

# MODEL PREDICTIVE CONTROL OF A MECHANICAL PULP BLEACHING PROCESS

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Abstract: In this paper we present and discuss all aspects of controlling a real-world delay-time system application, the pulp bleaching process at Irving Paper Ltd. The bleaching process was thoroughly studied and modelled. A delay-time estimator was designed to tackle the problem of the long variable delay time, which was considered the biggest challenge in this project. The model predictive control (*MPC*) strategy was chosen to control the bleaching process taking into account its constraints, which were handled by incorporating a state of the art optimization method, i.e., an interior point method, in the controller. The designed *MPC* controller was implemented in the Irving Paper mill, in order to test and demonstrate its performance and stability.

Keywords: Time delay systems, model predictive control, interior point methods, pulp bleaching process

## 1. INTRODUCTION

Pulp brightness is measured as reflectance in the blue portion of the visible spectrum. Complete reflectance provides a white color. Absorption of any part of the visible spectrum by a material will result in the perception of color by the eye. Pulp brightness is measured against a magnesium oxide (MgO) standard on a scale of 0-100, which defines the *ISO* standard. Bleached kraft (white printing paper), for example, has brightness values ranging from 86 – 94 %*ISO*, unbleached kraft (brown paper bags) has a brightness of 20 – 30 %*ISO*, and newsprint is around 55 %*ISO*. Pulp darkness is due to lignin and lignin degradation products, and the specific compounds which cause light absorption (and therefore a colored pulp) are termed chromophores (Dence and Reeve, 1996).

The objective in the bleaching of mechanical pulps is to selectively remove the color-contributing groups while simultaneously preserving a high pulp yield. This involves mainly the use of bleaching agents such as hydrogen peroxide and sodium hydrosulfite. Hydrogen peroxide is the most widely used oxidative bleaching agent in mechanical pulp bleaching, particularly where high brightness is desired.

The mechanical pulp bleaching process at Irving Paper is mostly manually controlled, which degrades the quality of the produced paper due to the variations in the pulp brightness. The objective of this research was to study the possibility of controlling the pulp bleaching process at the Irving Paper mill. This would improve the pulp quality by minimizing the final pulp brightness variability, and achieve some economical benefits by minimizing the consumption of the bleaching chemicals. In section 2 of this paper the bleaching process and its dynamics are discussed. Process

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modeling is thoroughly addressed in section 3. In section 4 the process controller design is covered. The simulation and implementation results are demonstrated in sections 5 and 6. The research conclusions are given in section 7.

## 2. PROCESS DESCRIPTION AND DYNAMICS

The bleaching of mechanical pulp with hydrogen peroxide is usually carried out by pretreating the pulp with pentasodium diethylenetriamine-pentaacetic (*DTPA*) solution (an alkaline solution) to remove transitional metal ions in pulp, then the pulp is mixed with an alkaline peroxide bleaching solution (bleach liquor). The mixture of pulp and liquor is then held in a bleaching tower for several hours at temperatures that range from 60° to 82°C. After exiting the tower, the pulp *pH* is lowered by adding sulphur dioxide (*SO*<sub>2</sub>) to prevent alkaline reversion and to decompose the residual peroxide. Then the pulp is sent to the paper machine or to dryers when produced as a market pulp (Persley and Hill, 1996). The single-stage medium-consistency peroxide bleach plant as shown in figure 1 (Persley and Hill, 1996) is typical.

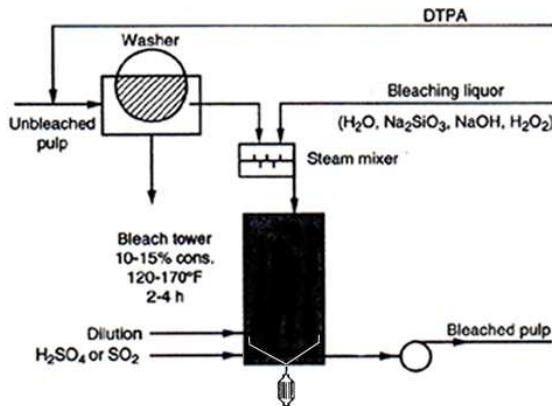


Fig. 1. Flowsheet for single-stage peroxide bleach plant.

