NONLINEAR CONTROL SYSTEM DESIGN BASED ON QUASILINEAR SYSTEM MODELS

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Abstract: In this paper, we report on recent progress in developing nonlinear control system design techniques based on sinusoidal-input describing function (SIDF) methods. This work includes the development of a new nonlinear compensator synthesis approach, and its application to a single-axis servo design problem of the sort encountered in robotics. This approach is capable of treating nonlinear systems of a very general nature, with no restrictions as to system order, number of nonlinearities, configuration, or nonlinearity type. The end result is a closed-loop nonlinear control system that is relatively insensitive to reference input amplitude.

1. INTRODUCTION

This paper treats recent work in the development of nonlinear control system design techniques based on sinusoidal-input describing function (SIDF) methods. The basis of this work has been established in earlier publications: Taylor (1,2) outlines the motivation for using a modern algebraic SIDF approach for control system design and establishes a systematic plan of attack, and both (2) and Taylor and Strobel (3) compare SIDF models with those based on the random-input DF technique. In addition, (3) develops one nonlinear compensator design method fully and demonstrates it by applying it to a significant nonlinear control design problem in robotics. The nonlinear compensator obtained using the approach in (3) is a PID compensator in series with a single static nonlinear operator that is directly synthesized; the application shows that the nonlinear compensator is capable of correcting instabilities caused by the amplitude dependence of the nonlinear plant without sacrificing performance.

The work described in this paper includes a new nonlinear compensator synthesis approach and its application to a nonlinear servo design problem from robotics. The synthesis technique is based on a set of amplitude-dependent SIDF models of the nonlinear plant. An intermediate step is the design of a linear compensator set based on these models; final synthesis of the nonlinear compensator is accomplished by SIDF inversion to determine the required compensator nonlinearities. The major extension in comparison with the research in (3) is that the compensator so obtained is fully nonlinear, i.e., there is a nonlinear operator associated with each term (proportional, integral, derivative) in the compensator. This approach can be extended readily to include other compensator types, e.g., lead/lag.

This approach is capable of treating nonlinear plants of a very general type, with no restrictions as to system order, number of nonlinearities, configuration, or nonlinearity type. Based on these results, the use of SIDF-based nonlinear compensator design methods should be substantially better understood and easier to apply. It is believed that this design approach will provide a framework for further developments in the realm of compensator design for nonlinear systems.

2. NONLINEAR COMPENSATOR DESIGN APPROACH

First, it is important to state the underlying premises of the SIDF design approaches that we have been developing (1-3):

a. The nonlinear system design problem being addressed is the synthesis of compensators that are effective for plants having frequency-domain input/output models that are sensitive to input amplitude (e.g., for plants that behave very differently for "small" and "large" input signals).

b. The primary objective of compensator design is to arrive at a closed-loop system that is as insensitive to input amplitude as possible.

This encompasses a limited but important set of problems, for which gain-scheduled compensators cannot be used and for which the desired response is similar to that provided by a linear system (in the sense of b. above).

The generation of sinusoidal-input describing function (SIDF) models that provide an amplitude-dependent input/output (I/O) characterization for a nonlinear plant has been dealt with in detail in (3). There are two basic approaches: solving the nonlinear algebraic equations derived from the principle of harmonic balance, and simulation coupled with fourier analysis.

The first method is not easy to apply, especially if it is desired to develop a general package that substitutes the appropriate SIDFs into the nonlinear algebraic equations and solves them. Also, the assumption is made that the input to each nonlinearity is approximately sinusoidal (refer to Atherton (4) or Gelb and Vander Velde (5)), which may leave the analysis open to question. However, there is an advantage to this approach: the SIDF model is obtained in a form that lends itself to further analysis such as finding the roots of the quasilinear characteristic equation.

The second technique is easier to implement, given a good package for integrating nonlinear differential equations, and avoids the need to justify the assumption that the inputs of every nonlinearity are nearly sinusoidal - there is no such assumption made using simulation. The only assumption is that a frequency-domain amplitude-dependent I/O model provides a good representation of the behavior of a nonlinear plant for control system design; we have discussed that issue in (2,3). In our opinion, while SIDF models are not exact, a set of SIDF models covering the range of input amplitudes that will be encountered provides an excellent basis for "robust design", in the sense that the sensitivity of the plant behavior to input amplitude is one of the most important issues in robustness, and the SIDF I/O model is the least conservative model that accurately takes this factor into account.

