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ROBUST NONLINEAR CONTROL BASED ON DESCRIBING FUNCTION METHODS

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Abstract: The robust control problem for nonlinear systems is discussed from the standpoint of the amplitude sensitivity of the nonlinear plant and final control system. Failure to recognize and accommodate this factor may give rise to nonlinear control systems that behave differently for small *versus* large input excitation, or perhaps exhibit limit cycles or instability. Sinusoidal-input describing functions (SIDFs) are shown to be effective in dealing with amplitude sensitivity in two areas: *modeling* (providing plant models that achieve an excellent trade-off between conservatism and robustness) and *nonlinear control synthesis.* In addition, SIDF-based modeling and synthesis approaches are broadly applicable. Several practical SIDF-based nonlinear compensator synthesis approaches are presented and illustrated via application to a position control problem.

Keywords: Nonlinear control systems; describing functions; frequency-domain modeling; fourier analysis; frequency-domain design; robust control; synthesis methods; PID control; fuzzy logic; position control.

1 INTRODUCTION

A major cause for concern in nonlinear control is the amplitude sensitivity of the nonlinear plant and final control system. It is well known that nonlinear control systems that are designed without accounting for this factor may not exhibit robust performance (e.g., may behave differently for small *versus* large input excitation, or perhaps exhibit limit cycles or instability). Modeling and synthesis methods that accommodate plant amplitude sensitivity may thus provide major benefits in the design of robust nonlinear control systems.

Various ways to deal with amplitude sensitivity exist in the context of models for control system design. These include replacing nonlinearities with linear elements having gains that lie in ranges based on:

- nonlinearity sector bounds,
- slope bounds,
- random-input describing functions (RIDFs), or
- sinusoidal-input describing functions (SIDFs).

It is argued in Section 3 that frequency-domain plant input/output models based on SIDFs provide an excellent trade-off between conservatism and robustness in this context. In particular, it is shown by example that sector and slope bounds may be excessively conservative, while RIDFs are generally not robust in the sense that a nonlinear control system design predicted to be stable based on RIDF plant models may limit cycle or be unstable. An important attribute of SIDF-based frequency-domain models is that they account for the fact that the effect of most nonlinear elements depends on frequency as well as amplitude; the other techniques do not capture both of these traits.

Design approaches based on SIDF models are all frequency-domain in orientation. The basic idea of all methods presented in Taylor (1983), Taylor and Strobel (1984, 1985), Taylor and Åström (1986) and Taylor and O'Donnell (1990) is to define a *frequency-domain objective* for the openloop compensated system and synthesize a nonlinear controller to meet that objective as closely as possible for a variety of error signal amplitudes (e.g., for small, medium and large input signals where the numerical values associated with the terms "small, medium and large" are based on the desired operating regimes of the final system). The designs are then at least validated in the time domain (e.g., step-response studies); recent approaches have added time-domain optimization to further reduce the amplitude sensitivity. The methods presented in Section 4 follow this paradigm.

Modeling and synthesis approaches based on these principles are broadly applicable. Plants may have any number of nonlinearities, of arbitrary type (even discontinuous or hysteretic), in any configuration. These methods are robust in several senses: In addition to dealing effectively with amplitude sensitivity, the exact form of each plant nonlinearity does not have to be known as long as the SIDF plant model captures the amplitude sensitivity with decent accuracy, and the final controller design is not specifically based on precise knowledge of the plant nonlinearities. The resulting controllers are simple in structure and thus readily implemented, with either piece-wise linear characteristics or fuzzy logic.

Several recent nonlinear compensator design approaches are presented and illustrated via an application to a position control problem. These synthesis techniques are based on amplitude-dependent SIDF models of the nonlinear plant coupled with the synthesis of controller nonlinearities via SIDF inversion, and represent extensions and improvements of methods and results reported previously (Taylor and Strobel, 1984 and 1985; Taylor and Åström, 1986). One breakthrough required for this work was devising a way to synthesize nonlinear controllers with rate feedback (Taylor and O'Donnell, 1990). Another was making a connection that permitted extension to the direct synthesis of fuzzy-logic controllers (Taylor and Sheng, 1996 and 1997).

