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On-line Optimal Power Flow by Energy Loss Minimization

S. Salamat Sharif James H. Taylor Eugene F. Hill
Electrical Engineering Department
University of New Brunswick
Fredericton, New Brunswick, CANADA E3B 5A3

Abstract: *A method for on-line application of optimal reactive power dispatch based on total energy loss minimization (ELM) is presented. In this approach the total energy loss from the present instant over the next hour is minimized. The method uses the load forecast during this period. All the continuous and discrete control variables are adjusted on an hourly basis. During the hour, any voltage constraint violations are removed by adjusting the vars/voltages of generators every 15 minutes.*

A detailed study of a sample network is given. The ELM and power loss minimization (PLM) methods are compared by using the sample network. As seen in simulation results, the voltage profile from the ELM method is more satisfactory than that from the PLM method. In addition, the total energy loss during the specified hour which is found from the ELM method is lower than that from the PLM method. Finally, the proposed method is more likely to find feasible solutions, while the PLM method can have difficulty doing so.

Keywords: optimal reactive power flow, optimization methods, power loss minimization

1 Introduction

The problem of reactive power dispatch has attracted a lot of attention during the last three decades [1]-[9]. The main objectives of this study address three important aspects in power systems. The first objective is to maintain the voltage profile of the network in an acceptable range ($V_{min} \leq V \leq V_{max}$) [1]. The second objective is to minimize the total power loss of the network while satisfying the first objective [2]. The third objective is to avoid excessive adjustment of the system configuration, i.e., to minimize transformer tap setting changes and generator var source switching.

The objective of acceptable voltage profile can be studied for steady-state conditions as well as contingency conditions [3]. During steady-state conditions, total power loss can be minimized by finding the optimal reactive power dispatch. The control variables for this study include vars/voltages of generators, the tap ratios of transformers and reactive power generation of var sources. The constraints include the var/voltage limits of generators, the voltage limits of load buses, tap ratio limits, var source limits, power flow balance at all buses, and security constraints.

This problem has been solved by linear programming, parametric linear programming, successive linear programming, quadratic programming, gradient method, and nonlinear quadratic programming [4].

Only a few papers have addressed the on-line application of optimal reactive power flow (ORPF) [5]-[8]. In the on-line application of ORPF, different objective functions can be minimized. These objective functions include minimization of total power loss, and minimum control shift or minimum number of control shifts for removing constraints violations. In the minimum control shift case, the summation of the magnitudes of the changes in the control variables is considered, while in the minimum number of control shifts the number of control variables which should be changed is minimized. In either case, the objective is to remove voltage constraint violations.

The objective mentioned in [5] and [8] is to minimize the total power loss while satisfying all the existing constraints. These two methods, from the application point of view, are quite different. The method in [5] minimizes the power loss exclusively on the basis of load forecast, while in [8] the power loss is minimized on the basis of real-time load conditions. The problem with the first method is that it doesn't consider the on-line load conditions in performing its off-line study for power loss minimization. In the second method the power loss is minimized without considering the load forecast during the next hour. In the method which is proposed in this paper, the total energy loss is minimized on the basis of the present load conditions and the load forecast during the next hour.

The rest of this paper is organized as follows: in section 2 of this paper the general idea of ELM is presented. In section 3 the formulation of the problem is given. In section 4 the application of the new method to a sample network is addressed. We conclude with a short summary of our results and their significance.

2 Energy Loss Minimization

In this section the two methods which are given in [5] and [8] are discussed. Afterward, the new proposed method and its advantages are given.

The method used in [5] consists of two stages, secondary and tertiary voltage regulation. In the first stage, secondary voltage regulation, an off-line optimal reactive power flow program minimizes the power loss on the basis of the load forecast for the most representative time intervals of the daily load diagram. The off-line study is run half an hour up to a few hours or a day in advance. In this study, the voltage of pilot nodes (important buses) and the reactive power generation of each section are specified. In the second stage, tertiary voltage regulation, the voltage deviation of pilot nodes and the deviation of the var generation of each area from their optimal values is minimized in an on-line study . The on-line study is done continuously (each 0.5 sec) and some of the control variables are adjusted on the basis of this study.

The method described in [8] divides the control variables into two sets, discrete control variables (capacitors, reactors, and tap ratios) and continuous control variables (vars/voltages of the generators). The optimal reactive power flow program is run with two different objectives: 1) power loss minimization, and 2) removing voltage constraint violations. The first run has a cycle of one hour, and uses all the control variables. The second run has a cycle of 15 minutes; in removing voltage constraint violations only the continuous control variables are employed. At each 15 minutes, the ORPF is run with minimum control shift or minimum number of control shifts for removing the constraints violations.

