

# Dynamic On-Line Energy Loss Minimization

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**Abstract:** *An efficient method for minimization of energy loss over intervals of time is presented. The proposed method uses different loading conditions during each given time interval instead of one single snapshot of the network. The given interval is divided into several shorter periods; the first load condition is a current snapshot and subsequent ones are forecasted.*

*By increasing the number of periods or load profiles, the dimension of problem rises substantially. This difficulty is handled by using the Generalized Benders Decomposition technique. Energy loss minimization and power loss minimization are compared by simulated application to the modified IEEE 30-bus system and the New Brunswick Power network. As seen in these simulation results, the proposed method not only improves the voltage profile, but it also decreases the total energy loss over the given time interval.*

**Keywords:** optimal reactive power flow, energy loss minimization, power loss minimization, optimization methods

## 1 Introduction

Optimal power flow (OPF) is a major issue in operation of power systems. This problem can be divided into two subproblems, MW and Mvar dispatch. In many cases, the optimal reactive power flow (ORPF) problem is considered independently [1], and in some others it is combined with MW dispatch [2]. ORPF addresses three important objectives: a) to keep the voltage profile in an acceptable range, b) to minimize the total transmission energy loss, and c) to avoid excessive adjustment of transformer tap settings and discrete var sources switching [1, 3]. In our study, the control variables for ORPF include the vars/voltages of generators, the tap ratios of transformers, and the reactive power generation of var sources. The constraints include the var/voltage limits of generators, the voltage limits of buses, tap ratio limits, var source limits, and power flow balance at the buses.

In most energy management systems (EMSs), a static var dispatch problem is solved [4]-[6]. However, the dynamic dispatch approach has been applied to optimal active power dispatch by several researchers [7]-[10]. In this paper, a new dynamic ORPF problem is proposed and solved. In this scheme, the total energy loss based on the on-line load conditions and the load forecast during an upcoming interval is minimized. The proposed method keeps the tap ratios and discrete var sources constant during the given interval, at settings that are optimal over the entire time. However, voltage constraint violations are eliminated at the beginning of shorter periods, and power loss is minimized to the extent possible by adjusting continuous controls such as generator vars/voltages.

In section 2 of this paper the static and dynamic dispatch methods are compared. In section 3

the formulation of the problem is presented. In section 4 the solution technique of the proposed method is given. In section 5 the application of the new method to several networks is addressed. A short summary of results and significant issues are given in Section 6.

## 2 Static Versus Dynamic OPF

In this section the application of static and dynamic dispatch to active and reactive OPF are addressed. Two methods of static ORPF are selected for discussion. In [4], the ORPF program is run with two different objectives: 1) power loss minimization (PLM), and 2) removing voltage constraint violations. The first run uses all the control variables, while in the second only the continuous control variables are generally employed. In [6], The power loss and/or constraint violations are minimized in three consecutive runs. In each run, one type of control variables will be used. The problem with these methods is that the optimization problem is solved for only one loading condition.

Several approaches for dynamic active power dispatch are reported in the literature. In [7], the optimal MW dispatch is simultaneously solved for on-line and twelve other load profiles in the upcoming hour. In [8], a dynamic dispatch for generation scheduling has been used. A time interval consisting of several one-hour subintervals is selected. The method comes up to an optimal generation scheduling for the whole time interval. In [9], a similar approach by using Generalized Benders Decomposition (GBD) and including the reactive power and voltage constraints is proposed. These methods do not consider ORPF, and are limited to MW dispatch or unit commitment problems. In [1, 11], an off-line study for finding fixed tap ratios and/or discrete var sources for different load profiles has been addressed. These methods are not suitable for on-line dynamic ORPF as proposed in this paper.

The proposed method is an on-line dynamic dispatch approach that has four main advantages:

1. Reduced energy loss - The proposed method minimizes the total energy loss during a given time interval as the main objective.
2. Reduced physical plant changes - This method keeps the tap ratios and discrete var sources constant during the whole interval. This reduces the number of physical plant changes and unnecessary equipment wear and life-cycle costs. This also results in an implicit economic benefit.
3. A decrease in the number of control variables - In this method, the number of controls at the beginning of each period except the first is restricted to the continuous variables.
4. Less chance of infeasible solutions - The probability of finding an infeasible solution with this method is much lower than with the Power Loss Minimization (PLM) method strictly applied as in [4].

