A FREQUENCY DOMAIN MODEL-ORDER-DEDUCTION ALGORITHM FOR LINEAR SYSTEMS

Bruce H. Wilson
Department of Mechanical Engineering
Northeastern University
Boston, MA

James H. Taylor
Department of Electrical Engineering
University of New Brunswick
Fredericton, NB CANADA

Bora Eryilmaz
Department of Mechanical Engineering
Northeastern University
Boston, MA

ABSTRACT

During the system design process engineers must choose the appropriate order of a model of a system of components, which is accomplished by determining the appropriate order of the component submodels. Several researchers have developed *model-order-deduction algorithms*, under the assumption that algorithms for synthesizing component submodels are available. Thus the issue of determining appropriate submodel order has been a principal topic. In previous efforts the primary means to determine model order adequacy has been to focus on the eigenvalues of the model. One algorithm, the model-order deduction algorithm (MODA), systematically increases the order of the system model until any subsequent increase results in adding system eigenvalues of magnitude outside a user-specified frequency range of interest $[0, \omega_{max}]$. Another algorithm, EXTENDED-MODA, begins with the same model that MODA synthesizes and increases the order further until the $|\lambda_i| \in [0, \omega_{max}]$ settle within some specified tolerance.

In this paper we develop a new algorithm, FD-MODA, that uses the convergence of the model in the frequency domain as its performance metric. The model's frequency response is based on zeros and poles (eigenvalues) of the system model, so it provides a more comprehensive indication of model performance than just the eigenvalues. It also seems to be more meaningful in the context of frequency-domain controller design methods, e.g. H_∞ optimization. FD-MODA's use is illustrated with two examples, and its efficacy in selecting an appropriate combination of component submodels is thereby demonstrated. In addition to using a broader performance metric than previous algorithms, FD-MODA also provides visual feedback – frequency response plots – of model convergence that are more indicative of model performance than a map of pole locations in the s-plane. We thus conclude that by focusing on frequency response as a model performance metric FD-MODA provides a useful and direct means of coordinating the automated model synthesis process.

INTRODUCTION

Modelers, both human and automated, must cope with a bewildering number and variety of decisions when developing a mathematical model of a physical system. The ultimate onus is, of course, on the human modeler to make these decisions wisely. However, it would be highly desirable for automated-modeling software to support this decision-making process to the extent possible.

One early decision is the type of modeling formalism to adopt. Continuum-based models, finite-element models, and discrete (lumped parameter) models all have wide and frequently overlapping applicability. Within a given formalism a modeler must contend with choices concerning the appropriate model order, the effects to include in a model, and how to represent (if at all) the inherent nonlinearities involved in most physical systems.

A great simplification results if the modeler represents a physical system in terms of a continuous-time, state-determined model, tacitly assuming that this lumped parameter formalism adequately describes the system under study. This representation (also called a "state-space" model) allows the system to be described with a finite (often low) number of state variables. The n state variables of such a representation are governed by a set of n ordinary-differential equations. Even with this simplification, however, a modeler still needs to: (i) select the effects to include in a model, (ii) determine the appropriate order for a model, and (iii) describe mathematically the important nonlinearities.

While a general technique for identifying which effects to include in a model is not available, considerable theory is available to address the problems of determining model order and choosing models for common nonlinear effects.

In our work, we are attempting to synthesize and extend existing theory to create general *model-order deduction algorithms* (MODAs) for linear and nonlinear systems. An important attribute of our approach is to consider a model's *frequency-domain behavior* to be a key measure of its "goodness". In the nonlinear case, preliminary results (based on describing-function techniques) are presented in Wilson and Taylor (1993); here we focus on an approach for linear systems called FD-MODA (frequency-domain model-order deduction algorithm).

BACKGROUND

The context of the present research in model-order deduction is as follows: We assume at the outset that the modeler is dealing with an electro-mechanical system that is assembled using a number of components (motors, gears, shafts), and that an automated modeling technique exists that can be used to generate a system model once the components are specified. In fact, this technique can produce a *number* of dynamic models, which differ in the level of detail used to model each component. In the linear case, the "level of detail" of each component submodel translates into its *order*; e.g., a more detailed motor model may include inductive lag (one additional state variable), or a more detailed shaft model might include one or several modes (adding two state variables to its lumped-parameter model per mode). The problem addressed by model-order deduction algorithms is deciding what *minimal* level of detail to use in each component submodel so that the resulting system model is suitably realistic.

Before this problem can be solved, one must specify what is meant by the term "suitably realistic." This consideration introduces the idea of a "model performance metric." We believe – and will attempt to justify – that in the context of control system analysis and design (at least) the appropriate metrics involve the system model's frequency response characteristics. In particular, we will focus on an H_∞ metric, i.e., we will use the *sup norm* of $\delta G(j\omega)$ (the normalized change in the model's frequency response) as the criterion for judging the suitability of the system model as we iteratively increase the order of each component submodel and look for convergence.