The remainder of this paper is organized as follows: Section 2 defines SIDF modeling; Section 3 outlines approaches and issues in modeling for nonlinear control system design; Section 4 presents several nonlinear controller configurations and relates the corresponding number of "degrees of freedom" (DOF) to the degree and form of the nonlinear plant amplitude sensitivity and then describes two nonlinear controller synthesis approaches; and Section 5 illustrates the application of SIDF-based approaches to a simply-stated but difficult position control problem.

2 SIDF MODELING

The basic idea of the describing function (DF) approach for modeling and studying nonlinear system behavior is to replace nonlinear elements with (quasi)linear descriptors whose gains are a function of input amplitude. These descriptors are governed by the form of input signal, which is assumed in advance. This technique is dealt with very thoroughly in a number of texts for the case of a single nonlinearity (Gelb and Vander Velde, 1968; Atherton, 1975); for systems with multiple nonlinearities in arbitrary configurations, the most general extensions may be attributed to Kazakov (1965) in the case of random-input DFs (RIDFs) and jointly to Taylor (1975, 1980) and Hannebrink et al (1977) for sinusoidal-input DFs (SIDFs). These developments have been presented in tutorial form in Ramnath, Hedrick and Paynter (1980).

The SIDF approach can be used for two primary purposes: limit cycle analysis and characterizing the input/output (I/O) behavior of a nonlinear plant in the frequency domain (cf. Taylor, 1980). It is the latter application - called SIDF *modeling* hereafter - that serves as the basis for the work presented here.

There are two methods for obtaining an SIDF I/O model of a nonlinear plant. One approach is to replace each static nonlinearity in the plant differential equation set with the corresponding describing function in analytic form and then set up and solve the equations of harmonic balance. The equations for determining an SIDF-based I/O model in this way are given in Taylor and Strobel (1984). Another, more direct, technique for obtaining an SIDF I/O model involves *simulation plus fourier analysis methods* (cf. Taylor, 1982). In the specific method described in Taylor and Lu (1993), fourier analysis is done in parallel with the simulation (fourier integrals are evaluated as the nonlinear plant is being integrated) which is very accurate and efficient. Using either approach, the designer can obtain the required SIDF I/O model in a straightforward manner. Finally, direct measurement of the frequency response of a nonlinear plant may be performed for a varity of amplitudes, using a spectrum analyzer.

The key equations necessary to define an SIDF I/O model are as follows: The nonlinear plant is assumed to be characterized by the general state-variable differential equation set and output equation

$$\dot{x} = f(x, u) \quad , \quad y = h(x, u) \tag{1}$$

where x is an n-dimensional state vector, u is a scalar input, and y is a scalar output variable. We are concerned with the behavior of the plant in the presence of sinusoidal signals, so we take u and y to have the form

$$\begin{aligned} u(t) &\triangleq u_0 + Re[a \exp(j\omega t)], \\ y(t) &\cong y_c + Re[c \exp(j\omega t)] \end{aligned} \tag{2}$$

where u_0 represents the operating point or "dc value" of u(t) and a is the sinusoidal component amplitude. In developing SIDF models the state variables and y are assumed to be approximately sinusoidal (this should be validated); then the corresponding components of y are characterized by y_c, the output *center value*, and c, the *complex amplitude* in standard phasor notation. The end result from either harmonic balance or simulation plus fourier analysis is that we neglect higher harmonics to approximate the sinusoidal-input I/O relation corresponding to Eqn. (1) as follows:

$$c(j\omega; u_0, a) \cong G(j\omega; u_0, a)a \tag{3}$$

where the SIDF I/O model is explicitly determined by u_0, a, as the notation indicates. Note that Eqn. (3) is exact in the fourier analytic sense ($c(j\omega)$ representing the first harmonic of y) if $G(j\omega, a)$ is obtained by simulation plus fourier analysis; in addition, there is no need to argue that the inputs to every nonlinearity are nearly sinusoidal in that case. To simplify notation, in cases where the operating point is zero the I/O relation is denoted $G(j\omega, a)$.

3 MODELS FOR CONTROL DESIGN

Several techniques for dealing with amplitude sensitivity were mentioned in Section 1. These include replacing each plant nonlinearity with a linear characteristic having a gain that lies in a range based on sector bounds, slope bounds, random-input describing functions (RIDFs), or sinusoidal-input describing functions (SIDFs). It was stated that SIDF I/O plant models provide an excellent trade-off between conservatism and robustness in this context; this is a vital point that merits detailed discussion.