The dynamic behavior of the bleaching process is very complex and quite nonlinear, since many variables affect the process dynamics such as chemical dosages, pulp alkalinity (*pH*), pulp consistency, retention time, production rate, and temperature. Basically the bleaching process dynamics can be divided into two parts. The first part involves the chemical kinetics of the bleaching reaction itself; compared with the time constants of the pulp mixing and transport processes this can be neglected.

The second part handles the dynamics of the pulp transport and mixing in the continuous flow system of the process, which mainly consists of the chemical mixer and the bleaching tower. The pulp stock is essentially in a state of plug flow inside the tower. The flow system can therefore be

represented by a continuous stirred tank reactor (*CSTR*) followed by a plug flow reactor (*PFR*). Consequently the mathematical model of the flow system assuming that the volume of the pulp inside the tower is constant, is given by (Wen and Fan, 1975):

$$C_{ko}^i(t - T_d(t)) = \frac{Q(t)}{V_{CSTR}} (C_{ki}(t) - C_{ko}(t - T_d(t))) \quad (1)$$

where  $C_{ki}$ ,  $C_{ko}$  are the chromophores concentrations at the tower inlet and outlet respectively,  $Q(t)$  is the pulp flow,  $V_{CSTR}$  is the volume of the mixing part of the tower, and  $T_d$  is the delay time resulting from the pulp travel inside the plug flow part of the tower.

Simulation studies (Qian and Tessier, 1997) on the mathematical model given by the kinetic model and equation 1 have shown that the bleaching process can be modeled by first order nonlinear dynamics plus a delay time. The peroxide dosage may be treated as the model input, and the other variables such as pulp consistency and initial brightness, *SO*<sub>2</sub> dosage, etc. can be considered as measured disturbances.

## 3. PROCESS MODEL IDENTIFICATION

Since the bleaching process is complex and time-varying, neither mathematical modelling nor standard system identification methods can be applied alone. Besides, if linear time-invariant models are used in the identification procedure, the time variability of the flow system will result in inaccurate linear time-invariant models with unacceptably high variance of the parameters estimated. It often happens that a model structure with a number of unknown parameters can be derived from physical laws in most real-world processes such as the bleaching process at Irving Paper mill. Identification methods can then be applied to estimate the unknown parameters.

Analysis of data records collected from the Irving Paper mill and modeling simulation results have shown that the bleaching process model can be interpreted as three separate dynamics (Ni, July 2000, Nov 2001; Li and Court, July 2000, Jan 2001, Nov 2001, April 2002):

- A pure gain  $K$  represents the linearized bleaching reaction kinetics, since the reaction is essentially complete by the time the pulp reaches the brightness sensor.
- A long variable delay time  $T_d$  results from the plug flow pattern of the bleaching tower.
- A first order dynamics with a time constant  $\tau$  due to the *SO*<sub>2</sub> mixing process at the outlet of the tower.

Zenger et al (Ylinen and Zenger, 1994; Zenger, 1995) introduced the concept of a variable delay function, which can be used to estimate the delay time even though the volume is varying. During

the time that a hypothetical concentration pulse stays in the vessel, the volume of material must pass through the vessel irrespective of the flow changes. This can be stated mathematically as:

$$\int_{t-T_d(t)}^t Q_{out}(\tau)d\tau = V(t - T_d(t)) \quad (2)$$

where  $t$  is the time at which the material exits the reactor,  $Q_{out}$  is the outflow, and  $T_d$  is the transport delay. Alternatively, the delay function can be expressed in terms of inflow  $Q_{in}$  by substituting  $\dot{V}(t) = Q_{in}(t) - Q_{out}(t)$  in equation 2:

$$\int_{t-T_d(t)}^t Q_{in}(\tau)d\tau = V(t) \quad (3)$$

An algorithm for time delay estimation can be developed by using either equation 2 or 3 as follows:

- (1) Store the inflow measurements over a time interval which equals the maximum retention time of the reactor, with sampling time  $h$ .
- (2) Measure the volume at time  $t$  and set the counter  $k = t - h$ .
- (3) Integrate the inflow backwards from  $k$  to  $t$ .
- (4) If the integration result equals the volume at time  $t$  then stop and  $T_d(t) = t - k$ .
- (5) Else set  $k = k - h$  and go to step 3.