Automated Modeling Framework

The model order deduction algorithm described in this paper, FD-MODA, is built within a framework of components, automated model synthesis procedures, and model evaluation techniques. To define the framework for this algorithm, we begin by specifying the set of components

$$C = \{\mathrm{c}_1, \mathrm{c}_2, \ldots, \mathrm{c}_n\}$$

from which *systems* (of components) can be constructed. In the domain of electro-mechanical systems, C will include components such as DC motors, torsional shafts, and gear pairs. Each component c_i has a set of relevant *attributes* that affect the dynamic response of that component, e.g. shaft length, diameter and material. We denote this component-specific set of attributes as:

$$d_i = \{\mathrm{d}_i^{(1)}, \mathrm{d}_i^{(2)}, \ldots, \mathrm{d}_i^{(l)}\}.$$

In addition, each component c_i has a corresponding *model generating function* (MGF) that maps the component into one of its submodels. The set of MGFs for C is

$$F = \{\mathrm{f}_1, \mathrm{f}_2, \ldots, \mathrm{f}_n\}.$$

Each f_i is capable of synthesizing one or more candidate submodels of the component c_i to which it corresponds. A non-negative integer, referred to as the *rank* of a component submodel, is used to specify its complexity. Depending on the component, the maximum rank is either unbounded or bounded. *Unbounded rank* components can be mapped into an infinite number of submodels, whereas *bounded rank* components can be mapped into only a finite (typically small) number of submodels. A flexible shaft is an example of the former case, while a motor would generally be of bounded rank.

The set of submodels corresponding to the i^{th} component is denoted as:

$$M_i = \{\mathrm{m}_i^{(1)}, \mathrm{m}_i^{(2)}, \ldots, \mathrm{m}_i^{(m)}\}.$$

Each submodel $\mathrm{m}_i^{(j)}$ in M_i has a rank and a set of coefficients that parameterize it. Without loss of generality, we assume the submodels $\mathrm{m}_i^{(j)}$ are ordered by increasing rank. We denote the corresponding set of coefficients as:

$$q_i^{(j)} = \{\mathrm{q}_{i,1}^{(j)}, \mathrm{q}_{i,2}^{(j)}, \ldots, \mathrm{q}_{i,k}^{(j)}\}.$$

There are m candidate submodels of each component, and thus m possible sets of submodel coefficients are associated with it. Note that these coefficient sets are explicitly determined by the physical attributes of the i^{th} component, i.e., $\mathrm{q}_{i,k}^{(j)} = \mathrm{q}_{i,k}^{(j)}(\mathrm{d}_i)$.

To clarify the above interrelationships, suppose we want to synthesize the second submodel (in terms of rank or complexity) of the first component. We would use the first MGF (f_1) and supply it with arguments 2, for the second-rank model, and d_1, denoting the attributes of the first component. In an algorithmic notation,

$$\mathrm{f}_1(2, \mathrm{d}_1) \rightarrow \mathrm{m}_1^{(2)}(q_1^{(2)}(\mathrm{d}_1)).$$

The repeated subscript 1 of the submodel, m_1, and the attributes and coefficients of this submodel, d_1 and q_1, are needed to indicate that both the submodel and its coefficients are derived from a specific type of component. For example, the first submodel of a motor is very different from the first submodel of a shaft. The running superscript (2) denotes the use of the second-rank submodel. Of course, the notation could be one level more complicated if we attempt to distinguish between shaft_1 and shaft_2! However, to avoid unnecessary notation, additional subscripts and superscripts will be avoided when the origin of the submodel is clear or generic.

Finally, we would like to emphasize that models synthesized using FD-MODA have *physical parameters*, that is, the state variables are generalized displacements and momenta, and inertial, capacitive, and dissipative elements appear as coefficients of these state variables. This is in stark contrast to a black box approach to modeling, in which information internal to a physical system is not part of the model.

As mentioned previously, the framework of FD-MODA also includes a means to assemble the component submodels into a system model and to synthesize state equations. Submodel assembly is accomplished by kinematically coupling adjacent inertial elements of the component submodels and lumping two or more inertias into a single inertia. This is quite standard, and is discussed in texts such as Rosenberg and Karnopp (1983) and Karnopp et al. (1990). In these same texts, algorithms are provided for synthesizing the state equations of a model expressed as a bond graph. For future reference, we denote such a system model as:

$$\mathcal{S}^N = A(\mathrm{m}_i^{(r_i)})\,; \;\; N = \sum_i r_i$$

where A informally denotes "assembly" in the above sense, i ranges over the set of components and N denotes the rank of $\mathcal{S}$, which is related to its order.

Model Order Deduction

One approach to modeling is to begin with a high-order model of a physical system and apply mathematical transformations to pare the high frequency modes from the model. Such a model order reduction approach is quite powerful and commonplace; however, its use presupposes the existence of a high-order model to start with. In contrast to this approach, *model order deduction* begins with a very low-order model of a system and *adds* state variables to the model – through the inclusion of inertial and compliant elements – until the performance predicted by the model no longer changes (appreciably) as more state variables are added to the model.