The main steps of the proposed method are shown in Figure 1, and explained below:

- **Step 1: selection of interval duration-** The on-line load profile and the load forecast for the upcoming hours are inspected. Depending on the size of load variations and the experience of the operator, an interval varying from one to several hours will be selected. By observing the same data, the interval will be divided into “ $N$ ” periods. The number and duration of periods depend on the anticipated load profile changes (see Figure 2). By increasing the number of periods, the accuracy and the dimensionality of the solution rise simultaneously.

- **Step 2: dynamic var dispatch-** At the beginning of each interval, a dynamic dispatch to minimize total energy loss for the interval will be executed. The on-line load condition and load forecasts for the remaining  $N - 1$  periods are included. All continuous and discrete control variables are adjusted at the beginning of the interval.
- **Step 3: static var dispatch-** At the beginning of each period after the first, a static ORPF run will be executed to correct for differences between forecast and actual conditions. At each run, the constraint violations for the on-line load conditions are removed. If no violations exist, the power loss is minimized. In these runs, only continuous control variables are allowed to vary.

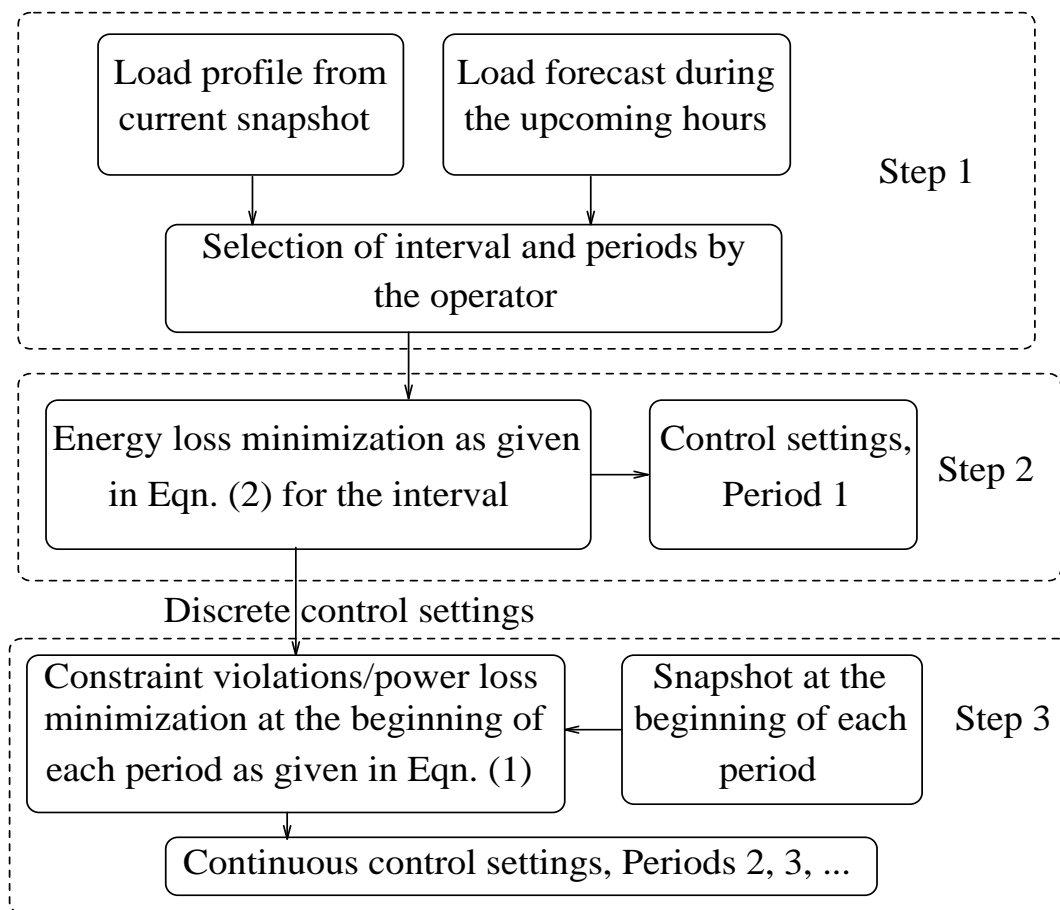


Figure 1: Flow chart for energy loss minimization

It should be noted that the version of PLM here is different from ELM only in Step 2, i.e., at the beginning of each interval PLM minimizes the power loss of Period 1 by using all the control variables. The proposed method, if accompanied with an accurate load forecast for the interval, gives not only a better voltage profile, but also lower energy loss than that given either by our version of PLM or that in [4] (which does not minimize losses every period), and a greater likelihood of finding feasible solutions. The last two comparisons hold if the discrete controls are adjusted only at the beginning of intervals. Of course, discrete control actions can be executed on an ad hoc basis to eliminate infeasible solutions for Periods 2, 3,  $\dots$ ,  $N$ , in which case PLM can find feasible solutions; ELM, however, achieves this result systematically.