Once the delay-time sequence has been estimated for a certain data set, a least square fitting procedure is then applied to estimate the model parameters. Figure 2 illustrates the identification of the bleaching process, where the delay-time sequence is estimated from the pulp inflow and pulp level data as shown in the top plots. Then the estimated delay-time sequence along with the peroxide dosage (input data) and the measured pulp brightness (output data) sequences were incorporated in a least square identification method to estimate the other models parameters as shown in the bottom plots. The actual brightness sequence (noisy trace) and the modeled brightness sequence (solid trace) are plotted in the same figure.

Simulation results have shown that the gain of the process is in the vicinity of 7, and the time constant is in the vicinity of 50 minutes. The delay-time estimation algorithm seems to have worked well, because the transient parts of both the estimated and the real brightness responses take place at nearly the same time, as illustrated in the bottom plots. The results are not entirely accurate because the effects of incoming pulp brightness and  $SO_2$  dosage were not taken into consideration. Identification results were significantly improved when the  $SO_2$  dosage was considered as a second input to the bleaching process model.

#### 4. CONTROLLER DESIGN

Model predictive control (*MPC*), an optimal control strategy, was chosen to control the mechanical pulp bleaching process because it can handle

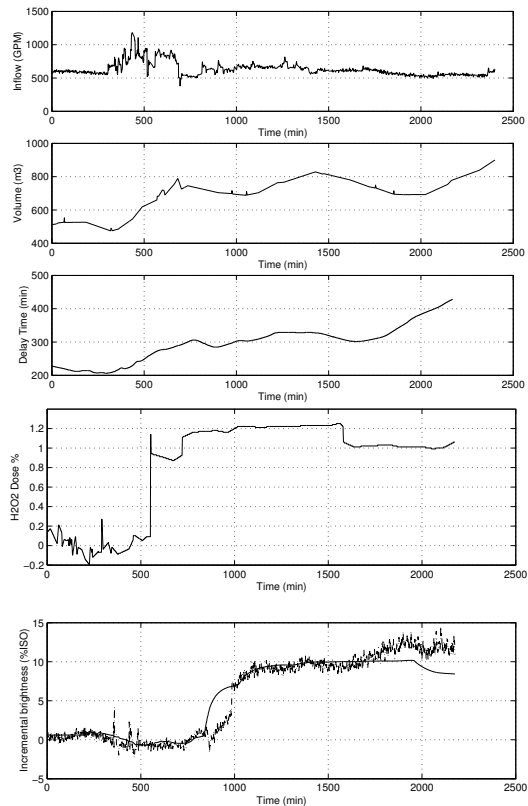


Fig. 2. Delay time estimation results (top set) and model parameter estimation results (bottom).

a great variety of processes, including multivariable processes and delay-time systems. *MPC* originated in the late seventies and has developed considerably since then. One may refer to many survey papers to get a better idea about the history of the *MPC* control strategy such as (Morari and Lee, 1999). Specifically, dynamic matrix control (*DMC*), see (Cutler and Ramarker, 1980), was chosen among the *MPC* methods because of its simplicity and efficiency. In fact, *DMC* has been applied in many successful industrial applications especially in the oil and chemical industries.

#### 4.1 Dynamic matrix control

*DMC* is an optimal control strategy that uses a step response model of a plant to predict the effect of an input profile on the evolving state of the plant. At each sampling instant, an optimal control problem is solved and its optimal plant input profile is implemented until another measurement becomes available. The updated plant information is used to formulate and solve a new optimal control problem, and the process is repeated.