3.1 SIDF and Linear Model Families

SIDF modeling gives rise to a family of models that corresponds to a range of input amplitude. Linear model families ($\dot{x} = Ax + Bu$) can be obtained by replacing each plant nonlinearity with a linear element having a gain that lies in a range based on its sector bound or slope bound. We will hereafter call these model families *sector* I/O *models* and *slope* I/O *models,* respectively. (Robustness cannot be achieved using *one* linear model based on the slope of each nonlinearity at the operating point for design, so that alternative is not considered.)

From the standpoint of robustness in the sense of maintaining stability in the presence of plant I/O variation due to amplitude sensitivity, it has been established that none of the model families defined above are sufficient basis for a *guarantee.* The idea that sector I/O models would suffice is called the Aizerman conjecture, and the premise that slope I/O models are useful in this context is the conjecture of Kalman; both have been disproven even in the case of a single nonlinearity (for discussion, see Narendra and Taylor, 1973). Both SIDF and RIDF models similarly can be shown to be inadequate for a robustness guarantee in this sense (see also Narendra and Taylor, 1973).

Despite the fact that these model families cannot be used to guarantee stability robustness, it is also true that in many circumstances they are conservative (e.g., a particular nonlinearity may pass well outside the sector for which Aizerman's conjecture would suggest stability yet the system is still stable). On the other hand, only very conservative conditions such as the circle criterion (Narendra and Goldwyn, 1964) and the off-axis circle criterion (Narendra and Cho, 1967) serve this purpose rigorously - however the very stringent conditions these criteria impose and the difficulty of extension to systems with multiple nonlinearities generally inhibit their use. Thus many control system designs are based on one of the model families under consideration as a (hopeful) basis for robustness. *It can be argued that designs based on* SIDF I/O *models that predict that limit cycles will not exist by a substantial margin is the best one can achieve in terms of robustness* (see also Atherton, 1981). In SIDF-based synthesis the frequency-domain design objective (see Section 4) must ensure this.

Returning to conservatism, considering a static nonlinearity and assuming that it is single-valued and its derivative exists everywhere, it can be stated that slope I/O models are always more conservative than sector I/O models, which in turn are always more conservative than SIDF models. This is because the range of an SIDF cannot exceed the sector range, and the sector range cannot exceed the slope range. An additional argument that sector and slope model families may be substantially more conservative than SIDF I/O models is based on the fact that only SIDF models account for the frequency dependence of each nonlinear effect. This is especially important in the case of multiple nonlinearities, as illustrated by the simple example depicted in Fig. 1: Denoting the minimum and maximum slopes of the gain-changing nonlinearities f_k by $\underline{m}_k$ and $\overline{m}_k$ respectively, we see that the sector and slope I/O models correspond to all linear systems with gains lying in the indicated rectangle, while SIDF I/O models only correspond to a "gain trajectory" as shown (the exact details of which depend on the linear dynamics that precede each nonlinearity). In many cases, the SIDF model will clearly prove to be a less restrictive basis for control synthesis.

3.2 SIDF and RIDF Models

There are two basic differences between SIDF and RIDF models for a static nonlinearity (Gelb and Vander Velde, 1968; Atherton, 1975): The assumed input amplitude distribution is different, and SIDFs can characterize the effective "phase shift" caused by multivalued nonlinearities such as those commonly used to represent hysteresis and backlash, while RIDFs cannot. These issues were discussed in some detail in Taylor (1983); in particular, it was shown that the input amplitude distribution issue is generally not a major consideration. The importance of the phase shift issue may be a matter of modeling judgment; hysteresis and backlash can be modeled without the use of multivalued nonlinearities, which would eliminate the second difference in distinguishing between the two DF methods.

However, there is a third difference (Taylor, 1983) that impacts the I/O model of a nonlinear plant in a fundamental way. This difference is related to how the DF is used in determining the I/O model; the result is that RIDF plant models (as usually defined, cf. Rajarao and Mahalanabis, 1970; Hedrick, 1976) do not capture the frequency dependence of the system nonlinear effects.

