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The Multivariate Noise-Predictive Delayed-Decision-Feedback Equalizer/Combiner for Multiuser Systems with Diversity

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Abstract

We describe in this paper how multiuser decision-feedback equalizers (DFE) can be generalized in order to allow a flexible temporal detection order for the received signals. The system consists of N users and one central base station. It is described by an equivalent N-input N-output discrete-time model. Only the reverse link is considered. The stationary channels between the users and the base station are assumed to be known. Additive Gaussian noise distorts the signals at the input of the receiver. The receiver is based on the noise-predictive decision-feedback equalizer for multiuser systems. This structure is generalized by individually delaying each output of the linear forward filter before a decision on the symbols is made.

1 Introduction

In wireless multiuser systems, several individual, spatially separated portables communicate simultaneously with a base station. An important and largely celebrated result for multiuser systems is that simultaneous detection of all users promises large performance gains compared to detecting each user separately while treating the other portables as unwanted interferers or noise. These gains are particularly significant for systems that are exposed to large amounts of multiple access interference (CDMA, SDMA and generalized diversity systems [1]).

The optimum multiuser detector is well known, however, it is too complex to implement in most practical systems. Less complex suboptimal approaches include the decorrelating and minimum mean-squared error (MMSE) detectors with or without decision feedback and multistage successive or parallel interference cancellation (MSIC, MPIC). A crucial idea in some feedback and interference cancellation methods is that the users with better performance are detected first. These decisions are then fed back in order to reduce the interference in the signals of the weaker users. The receiver proposed in this paper generalizes the multivariate noise-predictive decision-feedback equalizer (MNP-DFE) [2] by allowing an earlier detection of the strong users. This time-delayed decision method proves to be efficient especially if high symbol rates lead to frequency selective radio channels and if the received signal-to-noise ratios (SNR) of the users differ significantly. Such an environment is extremely challenging because future, present and past symbols from all active portables distort a specific symbol of the user of interest. Additionally, largely different user powers at the receiver are not only common in practice but also may reduce the performance of some detectors drastically (near-far effect). Furthermore, we consider the asynchronous case in which the signals from different users may arrive with arbitrary individual delays.

This paper analyzes the proposed multivariate noise-predictive delayed-decision-feedback equalizer (MNP-DDFE). The system model is described in Section 2. Expressions for the optimum infinite-length MMSE structure are derived in Section 3. Their evaluation involves the spectral factorization of a matrix, which is computationally very complex. In Section 4 we present an alternative method for finite-length filters that only requires the inversion of a matrix. Finally, we compare the performance of the proposed receiver to that of the same structure without delays (MNP-DFE) and the conventional linear MMSE equalizer/combiner (E/C) in Section 5.

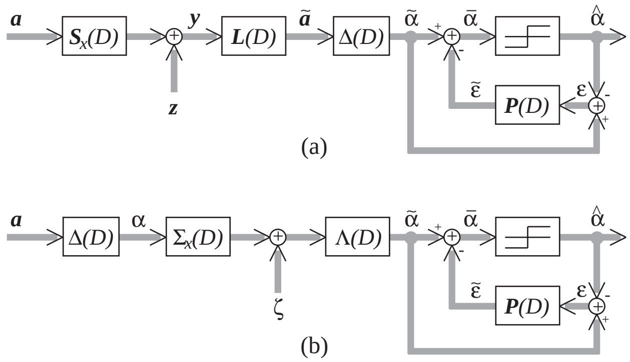

Figure 1: System model (a), and equivalent model (b)

2 System Model

The system is analyzed using the D-transform which is defined by $\boldsymbol{V}(D) = \sum_{n=-\infty}^{\infty} \boldsymbol{V}[n]D^n$, where $\boldsymbol{V}$ may be an arbitrary dimensional matrix. Consider the row vector $\boldsymbol{v} = [v_1, v_2, \ldots]$. Let us define the truncated sequence

$$\boldsymbol{v}_M[n] = \begin{cases} \boldsymbol{v}[n] & \text{, for } |n| \le M \\ 0 & \text{, for } |n| > M. \end{cases} \tag{1}$$