The first algorithm to apply this idea systematically was the "model order deduction algorithm" (MODA) (Wilson and Stein, 1992; Wilson and Stein, 1995) Since that contribution, several approaches have been proposed for its extension and improvement. One area that is clearly significant involves the definition of measures for a model's performance, since these govern the search for a "good" or "adequate" model and determine when such a process has converged. Therefore, a detailed discussion of the last element of the framework – model performance metrics – is needed to complete this background exposition.

Model Performance Metrics

Two basic truisms are: 1) a model should be accurate, and 2) an accurate low-order model is preferred over a high-order model of approximately the same accuracy. Quantifying accuracy is, however, somewhat elusive, because in the design stage no hardware is available against which the performance of a model can be evaluated. Even if hardware exists, the time and expense required to measure system performance may weigh against obtaining this data to evaluate a model.

Since model performance often cannot be evaluated against measured data, a reasonable alternative is to compare the performance of lower to higher-order models of the same system to get an indication of the sufficiency of model order. For example, if $\mathcal{S}^N$ provides approximately the same predicted behavior as higher-order model $\mathcal{S}^{N+1}$, we might conclude that $\mathcal{S}^N$ has sufficient order. Conversely, if a large difference exists in the predictions obtained from these two models, we would conclude that $\mathcal{S}^N$ has insufficient order and we should increase the order and compare models $\mathcal{S}^{N+1}$ and $\mathcal{S}^{N+2}$, and so on.

Previous model-order deduction algorithms have used the concept of a *frequency range of interest* (FROI) and eigenvalues as a performance metric. MODA used $|\lambda_i| \in [0, \omega_{max}]$ as a performance metric in the sense that it synthesized a low-order model that minimized the spectral radius, while simultaneously guaranteeing that any subsequent increase in model order would result in a spectral radius beyond ω_{max}. EXTENDED-MODA (Ferris and Stein, 1995) also focused on the $|\lambda_i| \in [0, \omega_{max}]$, but continued increasing model order until these eigenvalues no longer changed appreciably with subsequent increases in model order. In the sequel we will discuss MODA and EXTENDED-MODA in more detail and also demonstrate that the frequency response metric provides a more comprehensive measure of model performance than $|\lambda_i| \in [0, \omega_{max}]$.

Frequency Range of Interest.

The FROI is of considerable importance in systems engineering, and provides a context within which to formulate requirements on model performance. The FROI is the frequency band $[\omega_{min}, \omega_{max}]$ over which a model, in terms of steady-state input-output prediction, should give a reliable indication of system response. For convenience, zero may often be taken as the lower bound of the FROI ($\omega_{min} = 0$). The upper bound may be determined from an input specification, e.g., if the frequency content of commanded inputs or disturbance inputs is known then the model should be accurate to frequencies 2–5 times the highest input frequency (ω_{in}) (Karnopp et al., 1990); accordingly, $\omega_{max} = 5 \times \omega_{in}$. In the case of closed-loop system design, the open-loop crossover frequency drives the FROI in that the model should provide a reliable response at frequencies 1 to 2 decades beyond the crossover frequency ω_{co}, that is, $10 \times \omega_{co} \leq \omega_{max} \leq 100 \times \omega_{co}$. Both of these approaches are merely "rules of thumb"; the engineer must temper these suggestions with the particulars of the problem under consideration.

Eigenvalue-Based Model Performance Metrics.

MODA coordinates the search for the combination of component submodels that results in a model that, for a given order, minimizes the spectral radius and guarantees that any increase in model order will result in a spectral radius beyond the upper bound of the FROI $[0, \omega_{max}]$. This implies that a model synthesized using MODA contains only eigenvalues with magnitude in the FROI. Wilson and Stein (1992; 1995) use the term *Proper Model* to refer to a model with physically based parameters and state variables and that meets this performance metric.

A limitation of MODA cited by Ferris and Stein (1995) is the lack of a guarantee regarding the accuracy of the condition $|\lambda_i| \in [0, \omega_{max}]$ in a model synthesized using MODA. That different lumpings of continuous components in a larger system model will result in different eigenvalues is well known. In the case of MODA, an increase in the order of a Proper Model generally results in some shifting of $|\lambda_i| \in [0, \omega_{max}]$, which suggests that a more accurate prediction can be obtained by increasing the order of the Proper Model.

Ferris and Stein addressed the issue of eigenvalue migration by creating a new model synthesis algorithm that monitors the migration of the $|\lambda_i| \in [0, \omega_{max}]$, which they refer to as the *critical system eigenvalues.* The new algorithm EXTENDED-MODA synthesizes a Proper Model in the same manner as MODA; it then continues increasing model order until the $|\lambda_i| \in [0, \omega_{max}]$ remain approximately the same as the model order is increased. The degree of approximation can be controlled by a user-specified tolerance defining the acceptable percentage change. The claim by Ferris and Stein is that EXTENDED-MODA will synthesize a model of appropriate complexity that provides estimates of $|\lambda_i| \in [0, \omega_{max}]$ that have converged to some user-specified percentage.