This difference arises from the fact that the standard RIDF model is the result of *one* quasilinearization procedure carried out over a wide band of frequencies, while the SIDF model is obtained by quasilinearizing at a *number* of frequencies. This behavior is best understood via the simple example from Taylor (1983) involving a low-pass linear system followed by a saturation (unity limiter):

$$f(v) = \begin{cases} v\,, & |\,v\,| < 1 \\ \text{sgn}(v)\,, & |\,v\,| \geq 1 \end{cases} \quad (4)$$

- Considering sinusoidal inputs of amplitude substantially greater than unity, the following behavior is exhibited: Low-frequency inputs are only slightly attenuated by the linear dynamics, resulting in heavy saturation and reduced SIDF gain; however, as frequency and thus attenuation increases, saturation decreases correspondingly and eventually disappears, giving a response that approaches the output of the low-pass linear dynamics alone.
- A random input with rms value greater than one, on the other hand, results in saturation at all frequencies, so $G(j\omega, a)$ is identical to the linear dynamics followed by a gain less than unity.

Bode plots for this example were presented in Taylor and Strobel (1984). The SIDF approach captures both a gain change and an effective increase in the transfer function magnitude corner frequency, while the RIDF model shows only a gain reduction.

More striking differences were observed in a robotic arm modeling study (Taylor and Strobel, 1984). In that case, both experimental and simulation-derived RIDF and SIDF I/O models were obtained and the difference between the two DF models was large, especially at low frequencies where the magnitude was more than 12 dB higher for the SIDF model compared with the RIDF case. Combining these observations, it is safe to say that one can easily find or devise cases where SIDF models predict limit cycles or instability while RIDF models may erroneously predict a good margin of stability.

These examples show that RIDF and SIDF I/O models differ in an important way. More specifically, they demonstrate that control systems designed using SIDF I/O models are more likely to be robust than those designed using RIDF models. (Aside: By their very definition, RIDFs were not intended to serve as a basis for robust design; SIDFs, being Hopf bifurcation analysis tools, are more suitable.)

4 NONLINEAR CONTROL SYNTHESIS

The nonlinear controllers synthesized by the various SIDF-based methods discussed in Taylor and Strobel (1984, 1985) and Taylor and Åström (1986) are all simple in structure, being comprised of parallel paths made up of linear dynamics in series with static nonlinearities that are readily implemented (piece-wise linear functions of the input). Figure 2 depicts these and related controller configurations.

These controllers are characterized by "degrees of freedom" (DOFs), in the following sense: the 1-DOF compensator (Fig. 2a) realizes a *single* amplitude-dependent gain over all frequencies, while the 3-DOF cases (Figs. 2b, 2c) provide the ability to synthesize independent gain/amplitude relations at low, middle and high frequencies corresponding to the integral, proportional, and derivative terms respectively. (Most other compensator types, e.g., the lead/lag compensator, can be decomposed similarly.) It is clear that simple amplitude sensitivity problems can be addressed using the 1-DOF configuration; more degrees of freedom may be needed if the amplitude dependence is complicated.

A 1-DOF controller synthesis approach was described in Taylor and Strobel (1984) and employed in Taylor and Åström (1986); a more effective 3-DOF approach was presented in Taylor and Strobel (1985). Finally, we realized a more effective 3-DOF configuration, having the derivative term as rate feedback (Taylor and O'Donnell, 1990; see Fig. 2c). This is motivated by the practical consideration that taking the derivative of the reference input produces large plant input signals if the reference can make abrupt changes; since the final test of our SIDF-based synthesis approaches is a step-response study, the rate feedback configuration is definitely preferable.

The two most recent SIDF-based nonlinear control system synthesis approaches are presented below. The first yields a 3-DOF algorithm with nonlinear rate feedback, which is designed entirely in the frequency domain, and the second generates a fuzzy-logic controller based on the first result and refined by time-domain optimization. In Section 5, these methods are demonstrated on a position control problem.

Before proceeding, it is important to reiterate the premises of the sinusoidal-input describing function design approaches that we have been developing:

1. The nonlinear system design problem being addressed is the synthesis of controllers that are effective for plants having frequency-domain I/O models that are *sensitive* to input amplitude (e.g., for plants that behave very differently for "small", "medium" and "large" input signals).
2. Our primary objective in nonlinear compensator design is to arrive at a closed-loop system that is as *insensitive* to input amplitude as possible.