In the proposed method, the total energy loss is minimized on the basis of on-line load conditions and the load forecast during the next hour. This method minimizes the energy loss for the next hour by using all the control variables (continuous and discrete). The voltage constraint violations are removed by running the ORPF program every 15 minutes; in these runs only continous control variables are allowed to vary. The proposed method, if accompanied with an accurate load forecast for the next hour, gives not only a better voltage profile, but also lower energy loss than that given in [8], and a greater likelihood of finding feasible solutions.

3 Problem Formulation

The formulation of this problem is explained in two stages. In the first step, the minimization of total power loss is addressed. In the second stage the formulation for minimization of total energy loss is given.

3.1 Formulation for Power Loss Minimization

A decoupled model optimal power flow (OPF) formulation has been used to simplify the application of the proposed method. In this model the active power generation of all the generators except at the slack bus are constant. The objective function is total power loss. The control variables include generator vars/voltages, the tap ratios of tap-changing transformers, and reactive power generation of var sources (capacitive or inductive). The constraints of this problem are voltage limits on the load buses, var/voltage limits of the generators, tap ratio limits, and var source limits. Some other constraints such as power flow limits of branches, etc. can also be included. This objective function is given by:

$$\min\ P_L = \frac{1}{2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} g(i,j)[V_i^2+V_j^2-2V_iV_j cos(\delta_i-\delta_j)] \tag{1}$$

where:

P_L = total power loss of transmission lines,
N = number of buses,
$g(i,j)$ = the conductance of the line between bus i and bus j for $i \neq j$,
V_i = voltage magnitude at bus i,

δ_i = voltage phase angle at bus i.

The constraints of this problem are similar to the constraints which are given in equations (3-5) (for energy loss minimization) with the index k removed. This problem can be solved by any ORPF package.

3.2 Formulation for Energy Loss Minimization

Before stating the formulation for the energy loss minimization method, the strategy of control variable settings should be explained. In the setting procedure, each hour is divided into four 15 minutes periods. All the control variables (continuous and discrete) are set at the beginning of each hour. At the beginning of periods $2-4$ only the continuous control variables may be adjusted. This two-tier strategy is imposed to reduce the number of physical plant changes and thereby avoid unnecessary equipment wear and life-cycle costs.

In the formulation of the problem the total power loss (equation (1)) is replaced by the total energy loss over a given one hour interval. For this calculation, it is assumed that the bus loads remain constant during each 15-minute period. The bus voltages during each period are also constant, but differ from one period to the next. Therefore, by calculating the power loss at the beginning of the k^{th} period, P_L^k, the energy loss can be approximated by multiplying P_L^k by 1/4 hour (15 minutes) and summing. Therefore, the energy loss objective function can be formulated as:

$$\min E_L = \sum_{k=1}^{4} P_L^k/4 \tag{2}$$

where E_L is the energy loss for one hour. P_L^k for each period can be evaluated from equation (1) with the values of voltages for period k. The constraints of the problem are as follows:

(a) Continous-variable constraints

$$\begin{aligned} V_l^{min} &\le V_l^k \le V_l^{max} && \text{for } k=1,2,3,4 \\ V_g^{min} &\le V_g^k \le V_g^{max} && \text{for } k=1,2,3,4 \\ Q_g^{min} &\le Q_g^k \le Q_g^{max} && \text{for } k=1,2,3,4 \end{aligned} \tag{3}$$

(b) Discrete-variable constraints

$$\begin{aligned} Q_C^{min} &\le Q_C \le Q_C^{max} \\ Q_L^{min} &\le Q_L \le Q_L^{max} \\ T^{min} &\le T \le T^{max} \end{aligned} \tag{4}$$

(c) Power flow equality constraints

$$\begin{aligned} \Delta P^k &= 0 && \text{for } k=1,2,3,4 \\ \Delta Q^k &= 0 && \text{for } k=1,2,3,4 \end{aligned} \tag{5}$$

where the index k denotes values that are adjusted every 15 minutes, and:

V_l^k = voltage magnitude at load buses
V_g^k = voltage magnitude at generator buses
Q_g^k = var generation of generators
Q_C = var generation of shunt capacitors,
Q_L = var generation of shunt inductors,
T = transformer tap ratio,
ΔP^k = active power mismatch at all buses
ΔQ^k = reactive power mismatch at all buses.

We note again that the discrete control variables, Q_C, Q_L, T, are kept constant during the hour, to minimize equipment wear; for this reason, they are designated without the k index.