Frequency-Response–Based Model Performance Metrics.

The frequency response of a system, denoted $G(j\omega)$, is a useful performance metric and design aid. When plotted on the G-plane the frequency response is the basis for Nyquist stability analysis, and is used to compute gain and phase margins. When the magnitude and phase are plotted separately, as in Bode plots, frequency response is used for frequency-domain based compensator design.

We believe that considering a model's frequency response over a given frequency range provides a broader measure of its performance than monitoring eigenvalues within this same range. Frequency response is obtained by evaluating a transfer function, a ratio of polynomials, over a range of frequencies, i.e.

$$G(j\omega) = \frac{N(j\omega)}{D(j\omega)}; \quad \omega \in [\omega_{min}, \omega_{max}]. \tag{1}$$

The transfer function (1) is obtained from the state matrices using the well-known relation:

$$G(s) = C(sI - A)^{-1}B + D. \tag{2}$$

If we focus (for the present) on SISO systems, we can rewrite (2) as

$$G(s) = K\frac{\prod_{i=1}^{m}(s+z_i)}{\prod_{i=1}^{n}(s+p_i)}. \tag{3}$$

where $m \leq n$ and $m < n$ if $D = 0$. The poles of (3) are the eigenvalues of the system.

We now consider (3) in the context of the model order deduction algorithms described in the previous section. Comparing the frequency response of (3) over a FROI $[\omega_{min}, \omega_{max}]$ provides a more meaningful basis for deciding submodel rank and evaluating model performance than just focusing on the poles. There are several reasons for this assertion:

- Using eigenvalue convergence as a criterion for setting model order may have little bearing on the convergence of $G(j\omega)$. For example, the variation of $G(j\omega)$ for a 10% change in a real eigenvalue may be quite small, while a 10% change in a pair of very lightly damped eigenvalues may have a major impact.
- In the context of control system analysis and design, $G(j\omega)$ tells the "whole story" regarding gain and phase margin, loop-shaping requirements (for compensator design), and the like; eigenvalues convey only part of this information.

In the final analysis, we want the system model to provide a reliable prediction of model performance for $\omega \in [\omega_{min}, \omega_{max}]$. For this to occur, both the zeros and the poles of (3) must have converged sufficiently such that increases in model order do not cause appreciable changes in the frequency response. This cannot be done by considering only the eigenvalues.

FREQUENCY-DOMAIN METHOD FOR MODEL-ORDER DEDUCTION

The most succinct way to present the frequency-domain method for model-order deduction is to define the objective of the FD-MODA algorithm, and specify the logic used in its execution.

Objective

The input and the desired output specify the requirements of the algorithm. The input to FD-MODA includes:

- a system, $\mathcal{S}$, of serially-connected components, e.g. $\mathcal{S} = \{c_1, c_3, c_2, c_3, c_8\}$,
- a FROI $[\omega_{min}, \omega_{max}]$, and
- a frequency response convergence tolerance, `TOL`.

The output of FD-MODA is the set of ranks of the components of $\mathcal{S}$, i.e. $\mathcal{R} = \{r_1, r_3, r_2, r_3, r_8\}$. This set satisfies two conditions:

1. the frequency response over $[\omega_{min}, \omega_{max}]$ predicted by a model synthesized based on $\mathcal{R}$ has converged within `TOL`,
2. the sum of the ranks, $\sum_i r_i$, is minimum.

This provides a system model that is of appropriate complexity given the specification that the frequency response should be accurate over the FROI.

Model Order Determination Procedure

1. **Initialize**
   (a) Initialize all component ranks to zero ($r_i = 0$)
   (b) Specify a tolerance `TOL`
   (c) Specify a frequency of interest $[\omega_{min}, \omega_{max}]$
   (d) Create a grid of K points, $\omega_k \in [\omega_{min}, \omega_{max}]$
   (e) Synthesize a system model $\mathcal{S}$, based on current ranks
   (f) Compute $G(j\omega)$ for all ω_k
   (g) Set $G^*(j\omega) = G(j\omega)$
2. **Determine Most Sensitive Component**
   (a) $dG_{max} = 0$
   (b) Cycle over the components c_i:
      i. Increase rank of c_i
      ii. Synthesize system model, based on current ranks
      iii. Compute $G(j\omega)$ for all $\omega_k \rightarrow G(j\omega_k)$
      iv. $\delta G_k = (G^*(j\omega_k) - G(j\omega_k))/G^*(j\omega_k)$
      v. If ($\| \delta G_k \|_\infty = \max_k | \delta G_k |) > dG_{max}$ then:
         A. $dG_{max} = \| \delta G_k \|_\infty$
         B. $i^* = i$
      vi. Decrease rank of component c_i
3. **Evaluate Need to Increase Order**
   (a) If $dG_{max} >$ `TOL` then:
      i. Augment model order by increasing rank of component c_{i^*}
      ii. Synthesize system model, based on current ranks
      iii. Compute $G(j\omega)$ for all ω_i
      iv. $G^*(j\omega) = G(j\omega)$
      v. go to 2
   (b) else, continue below
4. **Output Results**
   (a) Output ranks of components
   (b) Output system model

ALGORITHM DEMONSTRATION

We will demonstrate FD-MODA with an example that involves synthesizing a model of a drive train. The section begins with a description of the drive train, followed by a discussion of the model generating functions that correspond to its components. With this background, the section culminates with a step-by-step description of FD-MODA-coordinated model synthesis.