This encompasses a limited but important set of problems, for which gain-scheduled compensators cannot be used (gain-scheduled compensators can handle plants whose behavior differs at different operating points but not amplitude-dependent plants) and for which other approaches (e.g., variable structure systems, model-reference adaptive control, global linearization) do not apply because their objectives are different (e.g., their objectives deal with *asymptotic* solution properties rather than *transient* behavior, or they deal with the behavior of *transformed* variables rather than physical variables).

4.1 3-DOF Nonlinear Controller Synthesis

An outline of the synthesis algorithm for the nonlinear PI plus rate feedback (PI+RF) controller is as follows:

1. Select sets of input amplitudes and frequencies that characterize the operating regimes of interest.
2. Generate sinusoidal-input describing function models of the plant corresponding to the input amplitudes and frequencies of interest.
3. Design amplitude-dependent rate-feedback gains using an extension of the D'Azzo and Houpis (1960) algorithm, devised by Taylor and O'Donnell (1990).
4. Convert these linear designs into a piece-wise linear characteristic, by sinusoidal-input describing function inversion (adjusting the slopes and breakpoints so the nonlinearity's sinusoidal-input describing function fits the gain / amplitude data from Step 3 with minimum mean square error).
5. Find sinusoidal-input describing function models for the nonlinear plant plus nonlinear rate-feedback compensation.
6. Design PI compensator gains using the frequency-domain sensitivity minimization technique described in Taylor and O'Donnell (1990).
7. Convert these linear designs into a piece-wise linear PI controller, also by sinusoidal-input describing function inversion.
8. Develop a simulation model of the plant with nonlinear PI+RF control.
9. Validate the design through step-response simulation.

Since these steps are described in some detail in Taylor and O'Donnell (1990) we do not provide more detail here.

4.2 Fuzzy-Logic Controller Synthesis

Fuzzy-logic-based or fuzzy control has a long record of development and application. For the purposes of this presentation, we merely point out the well-established connection between one standard form of fuzzy control – where the decision variables are "error" and "error-rate-of-change" – and proportional plus derivative (PD) control (Tong, 1977). In many applications, this idea is implemented by using operator heuristics to describe what the linguistic variables such as "large-negative", "small-negative", "zero", "small-positive", "large-positive" mean in terms of membership functions for error and error-rate-of-change, and then to describe the appropriate control action to take (level of plant input to apply) under various circumstances, e.g., "if error is large-negative and error rate is small-positive then control is small-positive". Of course, the details such as the number of linguistic variables etc. vary from application to application; however, this general idea is the basis for an important class of fuzzy-logic controllers (FLCs). A second type of FLC corresponds in the same analogous way with PI control.

As mentioned, in many applications the set of fuzzy rules is based on operator heuristics. In other cases, the control engineer may be the source of this knowledge. While these approaches have proven to be effective in numerous applications, there are many circumstances where the fuzzy logic rulebases are difficult or impossible to generate in this fashion. In these cases, a model-based approach for generating the set of fuzzy rules may be more appropriate. Situations where the system dynamics are too fast for a human operator to be able to cope and too nonlinear for the engineer to write a rulebase by intuition would seem to be two cases in point, and electromechanical systems with nonlinear friction would be a prime example (Section 5).

Our approach for fuzzy-logic controller synthesis uses the above purely sinusoidal-input describing function-based design as a starting point, exploiting the fact that a single-input / single-output fuzzy-logic rulebase can implement a piece-wise linear characteristic, if the membership functions are selected suitably. An example of this is shown in Fig. 3, for one of the nonlinearities used in the illustrative application. The following steps are added to the recipe provided above for the nonlinear PI+RF controller:

1. Modify Step 4 above, to generate a rate-feedback FLC instead of a piece-wise linear one.
2. Modify Step 7 above, to generate FLCs for the proportional and integral channels, instead of piece-wise linear ones.
3. Optimize and validate the design through recursive step-response simulation.