Let $\boldsymbol{u}$ be another row vector whose truncated sequence is defined according to Equation (1). The cross-power spectrum $\boldsymbol{S}_{uv}(D)$ of $\boldsymbol{u}[n]$ and $\boldsymbol{v}[n]$ is then equal to

$$E_M[\boldsymbol{u}^H(D^{-*})\boldsymbol{v}(D)] \stackrel{\text{def}}{=} \lim_{M\to\infty} \frac{E[\boldsymbol{u}_M^H(D^{-*})\boldsymbol{v}_M(D)]}{2M+1} \tag{2}$$

where 'E' is the expectation operator, the superscripts 'H', '$*$', '-1' denote the conjugate transpose, complex conjugate and inverse, respectively. The superscript '$-*$' shall be interpreted in the sense $D^{-*} = (D^{-1})^*$.

Our model for the reverse link of a multiuser system consists of N users transmitting the data sequences a_i $(i = 1, \ldots, N)$, a $N \times N$ channel response matrix $\boldsymbol{S}_x$ and an N-input N-output receiver. One Gaussian noise signal z_i is added at each receiver input. The complex baseband notation is used to describe the system. All signals and system responses are in general complex discrete-time functions.

The data sequences consist of symbols drawn from a finite alphabet of complex numbers ($a_i[n] \in \mathcal{A}$). Let us combine all sequences in the $1 \times N$ data vector

$$\boldsymbol{a} = [a_1, a_2, \ldots, a_N]. \tag{3}$$

Generally, we use boldtype lowercase letters (e.g. $\boldsymbol{u}$) for signal vectors. Each signal vector component is denoted by the respective letter in normal type including a subscript that indicates the number of the component (e.g. u_i). Let the noise signal vector be $\boldsymbol{z} = [z_1, z_2, \ldots, z_N]$. A model of the system is shown in Figure 1(a).

It is assumed that the input and noise are uncorrelated signals with zero mean. The auto- and cross-spectra of input and noise signals are then given by

$$\boldsymbol{S}_a(D) = E_M[\boldsymbol{a}^H(D^{-*})\boldsymbol{a}(D)] \tag{4}$$

$$\boldsymbol{S}_z(D) = E_M[\boldsymbol{z}^H(D^{-*})\boldsymbol{z}(D)] = \boldsymbol{S}_x(D) \tag{5}$$

$$\boldsymbol{S}_{az}(D) = E_M[\boldsymbol{a}^H(D^{-*})\boldsymbol{z}(D)] = \boldsymbol{O}_N \tag{6}$$

where $\boldsymbol{O}_n$ is the $n \times n$ zero matrix. The noise spectrum is equal to the channel response [1, 3]. It can be shown that this model describes a discrete-time communications system with continuous-time channel when the front end of the receiver consists of a matrix filter that whitens the continuous-time noise signals followed by a matched filter matrix matched to the channel responses [1]. This structure is motivated by the following two reasons. Firstly, the signal $\boldsymbol{y}$ at the output of the matched filter matrix forms a set of sufficient statistics about the input $\boldsymbol{a}$ [4, pp. 584–86]. Secondly, the optimum linear MMSE equalizer/combiner can be realized with this structure [1, 5].

The receiver consists of four parts. The first is a linear $N \times N$ forward filter matrix $\boldsymbol{L}(D)$. It is followed by the *delay matrix* $\boldsymbol{\Delta}$:

$$\boldsymbol{\Delta}(D) = \mathbf{D}\text{iag}\langle D^{\Delta_k}\rangle, \quad k = 1, 2, \ldots, N \tag{7}$$

where $\mathbf{D}\text{iag}\langle u_i\rangle$ is a diagonal matrix with diagonal elements u_i $(i = 1, 2, \ldots)$ and Δ_k is an integer greater than or equal to zero. $\boldsymbol{\Delta}$ delays the k-th input signal by Δ_k symbols. The third receiver part is a decision element that maps the continuous-valued input $\bar{\boldsymbol{a}}[n]$ into symbols from the finite alphabet $\mathcal{A}^N$. The decisions are fed back in order to calculate the linear estimation error which serves as input to the $N \times N$ noise prediction filter matrix $\boldsymbol{P}$.