Physical System

The drive train, shown in Figure 1, consists of a DC motor, a flywheel supported by roller bearings, a torsional shaft, and a second flywheel, also supported by roller bearings.

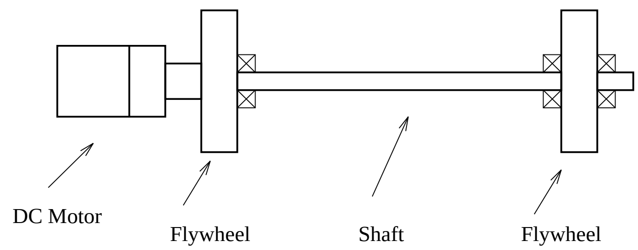

Figure 1: Demonstration system

Component dimensions, motor data, and material parameters for the system in Figure 1 are as follows:

DC motor torsional inertia J_a 3.6e-4 $kg{-}m^2$, inductance L_a 3.1e-3 H, resistance R_a 1.4 ohm, torque constant K_t 0.17 Nm/amp, viscous friction B_a 7.4e-5 $Nm/rad/sec$

Flywheel 1 diameter 0.2 m, thickness 0.0254 m, density 7775 kg/m^3, viscous friction coefficient 3e-5 $Nm/rad/sec$

Shaft diameter 0.02 m, length 1.0 m density 7775 kg/m^3, shear modulus 7.31e10 N/m^2

Flywheel 2 diameter 0.2 m, thickness 0.0254 m, density 7775 kg/m^3, viscous friction coefficient 3e-5 $Nm/rad/sec$

Component Submodels

Models are needed for each component in the drive train. As discussed, model generating functions are assumed to be available, and these can be used to synthesize component submodels. In that section, model generating functions were defined as taking an argument, *rank*, that dictates the complexity of the component submodel. Henceforth, we will call the model that results from the rank being set equal to n, the "rank-n" model.

For the DC motor, two models are available, a rank-0 model and a rank-1 model. As discussed in more detail in Wilson and Stein (1992), the rank-0 model does not include the inductance of the windings, and the rank-1 model

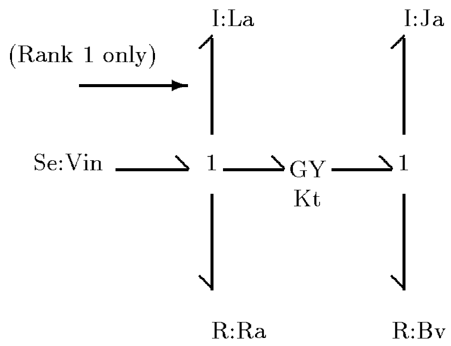

Figure 2: Bond graphs for rank 0-1 DC motor

includes the inductance. The rank-0 and rank-1 DC motor bond graphs are shown in Figure 2, and are standard (Karnopp et al., 1990).

The flywheel model and torsional shaft models are distinct in that only one model is available for the flywheel and an infinite quantity of models are available for the shaft. One model is available for the flywheel, an inertial element coupled to a dissipative element. No additional inertial or compliant elements can be used to augment the flywheel model. The model for the flywheel is standard (Rosenberg and Karnopp, 1983). In the case of the torsional shaft, it can be modeled as a torsional inertia, or as two rigid shafts separated by a torsional spring, or as $N+1$ rigid shafts separated by N torsional springs. As discussed in Wilson and Stein (1992), this latter model is the rank-N shaft model. Bond graphs for the flywheels, the rank-0 shaft, and the rank-1 shaft are shown in Figure 3; note the distinction between rank-0 and rank-1 shaft made in Figure 3.

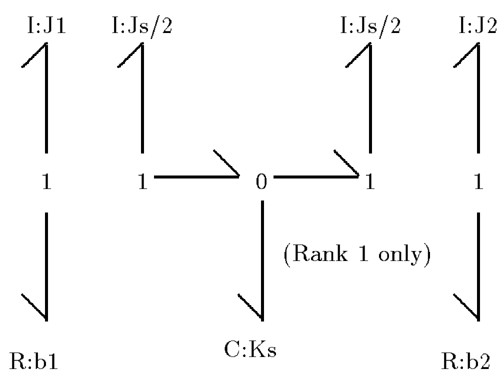

Figure 3: Bond graphs for flywheels, rank 0-1 shaft

Proper Model

In the FD-MODA framework model synthesis reduces to selecting the set of component ranks that result in a model that has converged over a specific FROI. Converge in this context implies that the model frequency response does not change appreciably when the rank of any component is increased. For the drive train in Figure 1 only the DC motor and the shaft can have their ranks changed. The flywheel models are strictly rank-0. The choices for the DC motor are rank-0 and rank-1, and the choices for the shaft range from rank-0 to rank-∞. Our model notation here is as follows: the (m,n) model corresponds to the rank-m motor (model) and the rank-n shaft. We assume a FROI of 1–1000 $\frac{rad}{sec}$, with a 1000-point grid, and a convergence tolerance of 0.2.