We have recently extended the nonlinear simulation package SIMNON (Elmqvist (6)) to perform SIDF I/O model generation for nonlinear system models. The basic idea is to drive the nonlinear plant with a sinusoid of the desired amplitude for the range of frequencies of interest, and evaluate Fourier integrals as the simulation proceeds. The simulation for a given frequency is stopped when the Fourier integrals have converged, and the I/O model is evaluated; for details, refer to Taylor (7). Henceforth, we will call this extended simulation package FRSIMN, and use the notation $G(j\omega, a)$ to designate the SIDF I/O model generated by driving a nonlinear system with the input $u(t) = a \cos(\omega t)$.

The method we have developed to synthesize a nonlinear compensator for a nonlinear plant proceeds as follows:

a. Select sets of input amplitudes $\{a_i\}$ and frequencies $\{\omega_k\}$ that cover the range of plant input amplitudes and frequencies of interest,

b. use FRSIMN to generate the corresponding set of I/O models $\{G_i(j\omega_k, a_i)\}$, or more compactly $\{G_i(j\omega, a_i)\}$,

c. select one of the I/O models $G^*(j\omega, a^*)$ to be the nominal case for which a preliminary linear compensation will be designed ($a^* \in \{a_i\}$),

d. design a PID compensator $C^*(j\omega)$ for the nominal model,

e. add a model of the PID compensator $C^*(j\omega)$ in series with the nonlinear plant model,

f. use FRSIMN with the compensator input signal $e = e^* \cos(\omega t)$, for a value of e^* chosen to be consistent with a^* and the PID gain near the crossover frequency, denoted $|C_{co}|$, i.e., $e^* = a^*/|C_{co}|$, and generate the SIDF I/O relation for the PID in series with the plant, denoted $CG^*(j\omega, e^*)$,

g. select a set of compensator input amplitudes $\{e_i\}$ that corresponds to the set $\{a_i\}$ in the same way, i.e., $e_i = a_i/|C_{co}|$,

h. design a set of linear compensators $\{C_i(j\omega)\}$ so that the error

$$E(j\omega) = 1 - C_i(j\omega)G(j\omega, e_i|C_i(j\omega)|)/CG^*(j\omega, e^*) \quad (1)$$

over the frequency set $\{\omega_k\}$ is minimized in the mean square sense; this yields a set of PID parameters for each value of e_i, denoted $\{K_P(e_i)\}$, $\{K_I(e_i)\}$, $\{K_D(e_i)\}$,

i. use the sets of PID parameters $\{K_P(e_i)\}$, $\{K_I(e_i)\}$, $\{K_D(e_i)\}$ with SIDF inversion to synthesize the nonlinearities $f_P(e)$, $f_I(e)$, $f_D(e)$,

j. develop a nonlinear PID compensator model that incorporates these nonlinear functions, and

k. simulate the closed-loop system comprised of the nonlinear compensator and the nonlinear plant to validate the design.

The first three steps require only a knowledge of the expected operating conditions of the nonlinear plant

frequency-domain I/O models. The design of a PID compensator based on G^* is a straight-forward application of classical control system design methods. Steps e and f involve adding the compensator to the simulation model and using the same package to obtain the frequency-domain I/O model of the compensated plant. The steps d - f may require iteration, in the sense that CG^* is not the same as $C^*(j\omega)G^*(j\omega, a^*(\omega))$, because the plant input amplitude is not constant; rather, it is $a(\omega) = e^*|C^*(j\omega)|$, so the initial design objectives (e.g., gain and phase margins) may not be met. Once the designer has obtained a satisfactory CG^*, steps g - i result in the direct synthesis of a nonlinear compensator that provides the best fit possible to CG^*, for the appropriate set of compensator input signal amplitudes $\{e_i\}$.