This problem can also be solved by any ORPF package. This problem is solved at the beginning of each hour for the minimization of energy loss during the next hour; discrete and continuous variables are set at the beginning of the hour from the results of the ELM run. The continuous control variables are set at each 15 minutes by the ORPF with the objective of minimum control shift for removing the violations of bus voltages constraints or for power loss minimization if violations do not exist.

4 System Studies

The Ward and Hale 6-bus system (Fig. 1) [9] and modified IEEE 30-bus system have been studied. The detailed study of the 6-bus system is given in this section. The line data and the bus data of the 6-bus system are given in Tables 1 and 2 respectively. The data in Table 2 corresponds to the full load conditions. The limits of bus voltages, tap ratios, shunt capacitors, and generators vars are given in Table 3. This system is studied for comparing the power and energy loss minimization methods.

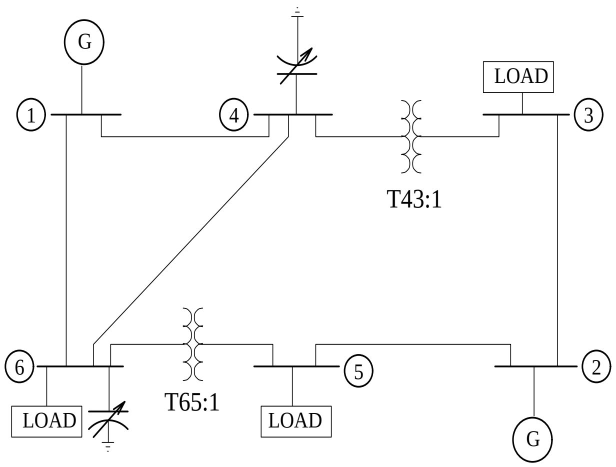

Figure 1: Ward-Hale 6-bus System

Table 1: Line data of the 6-bus system on 100 MVA base

Line Number	Bus Number		Impedance (per unit)		Tap Ratio
	From	To	R	X	
1	1	6	0.123	0.518	–
2	1	4	0.080	0.370	–
3	4	6	0.097	0.407	–
4	6	5	0.000	0.300	1.025
5	5	2	0.282	0.640	–
6	2	3	0.723	1.050	–
7	4	3	0.000	0.133	1.100

Table 2: Bus data of the 6-bus system in full load conditions

Bus Number	Voltage		Load	
	V (per unit)	δ (deg)	P (MW)	Q (MVAR)
1	1.04	0.0	0.0	0.0
2	1.11	-6.6	0.0	0.0
3	0.85	-14.0	55.0	11.0
4	0.95	-10.1	0.0	0.0
5	0.92	-13.6	30.0	18.0
6	0.91	-12.8	50.0	10.0

Table 3: Low and high limits of variables

Variable	Limits	
	Low	High
V_{g1}	1.00	1.10
V_{g2}	1.10	1.15
V_{l3}	0.90	1.00
V_{l4}	0.90	1.00
V_{l5}	0.90	1.00
V_{l6}	0.90	1.00
Q_{g1}	-20.0	100.0
Q_{g2}	-20.0	100.0
Q_{C4}	0.0	15.0
Q_{C6}	0.0	30.0
$T43$	0.9	1.10
$T65$	0.9	1.10

4.1 Power Loss Minimization

By running the ORPF program the total power loss of the 6-bus system (Fig. 1) under full load conditions is minimized. The objective function and constraints which are given in equations (1) and (3-5) (without the k index) have been used. The total power loss for the initial state of this system is 10.90 MW. The power loss after running the ORPF is given in period 1 of Table 5. The values of all variables after running ORPF program are given in period 1 of Table 6. As mentioned before, PLM does not consider forecast load variation so the "period 1" values are assumed to pertain to the entire hour.

The actual load conditions are assumed to be constant during each 15-minute period. The load diagram for all the four periods is given in Table 4. As mentioned in section 3.2, the only control variables that can be varied during the hour are Q_{g1} and Q_{g2}. Therefore, in the beginning of periods 2-4, the ORPF is run with only these two control variables used to remove any voltage constraint violations. Due to the variation of all variables during the hour, power loss also changes, as shown in Table 5. The total energy loss achieved by PLM can be determined as:

$$E_L = (P_L^1 + P_L^2 + P_L^3 + P_L^4)/4 = 6.70\ MWH.$$

The voltage magnitude and control variables which are calculated with the ORPF program for all the four periods are given in Table 6.