For the first pass of the algorithm, we begin with the (0,0) model and compare $\delta G(j\omega)$ of the (1,0) and (0,1) models. As noted in Table 1, the (0,1) model has a much larger dG_{max} than the (1,0) model, which implies that the shaft compliance has a larger effect on model performance than motor inductance. As the (0,1) model has the largest dG_{max}, this model is the first model for the second pass of the algorithm.

During the second pass of FD-MODA, we compare the (0,1) model with the (1,1) and the (0,2) models. As tabulated in Table 1, the (1,1) model has a much larger dG_{max} than the (0,2) model, which implies that the motor inductance has a more significant effect on model performance than a two-spring shaft model. Thus, the (1,1) model is the first model for the third pass of the algorithm.

The model converges after the second pass. Within the FROI [1,1000] $\frac{rad}{sec}$, there is little change between the performance predicted by the (1,1) model and the (1,2) model, which is the only available choice at this point, in terms of incrementally increasing complexity. As Table 1 indicates, dG_{max} for the (1,2) model is 0.16, which is less than the specified tolerance (0.2).

Pass	Iter.	Motor	Shaft	$G^*(j\omega)$	dG_{max}
1	0	0	0	(0,0)	—
1	1	1	0	(1,0)	0.9115
1	2	0	1	(0,1)	330.9
2	0	0	1	(0,1)	—
2	1	1	1	(1,1)	0.91
2	2	0	2	(0,2)	0.18
3	0	1	1	(1,1)	—
3	1	1	2	(1,2)	0.16

Table 1: Component ranks during drive train model synthesis

Although Nyquist plots were used to determine that the (1,1) model is the Proper Model for the drive train in Figure 1, Bode magnitude plots better illustrate the changes in predicted behavior as the model rank is increased.

Consider the magnitude plot for the (0,0)–(1,1) models shown in Figure 4. Note the large difference between

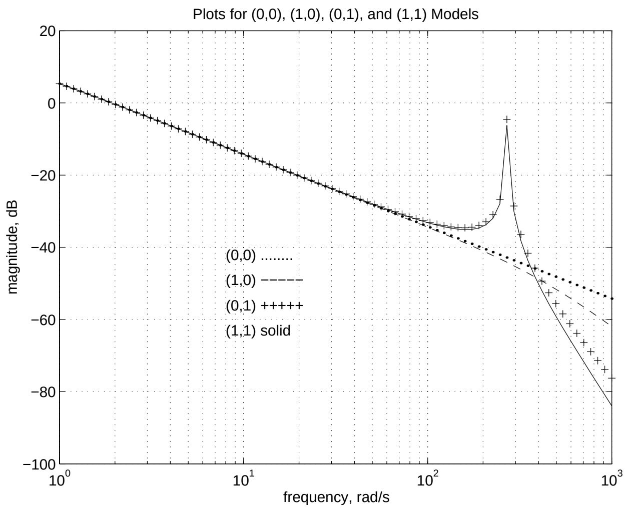

Figure 4: Bode magnitude plots for (0,0)–(1,1) models

the (0,0) and the (0,1) plots, which corresponds to the selection of the (0,1) model during the first pass. Note also the smaller, but still significant, difference between the (0,1) and the (1,1) plots, which corresponds to the selection of the (1,1) model during the second pass. The model converges during Pass 2, which is evident when the nearly identical Bode plots for the (1,1) and (1,2) models are plotted (not shown).

COMPARISON OF MODEL-ORDER-DEDUCTION ALGORITHMS

We now examine the synthesis of a model of a shaker system described by Ferris and Stein (1995). The shaker, shown in Figure 5, consists of a rod, a DC electro-mechanical linear actuator, a second rod, and a vibration isolator. Component dimensions and data for the system in Figure 5 are as follows:

Rod 1 density 7755 kg/m^3, elastic modulus 2.1e10 N/m^2, diameter 0.05 m, length 2.0 m,

Shaker gyrator modulus 1 N/A, resistance 1 ohm, inductance 5e-4 H, base mass 1 kg, armature mass 1 kg.

Rod 2 same as Rod 1

Isolator mass 1 0.5 kg, spring stiffness 10000 N/m, mass 2 0.5 kg.