The discrete-time response model of the plant is  $y(t) = \sum_{i=1}^{\infty} g_i \Delta u(t-i)$  where  $g_i$  are the sampled output values for the step input and  $\Delta u(t) = u(t) - u(t-1)$  is the input increment, so the prediction values along the horizon will be:

$$\hat{y}(t+k|t) = \sum_{i=1}^{\infty} g_i \Delta u(t+k-i) + \hat{n}(t+k|t) \quad (4)$$

Disturbances are considered to be constant  $\hat{n}(t+k|t) = \hat{n}(t|t) = y_m(t) - \hat{y}(t|t)$ , where  $y_m$  is the measured output. The prediction of the output sequence can be separated into two parts (Camacho, 1999). One of them, the free response  $f(t+k|t)$ , corresponds to the evolution of the present state of the process due to the past control moves. The other part, the forced response, is due to future control moves.

Now the prediction can be computed over the prediction horizon  $N_P$ , considering  $N_U$  control actions:

$$\begin{aligned} \hat{y}(t+1|t) &= g_1 \Delta u(t) + f(t+1|t) \\ \hat{y}(t+2|t) &= g_2 \Delta u(t) + g_1 \Delta u(t+1) + f(t+2|t) \\ &\vdots \\ \hat{y}(t+N_P|t) &= \sum_{i=1}^{N_P} g_i \Delta u(t+N_P-i) + f(t+N_P|t) \end{aligned}$$

hence the prediction can be expressed in terms of the system's dynamic matrix  $G$ , the control increments vector  $U$ , and the free response vector  $F$  as  $\hat{Y} = GU + F$ , where  $G$  is made up of  $N_U$  columns of the system's step response appropriately shifted down in order.

The objective of a *DMC* controller is to drive the future output sequence  $\hat{Y}$  as close to a specified future reference trajectory  $W$  as possible in a least square sense, with the possibility of the inclusion of a penalty term on the input moves. The cost function can be expressed as a function of the future control sequence  $U$  as:

$$J(U) = \frac{1}{2} U^T H U + b^T U + c \quad (6)$$

where  $H = 2(G^T G + \lambda I)$  is the hessian of  $J(U)$ ,  $b^T = 2(F - W)^T G$  is a  $1 \times N_U$  vector,  $c = (F - W)^T (F - W)$  is a constant, and  $\lambda$  is a positive constant that can be used to tune the *DMC* controller to meet the required performance.

The optimal control increments can be obtained analytically for the unconstrained case, which can be given by  $U = (G^T G + \lambda I)^{-1} G^T (W - F)$ . However most real-world processes are subject to constraints, which originate from amplitude limits in the control signal, slew rate of the actuator, and limits on the output signal. They can be expressed in terms of the control increments vector  $U$  as:

$$u_{min} l \leq T U + u(t-1) l \leq u_{max} l \quad (7a)$$

$$du_{min} l \leq U \leq du_{max} l \quad (7b)$$

$$y_{min} l \leq G U + F \leq y_{max} l \quad (7c)$$

where  $l$  is an  $N_U \times 1$  vector whose all elements are ones, and  $T$  is an  $N_U \times N_U$  lower triangular matrix whose non null entries are ones. In order to solve the optimal control problem imposed by the constrained *DMC* strategy, numerical optimization algorithms have to be implemented.

## 4.2 Interior point methods

It is easy to show that the *DMC* optimal control problem is a convex quadratic programming (*QP*) problem, since the cost function is quadratic with a positive definite hessian. Furthermore the constraints are linear inequalities which comprise a convex set. There are several classes of algorithms for solving the *QP* problem such as the barrier function methods and the feasible direction approaches (Luenberger, 1984).

One popular scheme for solving a quadratic program is the use of an interior point method. Since the presentation of the new polynomial-time algorithm by Karmarker in his landmark paper in 1984, the new field of interior point methods has witnessed rapid development and expansion (Potra and Wright, 2000). Interior point methods have been chosen to solve the *DMC* optimal control problem because they offer a number of advantages over the popular active set approach and other methods from a computational point of view (Potra and Wright, 2000; Wright and Nocedal, 1999): It is difficult for the active set algorithm to exploit any structure inherent in the *QP* problem without redesigning most of its complex linear algebra operations. An interior point approach, on the other hand, can exploit fully the properties of the system arising for each problem class. The active set approach is very efficient for small and medium scale problems, whereas the interior point approach is efficient for large scale problems.