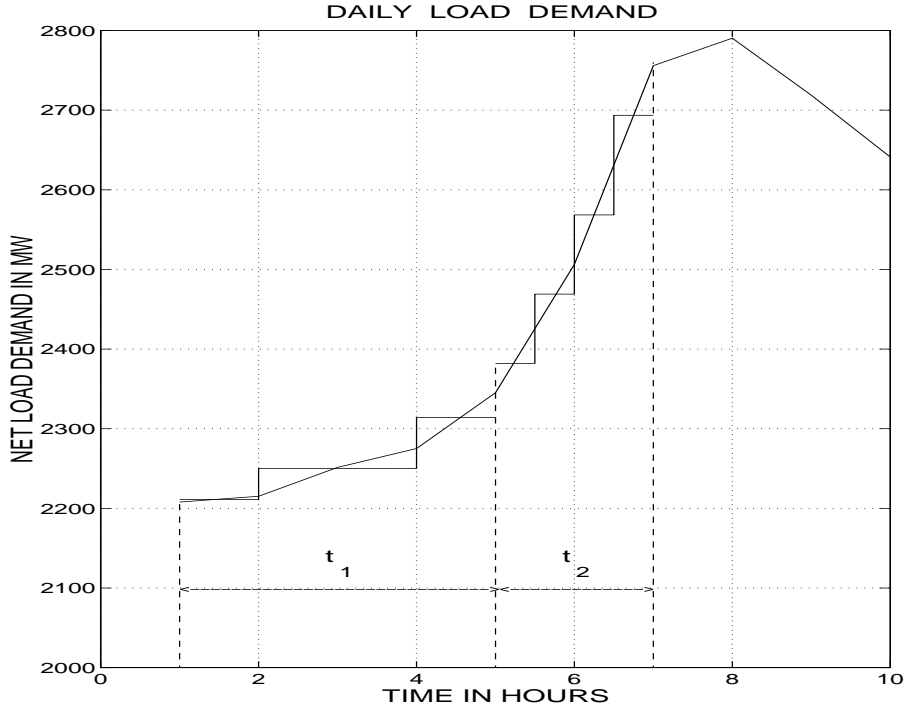


Figure 2: The selection of intervals and periods

### 3 Problem Formulation

The formulation of this problem is explained in two sections. In the first part, the minimization of power loss or constraint violations is addressed. In the second section, the formulation for minimization of total energy loss is given. The first and second formulations are used for the static and dynamic var dispatch purposes, respectively.

#### 3.1 Formulation for Constraint Violation and Power Loss Minimization

It is assumed that the optimal MW dispatch is already executed, and the active power generation of all the generators except at the slack bus are constant. With this assumption, the problem can be formulated as:

$$\begin{aligned}
 & \min_{\mathbf{x}} f(\mathbf{x}, \mathbf{y}^*) \\
 & \text{subject to:} \\
 & \quad \mathbf{h}(\mathbf{x}, \mathbf{y}^*) = 0, \\
 & \quad \mathbf{g}(\mathbf{x}, \mathbf{y}^*) \leq 0
 \end{aligned} \tag{1}$$

where  $f$  is the transmission power loss or the amount of constraint violations;  $\mathbf{x}$  is the vector of continuous variables;  $\mathbf{y}^*$  are the discrete variables, to be held constant during the period; the equality constraints,  $\mathbf{h}(\mathbf{x})$ , are related to power flow balance equations, and the inequality constraints,  $\mathbf{g}(\mathbf{x})$ , include functional and simple constraints on continuous variables. This formulation is used for static var dispatch at the beginning of Periods 2, 3,  $\dots$ ,  $N$ .

### 3.2 Formulation of Energy Loss Minimization (ELM) Method

The energy loss minimization problem is executed at the beginning of each interval. Each interval consists of  $N$  periods. The continuous variables have different values at each period, while the discrete variables have the same adjustment during the whole interval.