The most difficult step in nonlinear compensator synthesis is h. In effect, the original set of I/O models $\{G(j\omega, a_i)\}$ defines a complex-valued "surface" above the (ω, a) plane, which we call the G-surface; the synthesis of the set $\{C_i\}$ requires evaluating $C_i(j\omega)G(j\omega, a(\omega))$ where $a(\omega) = e_i|C_i(j\omega)|$, which implies that the values of $G(j\omega, a(\omega))$ are not immediately available from the original SIDF I/O model data. It would be possible, in principle, to obtain the necessary data by further use of FRSIMN; however, since these calculations take place inside a mean square error minimization algorithm, this would be very costly. Instead, we obtain the required data from the original G-surface data points by interpolation. The solution for the parameter set $\{K_P(e_i), K_I(e_i), K_D(e_i)\}$ that minimizes the error (1) in the mean square sense defines one PID compensator

$$C_i(s) = K_{P,i} + K_{I,i}/s + K_{D,i}s \quad (2)$$

in the set $\{C_i\}$; the error minimization is obtained using MINPACK (8).

The final synthesis step - SIDF inversion - amounts to determining the three nonlinearities whose SIDFs fit the amplitude-dependent gain data generated in step h. This we did with a routine which adjusts the parameters of a rather general piece-wise linear function until the corresponding SIDF has minimum mean square error with respect to the e-dependent gains obtained in h; again, we use MINPACK for this process. The nonlinearity is portrayed in Fig. 1; the four parameters adjusted in the fitting procedure are the two slopes S_1, S_2, break-point δ, and discontinuity D. This routine is also described in further detail in (7).

It is worth noting that SIDF inversion may require considerable insight on the part of the designer, because SIDFs cannot fit arbitrary gain/amplitude data: The selection of the set $\{e_i\}$ must be made so that the values are "well spread out", because SIDFs tend to be rather smooth and thus cannot fit substantial variations of gain over a closely-spaced set of amplitudes meaningfully. Also, the designer must be aware that rather different nonlinearities may have very similar SIDFs over a substantial range of input amplitudes (cf. the SIDFs for a saturation and for a relay with dead-zone, (5) Figs. B.1 and B.2); this may be confusing, but in fact gives the designer certain freedom in the final selection of the appropriate nonlinearity to implement.

This design approach is described in more detail in Taylor and Strobel (9), which is a companion to this paper.

3. APPLICATION TO A POSITION SERVO DESIGN PROBLEM

The nonlinear plant for which we wish to design a controller is depicted in Fig. 2. The servo motor saturation is modeled by a substantial reduction in gain; specifically, the parameters are $m_1 = 5.0$ Nm/v, $\delta = 0.5$ v, $m_2 = 1.0$ Nm/v. "Stiction" is modeled by the relation

$$T_m = \begin{cases} T_e - f_v\dot{\theta} - f_c \text{sign}(\dot{\theta}), & |T_e| > f_c \\ T_e - f_v\dot{\theta} - f_c \text{sign}(\dot{\theta}), & \dot{\theta} \neq 0.0 \\ 0.0, & |T_e| < f_c \text{ and } \dot{\theta} = 0.0 \end{cases} \quad (3)$$

where $f_v = 0.1$ Nm-s/rad, $f_c = 1.0$ Nm; the moment of inertia is $J = 0.01$ kg-m^2.

The nonlinear compensator synthesis procedure proceeded exactly as outlined in Section 2; the highlights are as follows:

a. The result of generating amplitude-dependent SIDF I/O models is shown in the Bode plots of Fig. 3. Note that the magnitude varies over a range of about 18 dB, and that the phase differs by as much as 50 deg.; this implies that it would be difficult to design a linear compensator that can achieve similar response to different amplitude input signals.

b. The Bode plots of CG^* are portrayed in Fig. 4; the linear PID was designed to achieve about 65 deg of phase margin at a crossover frequency of 37 rad/s.

c. Bode plots of $\{C_i(j\omega) * G(j\omega, a(\omega))\}$ are depicted in Fig. 5. Note that using the interpolation algorithm over the G-surface as described in step h above yields a linear PID compensator set $\{C_i\}$ that successfully achieves the objective of minimizing the mean square error with respect to CG^*. The PID compensator gains and corresponding numerator zeroes are given in Table 1; note that the zeroes for various values of e_i differ considerably, implying that this design procedure does more than merely attempt to cancel the saturation - rather, it provides substantially different frequency-domain behavior for different input amplitudes, to compensate for both saturation and stiction.

d. The nonlinearities obtained via SIDF inversion are shown in Fig. 6. Again, the different breakpoint values and slope ratios show that the compensator is not merely a saturation-inverter.