In 1989 Mehrotra described a practical implementation which is considered the most efficient algorithm for *LP* problems, see (Mehrotra, 1992). Mehrotra's predictor-corrector algorithm builds on the theory of all primal-dual interior point algorithms together with other ideas from optimization and numerical analysis. It also incorporates a number of heuristics that have been developed during ten years of computational experience. Mehrotra's algorithm can be extended to convex *QP* problems (refer to (Wright and Nocedal, 1999) for further mathematical details). This has made it attractive for many applications such as optimal control and model predictive control (Wright, 1997; Rao *et al.*, 1997). Mehrotra's algorithm has been incorporated in the *DMC* controller of the bleaching process.

## 5. SIMULATION RESULTS

The mechanical pulp bleaching process at Irving Paper mill is a very complex process. For the sake of simplicity, the bleaching process is handled as a *SISO* system whose input and output are the peroxide dosage and the final pulp brightness respectively. Since the bleaching process has a variable delay time, a delay-time estimator is embedded in the *DMC* controller. A combination of a Smith predictor and feed-forward techniques has been embedded in the controller to compensate for the incoming pulp brightness variations (i.e., disturbance), and hence improve the performance

of the controller. The objective of the simulation is to study the nominal and robust performances of the *DMC* controller.

As far as the nominal performance, when a stair signal is applied to the reference input at different points on the variable delay-time history, the final brightness exactly tracks the reference brightness signal at different delay times, as illustrated in figure 3 (top plots). This demonstrates that the *DMC* controller works well even though the delay time is varying.

In order to study the behavior of the system in the presence of disturbances, a disturbance in the incoming pulp brightness was applied at the time instant  $t = 2000 \text{ min}$  as shown in figure 3 (bottom plots). When there is no feedforward compensation included in the *DMC* controller, the effect of the disturbance appears on the final brightness response (dash-dotted line) after some delay time and lasts for a long time (more than 500 min) before it is completely rejected. However the disturbance is immediately rejected when the feedforward plus smith predictor technique is included in the *DMC* controller (solid line), and the system performance is improved in comparison with the previous case. This is obvious from the peroxide dosage response (control signal) where it responds to the disturbance earlier in the feedforward compensated case.

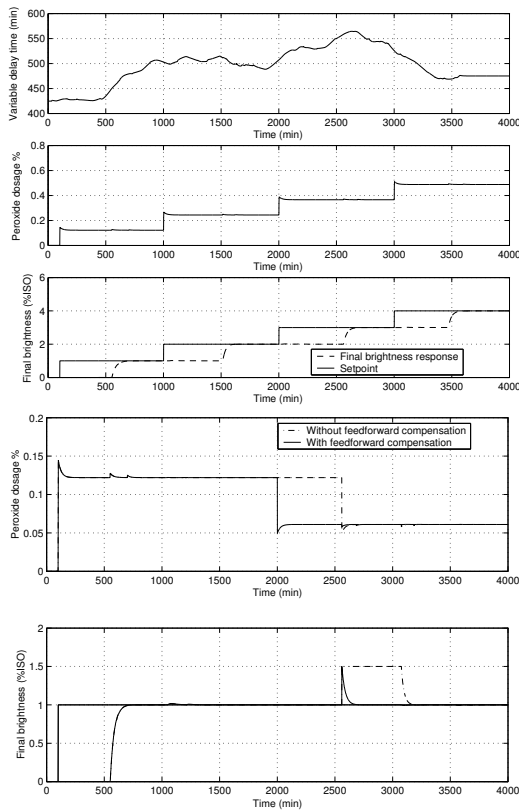


Fig. 3. Nominal performance: tracking behavior (top set), disturbance rejection (bottom set)

As for the robust performance, when the process has a gain uncertainty of  $\pm 20\%$ , the final bright-

ness response to a step at the reference input is markedly different from the nominal case. The final brightness does not track the reference for a fairly long time (an additional process delay time) as shown in figure 4 (top plots). It finally corrects itself, stepping up or down according to the sign of the uncertainty, and eventually gets back to track the reference. This implies that the *DMC* controller sets the peroxide dosage for the nominal case for some time, and then it corrects the dosage according to the sign and size of the gain uncertainty. The system response does not show much difference, when the time constant of the bleaching process is perturbed by  $\pm 20\%$ , as can be seen in figure 4 (bottom plots).

