal frequency is increased. Similar information is provided during the identification of significant nonlinear effects: If variations in frequency response caused by a specific component nonlinearity is excessive, then the engineer may decide to use a different type of bearing or gear train to alleviate the problem, for example.

4.3. Open Questions

The MODA approach in general represents a heuristic search strategy for determining an acceptably-accurate, suitably-low-order model of a system. We define "acceptable" in terms of a specific performance metric: frequency response having converged to within a user-defined tolerance. We do not guarantee accuracy in any absolute sense, since we assume that a high-order "truth model" is not available or is too difficult to produce, and thus we lack reference data for comparison.

We also do not guarantee that a lower-order model cannot be found to meet the same accuracy specification that the FD-MODANS-produced model satisfied. While augmenting the order of the model, FD-MODANS increases components' ranks by identifying the component that causes the largest change in the frequency response. Once the algorithm has committed to increasing the rank of a component, it will not backtrack and decrease it later. Thus, FD-MODANS does not conduct an exhaustive search. Conceivably, later increases in other components' ranks might make the rank of some component unnecessarily high. We do not have an example of this, but as yet lack a conclusive proof of the ability of FD-MODANS to synthesize minimal-order system models. Lacking such a proof, one might make a check at the end of the component-rank iterations by reducing the rank of each component to see if the frequency response remains within tolerance; however this would seem to be overly cautious.

Finally, we acknowledge that there may be cases where adding a set of significant nonlinearities to a system model might change the appropriate component ranks needed to satisfy the accuracy metric being used in the two searches. For example, the existence of significant stiction may exacerbate the resonant behavior of a flexible shaft, necessitating the use of a higher-rank submodel for that component. One way to address this problem would be to repeat the algorithm in Section 2 except for reversing the order of the two searches (determine "significant nonlinearities" first). If the results are the same, then we would be substantially more confident (but still not sure) that the resulting model is "acceptably-accurate, suitably-low-complexity". If the results are different, then we might suggest using the maximum component ranks and the union of the set of nonlinearities identified as needed in the two exercises.

Despite these open questions and lack of definitive answers, we feel confident that MODAS in general and FD-MODANS in particular represent a powerful, systematic attack on the difficult problem of producing accurate, low-complexity models. While we cannot guarantee "optimality", such an approach should be a valuable addition to an engineer's repertoire of techniques or to an automated modeling environment.

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