e. Bode plots depicting SIDF I/O models for the nonlinear PID compensator $(N(j\omega, e))$ and plant are provided in Fig. 7. While the spread of these plots is greater than that shown in Fig. 5, which can be accounted for by the approximate interpolation in step h and SIDF fitting in step i, the closeness to the original design objective in Fig. 4 is still satisfying. The closed-loop system comprised of the nonlinear PID compensator and plant is portrayed in Fig. 8.

f. The final validation of this design is given by simulating the performance of this system for step inputs. For the sake of comparison, the same test is applied to the closed-loop system having the linear PID compensator in the forward path. We see in Fig. 9 (a, b) that the effect of saturation and stiction are quite striking in the case with a linear compensator, while nonlinear compensation substantially reduces the effect of input-amplitude dependence. For example, the percent overshoot ranges from 12 to 57% for the linear PID, while the nonlinear PID keeps the percent overshoot in the range 32 to 38%. Also, there is severe stiction evident in the linear case; the nonlinear PID almost completely eliminates it.

Table 1. Linear PID Design Parameters

e_i	K_P	K_I	K_D	z_1	z_2
0.05	6.00	66.0	.0404	- 12.0	- 137.
0.065	5.41	59.7	.0569	- 12.7	- 82.3
0.08	4.89	53.0	.0605	- 12.9	- 67.9
0.16	4.12	41.7	.0649	- 12.6	- 50.8
0.32	6.02	57.9	.145	- 15.1	- 26.4
0.64	6.96	63.3	.195	- 17.9 +/- 2.5j	
1.28	7.40	64.0	.213	- 16.2	- 18.5
2.56	7.60	56.1	.220	- 10.7	- 23.9

4. SUMMARY AND CONCLUSIONS

The method outlined in Section 2 is a specific realization of the basic concept of using SIDF I/O models as the basis for nonlinear compensator design proposed in (1, 2). Based on the example presented in Section 3, we believe that this approach shows considerable promise in dealing with one of the more difficult problems in nonlinear systems design - the design of compensators to correct for the amplitude-dependence of nonlinear plants.

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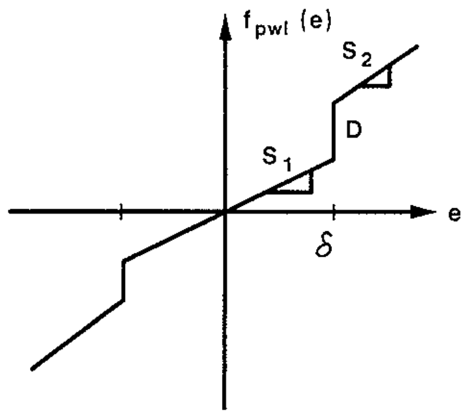