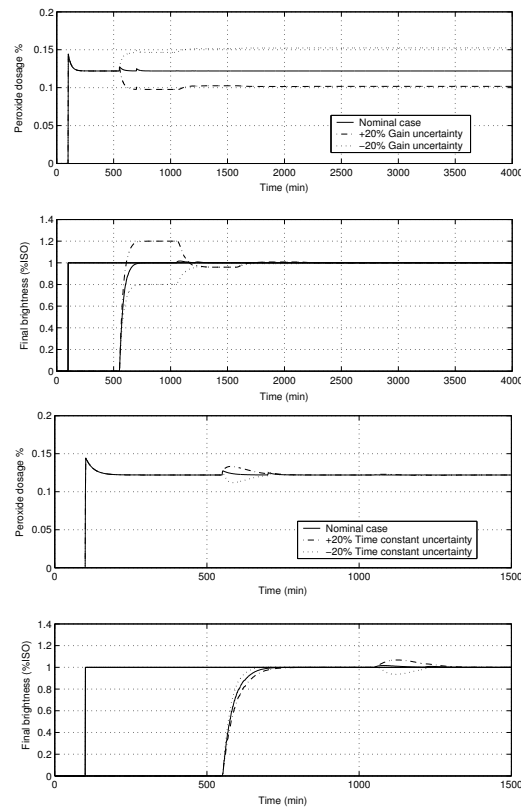


Fig. 4. Robust performance:  $\pm 20\%$  gain uncertainty case (top two plots),  $\pm 20\%$  time constant uncertainty case (bottom plots)

When introducing a delay-time uncertainty of  $\pm 5\%$  in the process, as can be observed in figure 5 (top two plots), the final brightness starts to exhibit peaking “blips” every 500 minutes, which only slowly decay. In order to explain those blips let’s consider the  $-5\%$  case, in which the brightness response occurs earlier than expected. This implies that the early measurement of the brightness will cause an error in the estimation of the free response in the *DMC* control algorithm. This in turn causes an error in the optimal control action in the form of a downward blip. Hence a blip in the brightness response will arise after some delay time that will result in another error, and the story will be repeated. Fortunately the blips decay after some time, which indicates that

the system is still stable. If the size of the delay-time uncertainty is increased to  $\mp 10\%$ , the amplitude of the oscillation in both the final brightness response and the peroxide dosage is increased. The system in this situation becomes unstable as illustrated in figure 5 (bottom plots). It can be concluded that the delay-time uncertainty must be small in order to obtain a high performance from the *DMC* controller and preserve its stability.

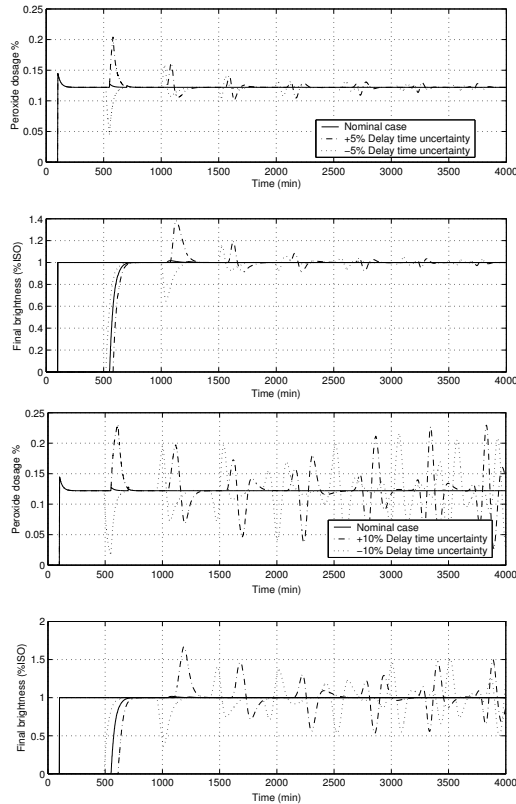


Fig. 5. Robust performance:  $\mp 5\%$  delay-time uncertainty case (top plots),  $\mp 10\%$  delay-time uncertainty case (bottom plots)

## 6. IMPLEMENTATION RESULTS

The *DMC* controller was implemented and tested on the bleaching process at the Irving Paper mill, to demonstrate its performance. It was implemented as an advisor, so the tests were made in a semiautomatic way, i.e., the control program was not allowed to write the optimal peroxide dosage directly to the process controller; rather, the test was done by applying a step change at the brightness reference input and having a mill operator change the peroxide dosage of the real bleaching process according to the recommended dosage of the control advisor.