Other load diagrams were also tested. One of the problems which was encountered during these studies is the infeasibility of solutions. It is possible that the bus loads in subsequent periods differ too much from those in period one. In these cases the ORPF

Table 4: The load diagram for the four periods

Load period	Period 1		Period 2		Period 3		period 4	
Bus Number	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)
3	55.0	11.0	50.0	10.0	45.0	9.0	44.0	9.0
5	30.0	18.0	26.0	16.0	23.0	14.0	22.0	14.0
6	50.0	10.0	45.0	9.0	40.0	8.0	40.0	9.0

Table 5: Power loss for the four periods from the PLM method in MW

Time Period	1	2	3	4
Power loss	8.47	6.93	5.79	5.62

program can't find any feasible solution by only adjusting continuous control variables, due to limited control action. However, the ELM method doesn't have this problem, since it uses the load forecast to set the discrete variables to values that suitably anticipate the expected load variations.

4.2 Energy Loss Minimization

The 6-bus system with the same load diagram (Table 4) is used for the minimization of energy loss. The energy loss is minimized by employing equation (2) under the constraints listed in equations (3-5). The total energy loss found in this method is equal to 6.59 MWH. This value is less than the energy loss found in the PLM method (6.70 MWH, see previous discussion). The bus voltages and control variables for this method for all the four periods are given in Table 7. By comparing the results of Tables 6 and 7 and other simulation studies, the following observations can be made:

1. The voltage profiles from energy loss minimization are more favorable (more nearly constant) than those from the PLM method. Thus, the stability of the system from a voltage point of view is much higher in the ELM method.
2. The energy loss in the ELM method for the above example is 1.6% less than that from the PLM method.
3. The advantages of the ELM method are more apparent when the load changes significantly. In cases where the load diagram is almost flat during the hour, ELM gives slightly better results.

Table 6: Bus and control variables from the PLM method

Variable	Time periods			
	1	2	3	4
V_{g1}	1.07	1.04	1.00	1.00
V_{g2}	1.11	1.10	1.10	1.10
V_{l3}	1.00	0.99	0.98	0.98
V_{l4}	1.00	0.99	0.97	0.97
V_{l5}	1.00	1.00	1.00	1.00
V_{l6}	1.00	1.00	0.98	0.98
Q_{g1}	19.37	11.49	0.26	0.56
Q_{g2}	9.15	7.55	10.60	10.48
Q_{C4}	10.0	10.0	10.0	10.0
Q_{C6}	30.0	30.0	30.0	30.0
$T43$	0.98	0.98	0.98	0.98
$T65$	1.05	1.05	1.05	1.05

4. In cases where the load changes during the next hour are large, coming to a feasible solution by the PLM method is not always possible. In these cases the ELM method is more likely to find a feasible solution. The reason is that the load conditions for all four periods have been considered in the load flow equations which are enforced as constraints in the ELM formulation. Therefore, the optimal values of the discrete control variables obtained using the ELM method can usually handle the load changes predicted by the load forecast. The only circumstance when ELM cannot find a feasible solution is when the load change is so large over the hour that discrete control variables must be adjusted to avoid voltage violations.

Table 7: Bus and control variables from the ELM method

Variable	Time periods			
	1	2	3	4
V_{g1}	1.07	1.06	1.04	1.04
V_{g2}	1.12	1.10	1.10	1.10
V_{l3}	1.00	1.00	1.00	1.00
V_{l4}	1.00	1.00	1.00	1.00
V_{l5}	0.99	0.99	1.00	1.00
V_{l6}	1.00	1.00	1.00	1.00
Q_{g1}	21.32	16.12	7.74	8.12
Q_{g2}	13.74	9.17	8.75	8.63
Q_{C4}	9.0	9.0	9.0	9.0
Q_{C6}	25.0	25.0	25.0	25.0
$T43$	0.98	0.98	0.98	0.98
$T65$	1.03	1.03	1.03	1.03

5 Conclusion

A new method for on-line optimal reactive power dispatch is proposed. The method minimizes the total energy loss during the next hour, while keeping the voltage profile within an acceptable range. By comparing simulation results, it is found that ELM gives a better voltage profile than that from the PLM method; in the example above, the method produced a nearly constant voltage profile during the specified hour. In addition, the energy loss was reduced at the same time, with the same number of discrete control variable changes as used by PLM.

The ELM method is based on the recognition that certain control variables should not be adjusted too often, as this may cause wear and shorten the life of the corresponding equipment. In this example, there are two categories of control variables established; the discrete and continuous control variables are adjusted at each hour, while during the hour, only the continuous control variables are adjusted. The number of categories could be increased, and the frequency of adjustment modified, to fit differing circumstances.

The probability of finding an infeasible solution with the ELM method is much lower than the PLM method. This advantage of the ELM method is obtained by considering the load forecast and making sure that anticipated load changes during the next hour can be accommodated.

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