Figure 1. Nonlinearity Class for SIDF Inversion

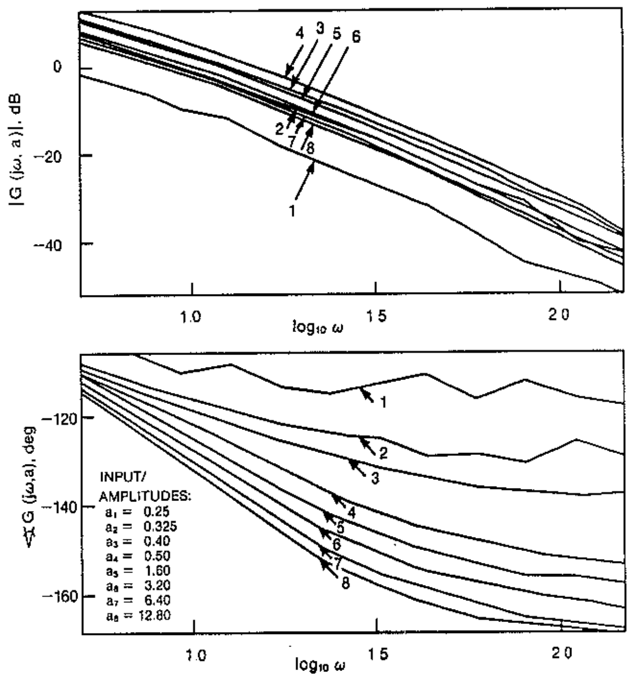


Figure 3. Plant SIDF I/O Models $\{G(j\omega, a_i)\}$

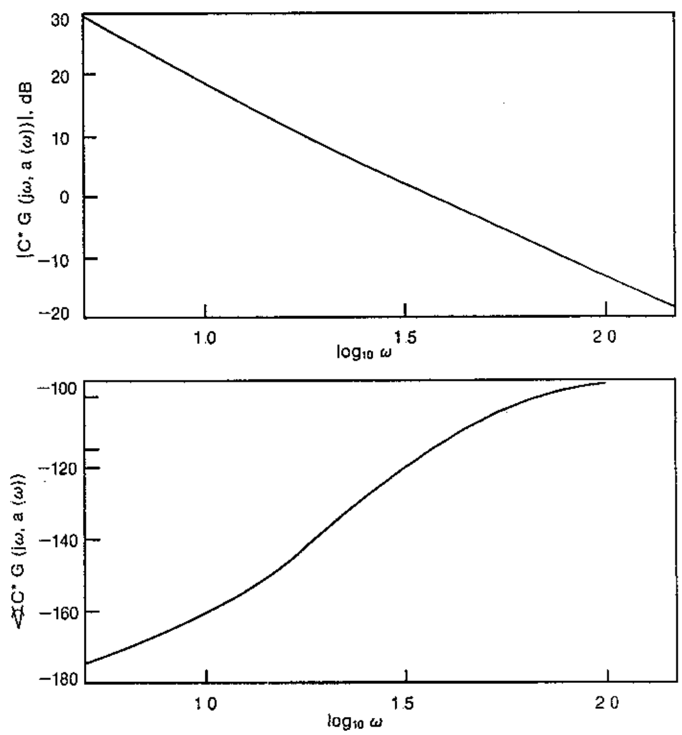


Figure 4. Nominal Design Objective $CG^*(j\omega, e^*)$

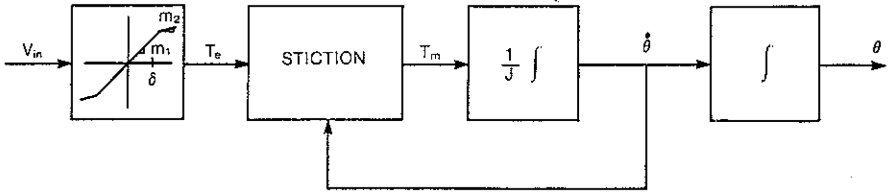


Figure 2. Position Servo Model Schematic

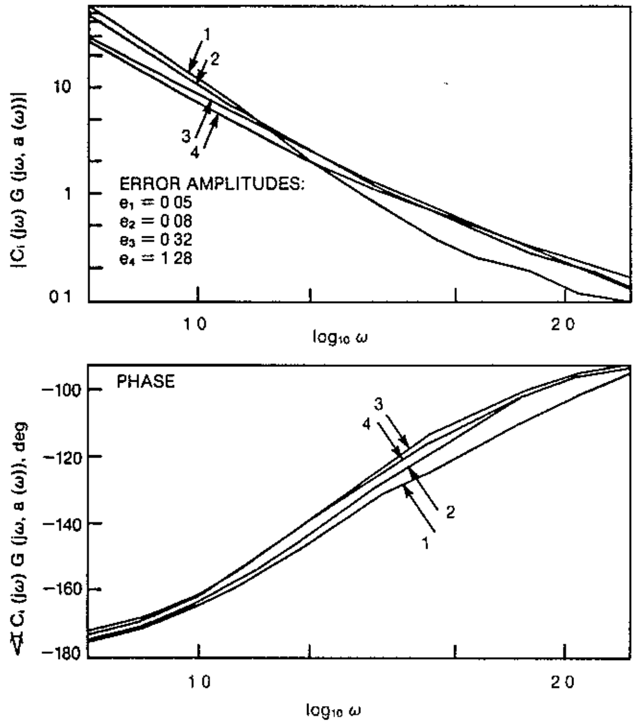


Figure 5. $C_i(j\omega)G(j\omega,\ a(\omega))$

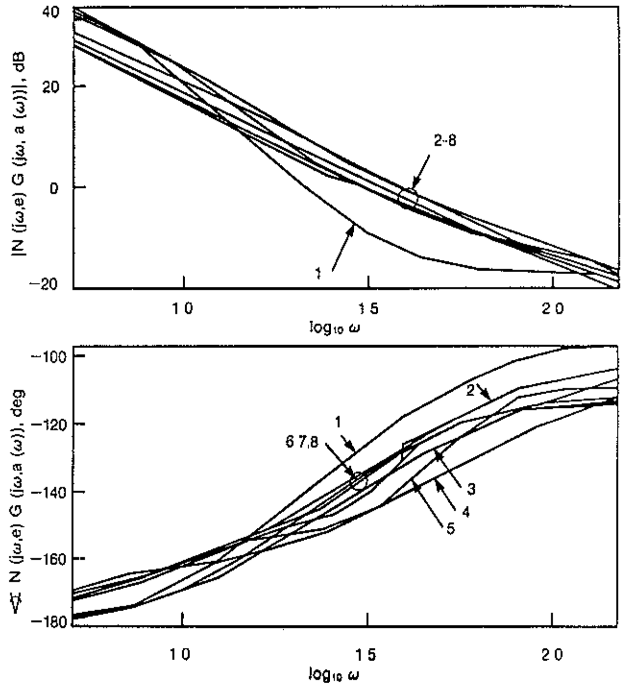