A step change of  $4\%$  *ISO* was applied at the reference input at time instant  $t = 418$  minutes, which caused in a change in the final brightness response at time instant  $t = 700$  minutes, as shown in figure 6 (bottom plots). Both the recommended peroxide dosage and the actual one (top plot in the bottom set) were simulated by using a brightness model

that has the same parameters used in the control advisor. The simulated brightness responses were then plotted along with the measured final brightness after removing the bias (bottom-most plot). The estimated delay time from the volume and flow data are also shown (top 3 plots).

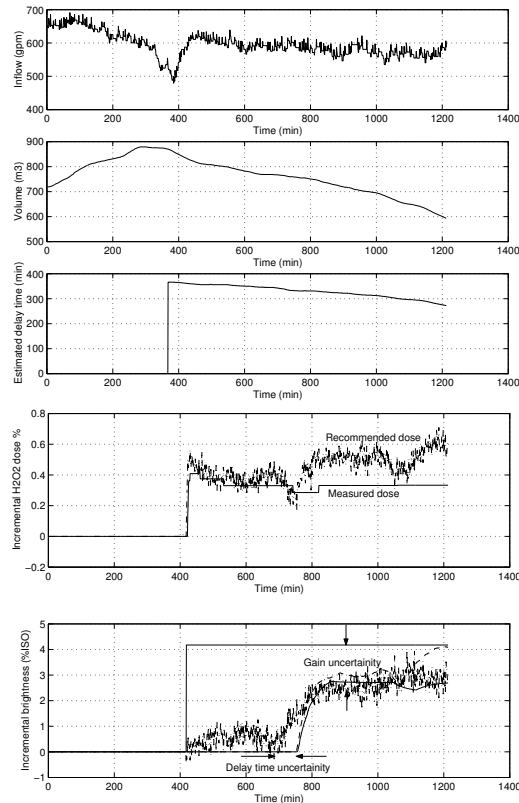


Fig. 6. Test results: the estimated delay time (top 3 plots), brightness and peroxide dosage responses (bottom plots)

Clearly the bottom plot reveals two interesting and yet significant observations:

- (1) Neither the simulated nor the measured brightness response tracked the set point. This is due to the inaccurate measurement of the final brightness sensor, which resulted in an apparent gain uncertainty in the bleaching process. However, the *DMC* controller increased the peroxide dosage at  $t = 770$  minutes to compensate for the gain uncertainty. As a result, the simulated brightness response was changed at time instant  $t = 1090$  minutes to attempt to track the set point. This indicates that the *DMC* controller can handle gain uncertainty, as the robust performance simulation has shown; however, the operator did not allow the real peroxide dosage to follow the recommended peroxide dosage, for economic considerations.
- (2) It is interesting to observe that the change in the real brightness response occurred earlier than it was supposed to. In other words, it should have occurred at the same time as the simulated brightness response. This implies

that there was delay-time uncertainty during the test. The effect of the uncertainty is also clear in the recommended peroxide dosage where a downward blip took place at the same time of the uncertainty. This result is identical to what was observed in the robust performance simulations. The reason for the delay-time uncertainty is because the pulp level inside the bleaching tower was decreasing during the test, which caused the brightness change to happen earlier than anticipated. This, in turn, raises questions about the performance of the delay-time estimator for use in real time control.

## 7. CONCLUSIONS

Control of the pulp bleaching process at the Irving Paper mill is a significant application, demonstrating the challenges and difficulties of dealing with a real-world delay-time process. The bleaching process was thoroughly studied and modeled as first order dynamics plus a variable delay time. A delay-time estimator was designed to tackle the problem of the long variable delay time. The estimator proved to be reliable for offline identification purposes and quite acceptable for real time control purposes under certain conditions.