The ELM method can be formulated as:

$$\begin{aligned} \min_{\mathbf{x}, \mathbf{y}} E_L &= \sum_{n=1}^N P_L^n * t^n \\ \text{subject to:} & \\ & \left. \begin{aligned} \mathbf{h}^n(\mathbf{x}^n, \mathbf{y}) &= 0 \\ \mathbf{g}^n(\mathbf{x}^n, \mathbf{y}) &\leq 0 \end{aligned} \right\} \text{ for } n = 1, 2, \dots, N \\ & \mathbf{y}_{\min} \leq \mathbf{y} \leq \mathbf{y}_{\max} \end{aligned} \quad (2)$$

where  $E_L$  is the total energy loss of interval;  $P_L^n$  is the power loss of period  $n$ ;  $t^n$  is the duration of period  $n$ ; and  $\mathbf{x}^n$  is the vector of continuous variables related to period  $n$ ;  $\mathbf{h}^n$  and  $\mathbf{g}^n$  are the equality and inequality constraints for period  $n$ , respectively. To emphasize that the discrete control variables,  $\mathbf{y}$ , do not have any  $n$  index, their inequality constraints are shown separately. The formulation for  $P_L$  is given in [3].

As we found through our discussions with the engineers and operators of NB Power company, they are not interested in any type of explicit modeling of switching costs. Based on their current practices, they allow the switching of capacitor/reactor banks twice a day and in extreme cases three or four times a day. The switching of transformer tap ratios are less restricted, and are permitted up to several times a day. Due to these reasons, no switching cost is included in the ELM objective function.

## 4 Solution Method

The power loss or constraint violations as given in (1) can be minimized by using the Newton method [6]. The ELM problem as formulated in (2) can be solved by using different decomposition techniques [12]. In this paper, the application of the Generalized Benders Decomposition (GBD) algorithm to ELM is proposed.

In GBD, the set of variables is divided into two subsets,  $\mathbf{x}$  and  $\mathbf{y}$ . The  $\mathbf{y}$  variables are termed as complicating variables. By fixing the  $\mathbf{y}$  variables the solution of the problem becomes much simpler. This algorithm is recommended for three types of problems [12]. In the type considered in this paper, the problem is transformed into  $N$  independent subproblems after fixing the  $\mathbf{y}$  variables. In GBD, the optimization of  $\mathbf{x}$  and  $\mathbf{y}$  variables are decomposed into two different subproblems, primal and master. In the primal subproblem, the optimization is performed by using the  $\mathbf{x}$  variables. In the master subproblem, the problem is optimized over the  $\mathbf{y}$  variables. The master and primal subproblems are solved alternatively until the convergence criteria are met [12]. It should be noted that the convexity requirements for the GBD technique [12] have not been proved for (2), so we cannot guarantee the convergence of the solution. Other optimization algorithms may be applied for ELM, although GBD is particularly well suited. During this study, all the simulated cases converged successfully, however. In the following sections, the formulation of master and primal subproblems will be discussed.

## 4.1 Primal Subproblem Formulation

The formulation of the primal subproblem is similar to Equation (2) except that the values of  $\mathbf{y}$  are replaced by constant values. By fixing the  $\mathbf{y}$  variables, the primal subproblem is decomposed into  $N$  independent subproblems each involving a different vector of  $\mathbf{x}$  as:

$$\left. \begin{array}{l} \min_{\mathbf{x}^n} E_L^n = P_L^n * t^n \\ \text{subject to:} \\ \mathbf{h}^n(\mathbf{x}^n, \mathbf{y}^*) = 0 \\ \mathbf{g}^n(\mathbf{x}^n, \mathbf{y}^*) \leq 0, \end{array} \right\} \text{ for } n = 1, 2, \dots, N, \quad (3)$$

where  $\mathbf{y}^*$  is fixed at its initial values (first iteration) or optimal values found in the previous master subproblem.

In the primal subproblem, the  $N$  subproblems in (3) are solved independently (in series or parallel). In subproblem  $n$  the total energy loss during period  $n$  is minimized by using  $\mathbf{x}^n$ .