The models for the rods are standard translational shaft models, where rank-0 corresponds to a rigid shaft, *rank-1* corresponds to two rigid shafts separated by one (*1*) spring,

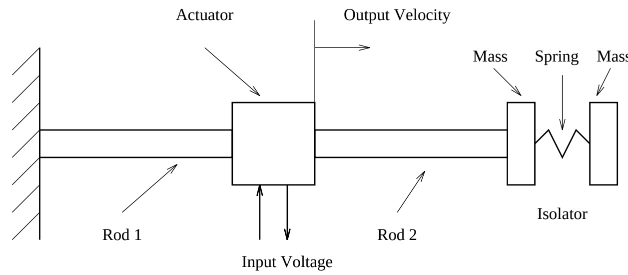

Figure 5: Shaker system

etc. The rank-0 model of the shaker omits the inductance of the shaker, while the rank-1 model includes the inductance (similar to a DC motor). Finally, the rank-0 model of the isolator is a single mass of 1 kg, and the rank-1 model consists of two masses separated by a spring. Note that the model of the second rod is different than the one used by Ferris and Stein (1995), which was obtained from a closed-form solution of a continuum based model of a translational shaft. The model used by Ferris and Stein appears to assume force-free boundary conditions, a condition that we believe does not accurately reflect the forces to which the second (right) rod in Figure 5 is subjected.

FD-MODA was used to synthesize a model of the shaker system with a FROI $[1, 1000]\ \frac{rad}{sec}$ and a tolerance of 0.2. The model synthesis process is summarized in Table 2.

P.	It.	R1	Shak.	R2	Isol.	$G^*(j\omega)$	dG_{max}
1	0	0	0	0	0	(0,0,0,0)	—
1	1	1	0	0	0	(1,0,0,0)	2.1e-4
1	2	0	1	0	0	(0,1,0,0)	4.5e-1
1	3	0	0	1	0	(0,0,1,0)	6.4e-1
1	4	0	0	0	1	(0,0,0,1)	3.0
2	0	0	0	0	1	(0,0,0,1)	—
2	1	1	0	0	1	(1,0,0,1)	2.1e-4
2	2	0	1	0	1	(0,1,0,1)	4.5e-1
2	3	0	0	1	1	(0,0,1,1)	6.0e-1
3	0	0	0	1	1	(0,0,1,1)	—
3	1	1	0	1	1	(1,0,1,1)	2.1e-4
3	2	0	1	1	1	(0,1,1,1)	4.5e-1
3	3	0	0	2	1	(0,0,2,1)	1.8e-1
4	0	0	1	1	1	(0,1,1,1)	—
4	1	1	1	1	1	(1,1,1,1)	1.8e-4
4	2	0	1	2	1	(0,1,2,1)	1.8e-1

Table 2: Component ranks and performance during shaker model synthesis

As in the previous example, although Nyquist plots were used to determine that the (0,1,1,1) model is the Proper Model for the shaker in Figure 5, Bode magnitude plots better illustrate the changes in predicted behavior as the model rank is increased. Following the steps of the model synthesis, the bode magnitude plots for the (0,0,0,0)–(0,1,1,1) models are shown in Figure 6.

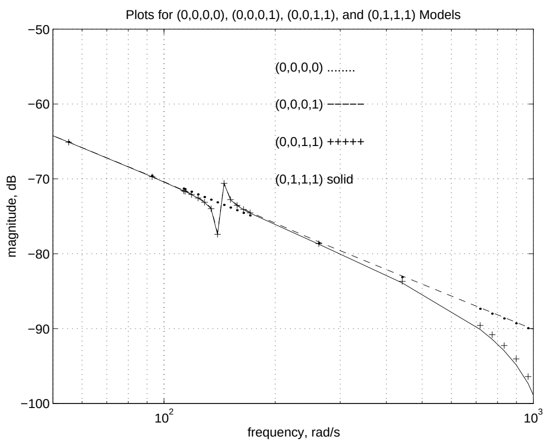

Figure 6: Bode magnitude plots for (0,0,0,0)–(0,1,1,1) models

Observe the large difference between the (0,0,0,0) and the (0,0,0,1) plots, which caused the selection of the (0,0,0,1) model during the first pass. Note also the smaller, but still significant, differences during the next three passes, until the model converges during the fourth pass. The characteristic "dip" in the Bode magnitude plot results from the vibration isolator acting as a dynamic vibration absorber, with a natural frequency of about 150 rad/sec.

Although space does not permit details to be included here, models were also synthesized using EXTENDED-MODA. With a FROI of $[0, 1000]$ and an eigenvalue tolerance of 0.1%, model synthesis *terminates* at the (0,0,0,1) model, even though, as indicated in Figure 6, the frequency response continues to change significantly over the FROI as the model order is increased further.

SUMMARY AND CONCLUSIONS

Significance

The FD-MODA algorithm results in an efficient search for the set of component ranks that produce a model that meets a frequency-domain performance criterion. The search uses a systematic means of testing the effect on system performance of more complex component submodels. The search is efficient because it adopts a gradient approach (Rich and Knight, 1991) to seek a model of sufficient complexity that meets the performance criterion, i.e. the Proper Model. This technique avoids the combinatorial explosion that would result from an exhaustive search for the component ranks that result in a Proper Model. Specifically, for n_c components this gradient search evaluates $n_c \times (1 + \sum_{i=1}^{n_c} r_i)$ models, where r_i is the final rank of component c_i. While this quantity may seem large, it is much smaller than the number that would result from an exhaustive strategy such as breadth-first.