Figure 7. SIDF I/O Model for Nonlinear PID and Plant

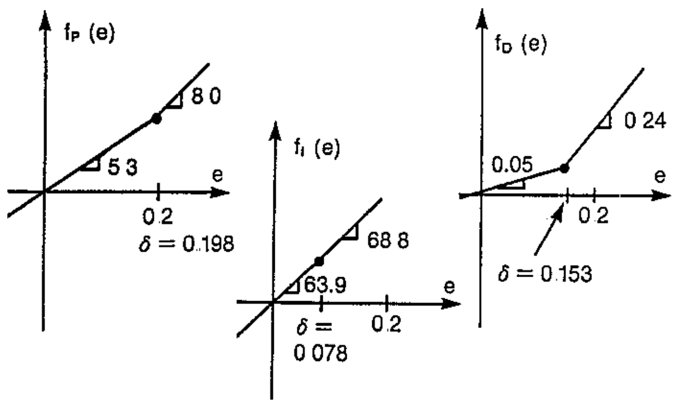


Figure 6. Controller Nonlinearities Synthesized by SIDF Inversion

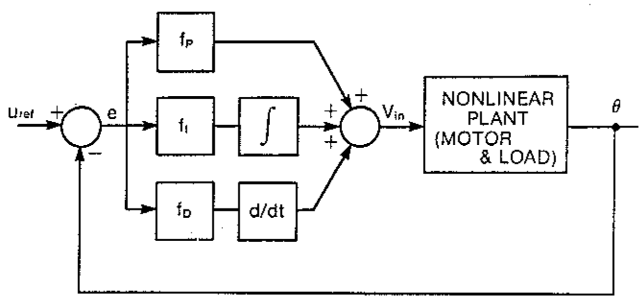
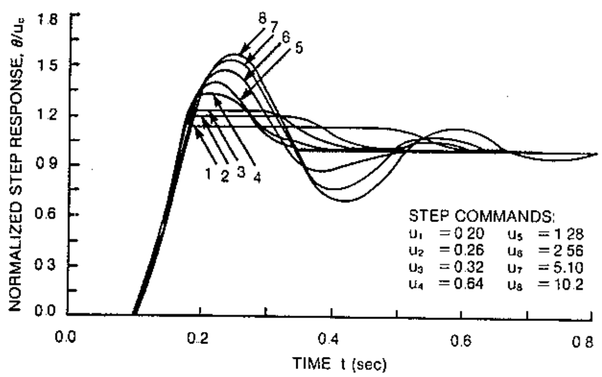
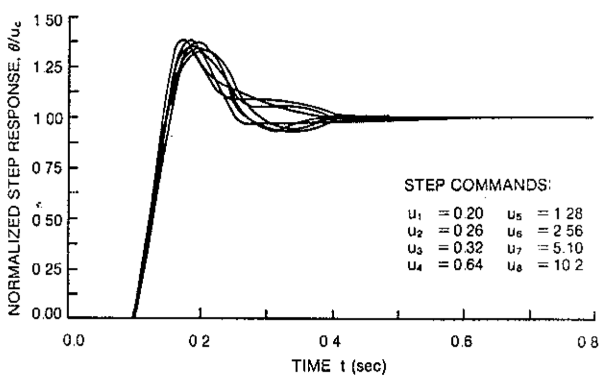


Figure 8. Final Nonlinear Control System



(a.) Linear PID and Nonlinear Plant



(b.) Nonlinear PID and Plant

Figure 9. Step Response Plots