## 4.2 Master Subproblem Formulation

In the master subproblem, the  $\mathbf{x}$  variables are fixed at their optimal values found in primal subproblems,  $\mathbf{x}^*$ . In this subproblem, the total energy loss over the interval is optimized by adjusting the  $\mathbf{y}$  variables. This subproblem can be formulated as:

$$\left. \begin{array}{l} \min_{\mathbf{y}, L_M} L_M \\ \text{subject to:} \\ L(\mathbf{x}_i^*, \mathbf{y}, \lambda_i, \mu_i) \leq L_M \quad \text{for } i = 1, 2, \dots, I \\ \hat{L}(\mathbf{x}_j^*, \mathbf{y}, \hat{\lambda}_j, \hat{\mu}_j) \leq 0 \quad \text{for } j = 1, 2, \dots, J \\ \mathbf{y}_{\min} \leq \mathbf{y} \leq \mathbf{y}_{\max}, \end{array} \right\} \quad (4)$$

where  $I$  and  $J$  are the iteration counts for feasible and infeasible primal subproblems, respectively; and:

$$\begin{aligned} L(\mathbf{x}_i^*, \mathbf{y}, \lambda_i, \mu_i) &= E_L(\mathbf{x}_i^*, \mathbf{y}) + \sum_{n=1}^N \lambda_i^n h^n(\mathbf{x}_i^{n*}, \mathbf{y}) + \\ &\quad \sum_{n=1}^N \mu_i^n g^n(\mathbf{x}_i^{n*}, \mathbf{y}) \\ \hat{L}(\mathbf{x}_j^*, \mathbf{y}, \hat{\lambda}_j, \hat{\mu}_j) &= \sum_{n=1}^N \hat{\lambda}_j^n h^n(\mathbf{x}_j^{n*}, \mathbf{y}) + \sum_{n=1}^N \hat{\mu}_j^n g^n(\mathbf{x}_j^{n*}, \mathbf{y}) \end{aligned}$$

where  $\lambda_i$  and  $\mu_i$  are the equality and inequality Lagrangian multiplier vectors respectively obtained from the feasible primal subproblem; and  $\hat{\lambda}_j$  and  $\hat{\mu}_j$  are the equality and inequality Lagrangian multiplier vectors respectively obtained from the infeasible primal subproblem; in all cases superscript  $n$  stands for period number.

## 5 System Studies

For comparing the power and energy loss minimization methods, several small and large size networks have been studied. Due to space limitations, only the summary of results for the modified

Figure 3: NB Power System is not available

IEEE 30-bus [13] and the NB (New Brunswick) Power [14] systems are given in this section. The line data and the bus data of the modified IEEE 30-bus system are same as those given in [13]. The NB Power network (Figure 3) comprises 277 buses, 223 lines, 35 generators, 45 on-load tap changing (LTC) transformers, 37 switched shunts, 6 fixed shunts, 15 tie lines, and 2 HVDC links.

For the modified IEEE 30-bus system, a time interval of two hours with four equal periods is studied. The load level in period one is equal to the peak load, and is reduced by 12.5% in each subsequent period. For the NB Power network, an actual two-hour time interval during the daily load pick-up is selected. This time interval is divided into three equal periods. The total net load in Period one is 1900 MW, and is increased to 2200 and 2500 MW in Periods two and three, respectively.

PLM and ELM are compared in the following sections. In both methods, all the control variables are set at the beginning of the interval, and the continuous variables are adjusted at the beginning of each subsequent period. At the beginning of interval, in PLM the discrete and continuous variables are set to minimize  $P_L^1$ , while in ELM all control variables are set to minimize  $E_L$ .

### 5.1 Power Loss Minimization

The power losses found for the modified IEEE 30-bus system for the four Periods are given in Table 1. The optimal values of bus voltages are given in Tables 2. The total energy loss achieved by PLM was determined as:

$$E_L = (P_L^1 + P_L^2 + P_L^3 + P_L^4)/2 = 48.64 \text{ MWH}.$$

The power losses found for the NB Power network for the three periods are given in Table 3, and the total energy loss for the NB Power network obtained from the PLM method is:

$$E_L = (P_L^1 + P_L^2 + P_L^3) * 2/3 = 98.90 \text{ MWH}.$$

Table 1: Power loss for the four periods from PLM, modified IEEE 30-bus system

Time Period	1	2	3	4
Power loss (MW)	39.42	28.40	18.45	11.64

Other load profiles were also tested. One of the problems which was encountered during these studies is the infeasibility of solutions. It is possible that the bus loads in subsequent periods differ too much from those in period one. In these cases a strict application of the PLM method can not find any feasible solution by only adjusting continuous control variables, due to limited control action. However, the ELM method does not have this problem, since it uses the load forecast to set the discrete variables to values that suitably anticipate the expected load variations.

### 5.2 Energy Loss Minimization

The modified IEEE 30-bus and NB Power systems with the same load profile mentioned above are used for the minimization of energy loss (Equation (2)).