This performance metric is comprehensive in that it reflects the frequency domain behavior of the entire system, not just the behavior of its poles. This observation is particularly important in the context of modeling plants for control system design. Note that this metric can also be used for closed-loop systems, since a change in closed-loop frequency response due to an increase in model order can be evaluated as easily as open-loop frequency response. Finally, the frequency response reflects system performance by concisely depicting input-output behavior over $\omega \in [\omega_{min}, \omega_{max}]$. Engineers have found this type of information meaningful for over half a century and continue to employ it in modern techniques such as H_∞-based design.

In this paper we've elected to compare models synthesized using FD-MODA and using another model synthesis algorithm, EXTENDED-MODA. We chose to compare the algorithms using the same system that Ferris and Stein (1995) used to demonstrate EXTENDED-MODA; however, unlike Ferris and Stein, we modeled a continuous component with a finite segment model, rather than with a finite model based on (we believe) invalid boundary conditions. This comparison is a relatively minor part of this paper, and is intended to demonstrate the utility of the comprehensive model performance metric used by FD-MODA, as contrasted with the eigenvalue based approach used by EXTENDED-MODA. The main contribution is a new modeling algorithm, FD-MODA, and not the example involving the comparison.

Open Issues

As noted in the previous section, we believe that FD-MODA provides an efficient means of synthesizing a linear state-space model that meets a useful performance criterion. However, we have identified several open research issues that merit comment:

- All of the elements of the FD-MODA framework presented in Section appear to be sound. What remains an open question is proof that FD-MODA indeed synthesizes a model of *minimal* order that has converged to a final frequency response with accuracy within some tolerance. We have not been able to prove this, and thus limit our claim to the following:

  > FD-MODA provides an effective heuristic for coordinating the synthesis of a model that has converged, relative to other lower-rank models of the same system, to a user-specified tolerance over a user-specified frequency range $[\omega_{min}, \omega_{max}]$.

- We consider finite-segment (lumped-parameter) models of the (potentially) distributed-parameter components in the system. One might also use a continuum-based model for the same continuous element. It is not clear which approach is preferable, as each has its problems:
  - Finite-segment models are simple to create and simple to use in a state-determined system model. However, it is well known that care must be taken to include sufficient degrees of freedom to achieve desired accuracy over a FROI.
  - Continuum-based models are more accurate but more difficult to obtain. They can be represented with a finite-mode bond graph (Karnopp et al., 1990) and used in building the system model; however, even though standard "solved" models exist for ideal continuous elements in isolation (e.g. a torsional shaft with no discrete inertias attached to it), the use of these solved models inside of a larger system model without consideration for adjacent elements may result in serious modeling errors. While procedures to synthesize continuum-based models of non-uniform elements (such as a shaft with discrete inertias) do exist (Gizhong and Wilson, 1994) they are tedious to use.

  The use of either type of model depends on the requirements of the model, but specific guidelines for selecting the appropriate model are not currently available.

- Finally, the generality of FD-MODA could be greatly broadened by extension to nonlinear systems. In this context, describing-function methods provide a direct approach for achieving this goal, and frequency-domain performance metrics are even more appropriate than in the linear case (cf. Taylor, 1983; Taylor and O'Donnell, 1990) for examples of using DF-based frequency-domain modeling for nonlinear control system design). Preliminary research in extending MODA for nonlinear systems was reported in Wilson and Taylor (1993) and resulted in MODANS, a MODA for nonlinear systems; however only eigenvalues were considered as the performance metric in that work. The corresponding frequency-domain extension, FD-MODANS is described in Taylor and Wilson (1995). The basic idea underlying generating *amplitude-dependent* frequency-domain models to be used in this context is as follows (refer also to Taylor, 1983, and Wilson and Taylor, 1993):
  - Create a fully nonlinear model of the electro-mechanical system;
  - augment it by adding states to serve as Fourier integrals;
  - excite the model with a sinusoidal input of suitable frequency ω and appropriate amplitude a;
  - numerically integrate the model for a sufficient number of cycles so that the Fourier integrals have time to converge; and
  - use the Fourier integrals as the basis for evaluating $G(j\omega, a)$.

  Once a set of frequency-domain models is obtained for the FROI and amplitude range of interest one may not only deduce how many modes to use in modeling each continuous component, but also decide which nonlinearities are important for realistically characterizing the behavior of the system.

Conclusion

Automated modeling approaches offer great promise for expediting the development process for electro-mechanical systems. In many contexts, most especially in developing controls for such systems, FD-MODA provides a useful technique for synthesizing a low-order yet accurate state-space model for the electro-mechanical assembly that is to be analyzed and designed.

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