Again, these steps are described in some detail in Taylor and Sheng (1996, 1997), so we do not provide much detail here. The main area of difference is in the use of *time-domain optimization* to refine the behavior of the preliminary FLC design generated as outlined in Section 4.1. It should be emphasized that the availability of this preliminary design which already closely achieves the objective of reduced sensitivity to input amplitude is critical to the success of the optimization step – without this starting point, there is little hope that optimization by itself could achieve a viable design.

Even with a good preliminary design, it is important to pose the optimization problem correctly to achieve the desired result and a solvable problem. First, the objective function Φ is expressed in terms of a target insensitive unit step response $h^*(t)$ as:

$$\Phi = \sum_{i=1}^{I} \sum_{k=1}^{K} w_k (h(t_k; a_i)/a_i - h^*(t_k))^2 \qquad (5)$$

where $h(t_k; a_i)$ denotes the step response of the control system to a step of amplitude a_i at the integration time steps t_k, and w_k are weighting factors to permit trade-offs between features of the transient response (e.g., overshoot) and those of the steady-state solution (e.g., the amount of steady-state offset).

The amplitudes a_i, $i = 1, 2, \ldots I$ may be selected to be the same as in the sinusoidal-input describing function design stage, or they may differ; it is important, however, that they be roughly consistent. We found in the application outlined below that weights had to be higher after the initial transient, to alleviate the effect of "sticking"; in general, the selection of these weights will be application and scenario specific. Most importantly, the target insensitive step response $h^*(t)$ must be approximately achievable by the controlled plant. Here we have adopted a simple strategy for selecting it appropriately: define $h^*(t)$ to be the average of the normalized step responses achieved by the preliminary FLC. More advanced strategies are presented in detail Taylor and Sheng (1998).

5 POSITION SERVO EXAMPLE

The nonlinear plant from Taylor and Strobel (1985) was used to provide a test case for the approaches outlined above. In brief summary:

- The nonlinear plant is a simple model of a position control system with torque motor saturation and stiction (Fig. 4); these effects are notoriously difficult to handle.
- The I/O behavior of a feedback system with the first-cut linear PID controller (Taylor and Strobel, 1985) is shown in Fig. 5, top half. These time-histories show the response of the system to step inputs of different amplitudes (the responses are normalized by dividing the response by the input step amplitude); clearly there is substantial amplitude dependence, i.e., "sticking" for small amplitude inputs and excessive overshoot caused by saturation and integral wind-up for large step inputs. These problems cannot be solved by a linear change of the loop gain: if the gain is reduced, then the offsets due to sticking will be worse; if the gain is increased, then the excessive overshoot caused by integral windup will be aggravated.
- The I/O performance of the SIDF-based nonlinear controller (Taylor and O'Donnell, 1990) is depicted in Fig. 5, bottom half. While the design procedure only focussed on minimizing open-loop sensitivity in the frequency domain, the substantial reduction in closed-loop step-response amplitude sensitivity is noteworthy.
- Finally, the behavior of a feedback system with a nonlinear FLC synthesized by the combined SIDF and time-domain optimization approach is portrayed in Fig. 6. Since this synthesis method is based explicitly on the final objective of making the closed-loop system insensitive to amplitude in the time domain, it is not surprising that it exhibits even less amplitude sensitivity than the purely SIDF-based design.

6 SUMMARY AND CONCLUSION

A detailed discussion of sinusoidal-input describing function modeling as the basis for robust nonlinear control system design was presented. This was done to provide a perspective from which to compare and evaluate various modeling approaches in this context.

Several recently-developed nonlinear control system synthesis approaches based on SIDF modeling and SIDF inversion were presented and shown by example to provide substantial benefit in terms of producing nonlinear control system designs that are insensitive to the amplitude of the reference input. The resulting nonlinear controllers have three degrees of freedom (3-DOF), i.e., three nonlinear elements that can independently influence the amplitude sensitivity of the compensated system at high, middle and low frequencies, thus providing a good deal of flexibility in meeting this objective; the use of nonlinear rate feedback also provided improved performance compared with earlier results. In terms of generality, *observe that these approaches only require that the nonlinear plant model can be simulated with sinusoidal inputs and that it should produce a periodic response* – a mild restriction indeed.

Finally, we observe that the stated objective of amplitude insensitivity in terms of frequency- and step-response addresses one specific but common control system requirement, which might aptly be called *performance robustness.* This is merely one aspect of robust nonlinear control, and thus it is not particularly meaningful to compare the scope and efficacy of these approaches with other techniques such as model reference adaptive control, sliding mode control, and linearizing transforms which do not and *can* not address this problem.