Table 2: Several bus voltages from the ELM (PLM) method in per unit, modified IEEE 30-bus system

Variable	Periods			
	1	2	3	4
$V_{g1}^+$	1.05 (1.05)	1.05 (1.05)	1.05 (1.05)	1.04 (0.99)
$V_{g2}$	1.04 (1.04)	1.04 (1.03)	1.03 (1.02)	1.02 (0.97)
$V_{g5}$	0.99 (0.99)	1.00 (1.00)	1.00 (0.97)	0.98 (0.95)
$V_{g8}$	1.01 (1.01)	1.01 (0.99)	1.00 (0.97)	0.99 (0.96)
$V_{l12}^*$	1.05 (1.05)	1.05 (1.04)	1.05 (1.01)	1.02 (1.01)
$V_{l14}$	1.03 (1.03)	1.03 (1.02)	1.03 (1.00)	1.01 (1.00)
$V_{l15}$	1.02 (1.03)	1.03 (1.02)	1.03 (1.00)	1.02 (1.00)
$V_{l28}$	1.00 (1.01)	1.01 (1.00)	1.00 (0.99)	0.99 (0.97)

+:  $V_{gi}$  is the generator bus voltage at bus  $i$ ;

\*:  $V_{lj}$  is the load bus voltage at bus  $j$ .

Table 3: Power loss for the three periods from PLM, NB Power network

Time Period	1	2	3
Power loss (MW)	35.10	46.65	66.60

For the modified IEEE 30-bus system, the total energy loss found by ELM is equal to 47.78 *MWH*. This value is 1.8% less than the energy loss found in the PLM method (48.64 *MWH*, see discussion in section 5.1). For the NB Power network, the total energy loss over the given time interval is equal to 95.90 *MWH* which is 3.0% less than that of PLM method, 98.90 *MWH*. By comparing the simulation results of these cases and other simulation studies, the following observations can be made:

1. The voltage profile from energy loss minimization are smoother than those from the PLM method (Table 2).
2. The energy loss from ELM for the above systems is 1.8% to 3% less than that from PLM.
3. The advantages of the ELM method are more apparent when the load changes significantly. In cases where the load profile is almost flat during the given time interval, ELM gives only slightly better results.
4. In cases where the load changes over the time interval are large, coming to a feasible solution by the PLM method is not always possible unless ad hoc adjustments of discrete controls are made. In these cases ELM is substantially more likely to find a feasible solution without such adjustments. The reason is that the load conditions for all periods have been considered in the load flow equations which are enforced as constraints in the ELM formulation. Therefore, the



optimal values of the discrete control variables obtained using the ELM method can usually handle the load changes predicted by the load forecast. The only circumstance when ELM cannot find a feasible solution is when the load changes over the given time interval is very different from the predicted values. In these cases, the discrete control variables must be adjusted to avoid voltage violations. This could be avoided by choosing a shorter interval.

5. The execution time of the ELM and PLM methods are similar for the runs at the beginning of Periods 2, 3,  $\dots$ ,  $N$ . However, at the beginning of each interval, the ELM problem has a larger size than that from the PLM method. The number of periods ( $N$ ) and the topology of the network are the most two important factors in this regard. In the GBD solution of ELM, the primal-master iterations can be as low as 2 for small networks and as large as 5-7 for bigger networks. The primal problems includes  $N$  optimization subproblems, which each of them has a smaller size than PLM problem. The CPU time for the NB Power network on a Vax Station 4000/96 computer are as follows: 1) at the beginning of Periods 2, 3,  $\dots$ ,  $N$ , the PLM and ELM methods need less than 2 seconds; and 2) at the beginning of each interval, PLM approximately needs three seconds, and ELM may need 10 times more in average.

## 6 Conclusion

A new strategy for on-line optimal reactive power dispatch is proposed. The method minimizes the total energy loss during the upcoming interval, while keeping the voltage profile within an acceptable range. Furthermore, by comparing simulation results, it is found that ELM gives a smoother voltage profile than that from the PLM method; in Table 2, we see that ELM produced a nearly constant voltage profile during the given interval. In addition, the energy loss was reduced at the same time, with the same number of discrete control variable changes as used by PLM. Finally, ELM achieves these benefits while ensuring that certain control variables are not be adjusted too often, thus saving wear and lengthening the life of the corresponding equipment. The frequency of adjustment could be modified, to fit differing circumstances.

The probability of finding an infeasible solution with ELM is much lower than with PLM. This advantage of the ELM method is obtained by considering the load forecast and making sure that anticipated load changes during the upcoming interval can be accommodated.

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