A model predictive control (*MPC*) strategy was chosen to control the bleaching process taking into account its constraints, which were handled by incorporating a state-of-the-art of optimization method, i.e., an interior point method, in the controller. The designed *MPC* controller was implemented in the real-world bleaching process at the Irving Paper mill, in order to test and demonstrate its performance and stability. Implementation and simulation results were in excellent agreement, and showed that the *MPC* controller can be successfully applied to processes that have delay-time uncertainty less than  $\pm 7\%$ .

Several strategies could be implemented to improve the performance of the *DMC* controller during periods of considerable delay-time uncertainty (i.e., when the pulp level in the bleaching tower is rapidly varying). One scenario would be to schedule the pulp outflow in the future, so that the prediction of the pulp outflow would be practically realizable. In terms of operational flexibility, this may not be desirable. Alternatively, a state event handler could be incorporated in the controller, so if the pulp flow is steady then the handler would switch the *DMC* controller on, otherwise, the process would be controlled manually. Finally, a new delay-time estimator, which integrates the pulp outflow forward in time and thereby can anticipate the time of response to a set point change could be incorporated in the controller, to eliminate the “blips” observed in figure 5.

## REFERENCES

- Camacho, Eduardo F. Bordons, Carlos (1999). *Model predictive control*. Springer. Berlin, New York.
- Cutler, C. R. and B. L. Ramarker (1980). Dynamic matrix control – a computer control algorithm. *Proceedings of the joint automatic control conference*.
- Dence, Carlton W. and Douglas W. Reeve (1996). Brightness: basic principles and measurement. In: *Pulp Bleaching Principles and Practice*. TAPPI Press. Atlanta, Ga. pp. 697–716.
- Li, Zhiqing and George Court (July 2000, Jan 2001, Nov 2001, April 2002). Private communication at Irving Paper Ltd., St. John, N.B.
- Luenberger, David (1984). *Linear and nonlinear programming*. 2nd ed.. Addison-Wesley. Reading, Mass.
- Mehrotra, S. (1992). On the implementation of primal dual interior point method. *Siam journal on optimization* **2**(4), 575–601.
- Morari, Manfred and Jay H. Lee (1999). Model predictive control: past, present, and future. *Computers and chemical engineering* **23**(4–5), 667–682.
- Ni, Yonhgao (July 2000, Nov 2001). Private communication at Pulp and Paper Research Center, Fredericton, N.B.
- Persley, J. R. and R. T. Hill (1996). Peroxide bleaching of (chemi)mechanical pulps. In: *Pulp Bleaching Principles and Practice*. TAPPI Press. Atlanta, Ga. pp. 457–489.
- Potra, F. A. and S. J. Wright (2000). Interior point methods. *Journal of Computational and Applied Mathematics* **124**(1–2), 281–302.
- Qian, X. and P. Tessier (1997). Dynamic modelling and control of a hydrogen peroxide bleaching process. *Pulp and Paper Canada* **89**(9), 81–85.
- Rao, C., S. J. Wright and J. Rawlings (1997). Applications of interior point methods to model predictive control. Preprint ANL/MCS-P664-0597. Mathematics and computer science division, Argonne National Laboratory.
- Wen, C. Y. and L. T. Fan (1975). *Models for flow systems and chemical reactors*. Marcel Dekker, Inc.. New York.
- Wright, S. J. (1997). Applying new optimization algorithms to model predictive control. In: *Proc. of Chemical process control-V*. Vol. 93 of *AIChE Symposium series no.316*. CACHE Publications. pp. 147–155.
- Wright, S. J. and J. Nocedal (1999). *Numerical optimization*. Springer. New York.
- Ylinen, R. and K Zenger (1994). Simulation of variable delays in material transport models. *Mathematics and computers in simulation* **37**, 57–72.
- Zenger, K (1995). Time-variable models for mixing processes under unsteady flow and volume. In: *Proc. of 4th IFAC Symp. on dynamics of chemical reactors, distillation columns, and batch reactor* (J. B. Rawlings, Ed.). Pergamon Press. Oxford, UK. pp. 57–63.