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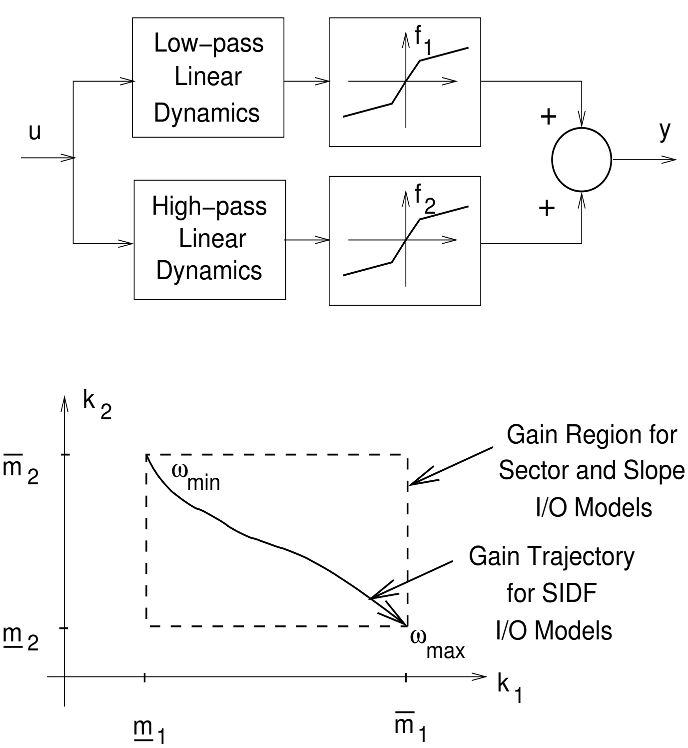


Figure 1: Conservatism of Model Families

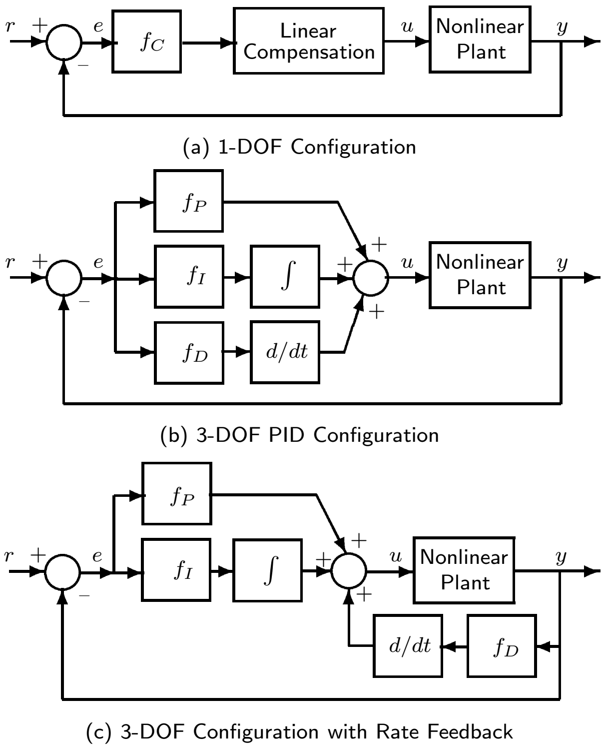


Figure 2: Controller Configurations

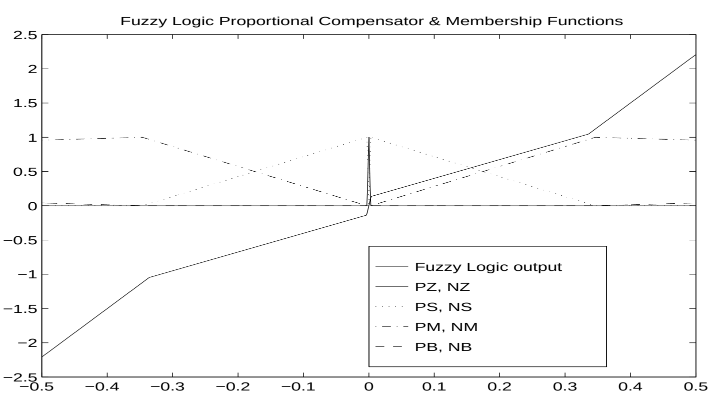
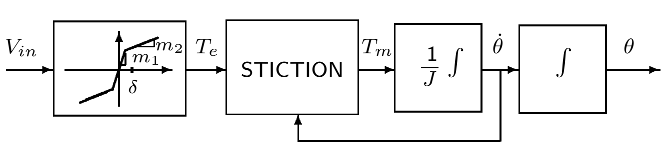


Figure 3: Piece-wise Linear FLC Characteristic



$$T_m = \begin{cases} T_e - f_v\dot{\theta} - f_c\,\mathrm{sign}(\dot{\theta})\,, & \dot{\theta} \neq 0 \text{ or } |T_e| > f_c \\ 0\,, & \text{otherwise} \end{cases}$$

Figure 4: Nonlinear Positioning Plant Model

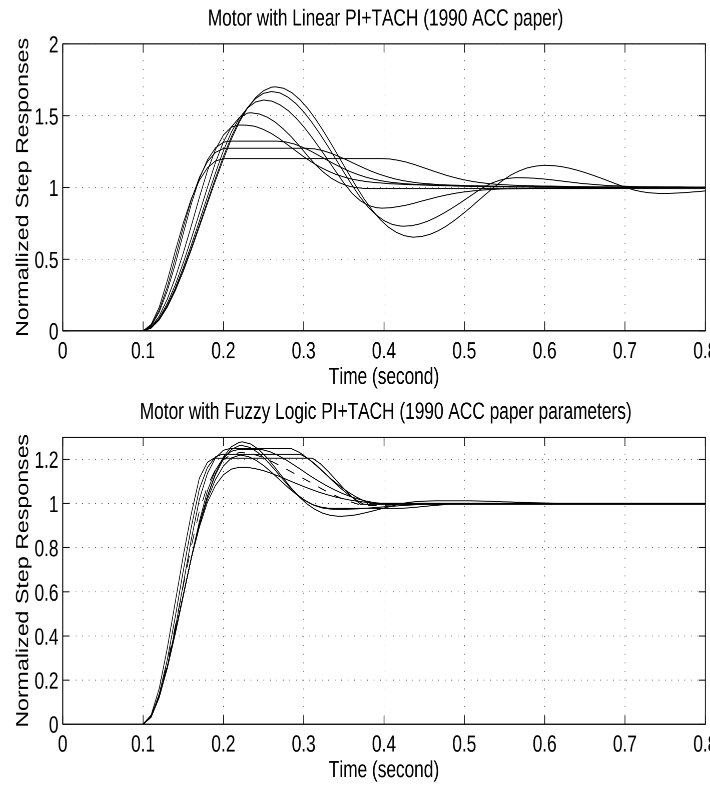


Figure 5: Control System Performance with Linear and Nonlinear (SIDF-based) Compensator (Fig. 2c)

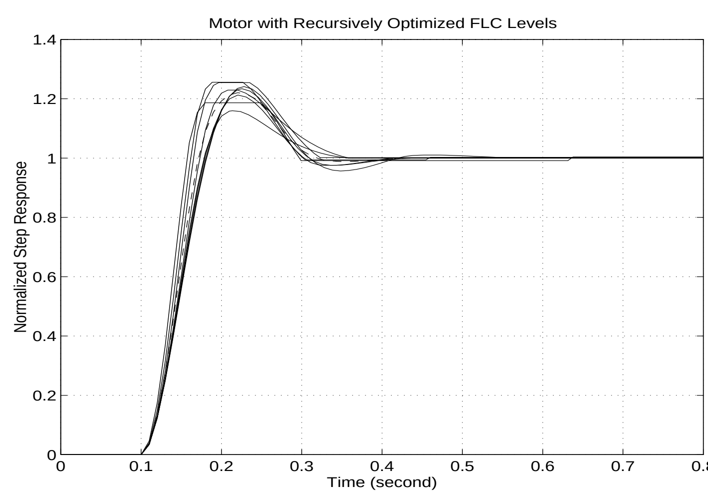


Figure 6: Control System Performance with Optimized FLC