According to Figure 1(a), the output of the delay matrix $\tilde{\boldsymbol{\alpha}}$ and the input to the decision element $\bar{\boldsymbol{\alpha}}$ can be written as

$$\tilde{\boldsymbol{\alpha}}(D) = [\boldsymbol{a}(D)\boldsymbol{S}_x(D) + \boldsymbol{z}(D)]\,\boldsymbol{L}(D)\boldsymbol{\Delta}(D) \tag{8}$$

$$\bar{\boldsymbol{\alpha}} = \tilde{\boldsymbol{\alpha}} - \tilde{\boldsymbol{\varepsilon}} \tag{9}$$

where $\tilde{\boldsymbol{\varepsilon}}$ is the predicted estimation error in $\tilde{\boldsymbol{\alpha}}$. Let $\boldsymbol{\alpha}$ and $\hat{\boldsymbol{\alpha}}$ denote the delayed input signal and the delayed estimate, respectively:

$$\boldsymbol{\alpha}(D) = \boldsymbol{a}(D)\boldsymbol{\Delta}(D) \tag{10}$$

$$\hat{\boldsymbol{\alpha}}(D) = \hat{\boldsymbol{a}}(D)\boldsymbol{\Delta}(D) \tag{11}$$

where $\hat{\boldsymbol{a}}$ is the estimate of the input signal $\boldsymbol{a}$. For analytical purposes we assume from now on that all decisions are correct, i.e. $\hat{\boldsymbol{\alpha}}(D) = \boldsymbol{\alpha}(D)$. The input to the prediction filter matrix $\boldsymbol{P}$ is then given by

$$\boldsymbol{\varepsilon} = \tilde{\boldsymbol{\alpha}} - \boldsymbol{\alpha}. \tag{12}$$

The output of $\boldsymbol{P}$ is the predicted error

$$\tilde{\boldsymbol{\varepsilon}}(D) = \boldsymbol{\varepsilon}(D)\boldsymbol{P}(D). \tag{13}$$

Note that the prediction filter $\boldsymbol{P}$ has to be *purely causal* [3], i.e. $\boldsymbol{P}(D) = \boldsymbol{P}[0] + \boldsymbol{P}[1]D^1 + \boldsymbol{P}[2]D^2 + \cdots$, and $\boldsymbol{P}[0]$ is restricted to be an upper triangular matrix with zeros along the diagonal.

An indicator for the system performance is the error $\boldsymbol{\epsilon} = \bar{\boldsymbol{\alpha}} - \boldsymbol{\alpha}$ of the input signal to the decision element. Using Equations (9) and (12), it can easily be shown that $\boldsymbol{\epsilon}$ is equal to the prediction error:

$$\boldsymbol{\epsilon} = \boldsymbol{\varepsilon} - \tilde{\boldsymbol{\varepsilon}}. \tag{14}$$

3 The Optimum Delayed-Decision-Feedback Equalizer

It is easy to show that the systems in Figure 1(a) and Figure 1(b) are equivalent if

$$\boldsymbol{\zeta}(D) = \boldsymbol{z}(D)\boldsymbol{\Delta}(D) \tag{15}$$

$$\boldsymbol{\Sigma}_x(D) = \boldsymbol{\Delta}^{-1}(D)\boldsymbol{S}_x(D)\boldsymbol{\Delta}(D) \tag{16}$$

$$\boldsymbol{\Lambda}(D) = \boldsymbol{\Delta}^{-1}(D)\boldsymbol{L}(D)\boldsymbol{\Delta}(D) \tag{17}$$

where $\boldsymbol{\Delta}^{-1}(D) = \boldsymbol{\Delta}(D^{-1})$ is the inverse of the delay matrix. Let us decompose the inverse of the forward filter matrix via matrix spectral factorization:

$$\boldsymbol{\Lambda}^{-1}(D) = \boldsymbol{\Psi}^H(D^{-*})\boldsymbol{\Psi}(D) \tag{18}$$

where $\boldsymbol{\Psi}(D) = \boldsymbol{\Psi}[0] + \boldsymbol{\Psi}[1]D^1 + \boldsymbol{\Psi}[2]D^2 + \cdots$, and $\boldsymbol{\Psi}[0]$ is an upper triangular nonsingular matrix. Let us further define the notation for a diagonal matrix

$$\boldsymbol{V}^d \stackrel{\text{def}}{=} \mathbf{D}\text{iag}\langle V_{i,i}\rangle, \quad i = 1, 2, \ldots \tag{19}$$

where $V_{i,i}$ is the i-th diagonal element of the matrix $\boldsymbol{V}$. Except for the delay matrix at the front, the structure in Figure 1(b) is equal to the MNP-DFE. It is shown in [2] that the forward and feedback filters of the optimum MNP-DFE (in the MMSE sense) are given by

$$\boldsymbol{\Lambda}(D) = \left[\boldsymbol{S}_\alpha(D)\boldsymbol{\Sigma}_x(D) + \boldsymbol{I}_N\right]^{-1}\boldsymbol{S}_\alpha(D) \tag{20}$$

$$\boldsymbol{P}(D) = \boldsymbol{I}_N - \boldsymbol{\Psi}(D)\left[\boldsymbol{\Psi}^d[0]\right]^{-1} \tag{21}$$

where $\boldsymbol{I}_N$ is the $N \times N$ identity matrix and $\boldsymbol{S}_\alpha(D) = E_M[\boldsymbol{\alpha}^H(D^{-*})\boldsymbol{\alpha}(D)]$ is the spectrum of the delayed data signal.

The optimum MNP-DDFE shown in Figure 1(a) can now be determined. The prediction filter matrix $\boldsymbol{P}$ is the same for both the MNP-DFE and the MNP-DDFE. Using Equations (16), (17) and (20), it can be verified that the optimum forward filter matrix $\boldsymbol{L}$ is identical to the linear MMSE E/C [1, 2, 5, 3]:

$$\boldsymbol{L}(D) = \left[\boldsymbol{S}_a(D)\boldsymbol{S}_x(D) + \boldsymbol{I}_N\right]^{-1}\boldsymbol{S}_a(D). \tag{22}$$

4 Finite-Length Predictor

Assume that the input signals a_i of different users are mutually independent and that samples of the same sequence are uncorrelated with zero mean and variance $\mathcal{E}_a$:

$$E\left[a_i[n]a_k[m]\right] = \mathcal{E}_a\delta[i-k]\delta[n-m] \tag{23}$$

where $\delta[k]$ is the Kronecker delta sequence. In this case, the spectrum of the input signal reduces to $\boldsymbol{S}_a(D) = \mathcal{E}_a\,\boldsymbol{I}_N$.

According to Equation (22), the forward filter $\boldsymbol{L}$ can be calculated by matrix inversion. However, in order to obtain the optimum predictor $\boldsymbol{P}$, the computationally intensive spectral factorization (18) has to be performed. Alternatively, $\boldsymbol{P}$ may be approximated with a finite impulse response (FIR) filter. The coefficients of the FIR filter can be determined by standard techniques which minimize the squared prediction error (14). Those methods only require the inversion of a matrix, and are, therefore, much less computationally complex.

We determine now an FIR approximation of $\boldsymbol{P}$. Suppose the length of the FIR predictor is L_p, i.e. $\boldsymbol{P}(D) = \boldsymbol{P}[0] + \boldsymbol{P}[1]D^1 + \cdots + \boldsymbol{P}[L_p-1]D^{L_p-1}$. Let us define the $1 \times (NL_p)$ input vector $\boldsymbol{\varepsilon}_\#[n]$ and the extended predictor matrix $\boldsymbol{P}_\#$:

$$\boldsymbol{\varepsilon}_\#[n] = [\boldsymbol{\varepsilon}[n-L_p+1], \boldsymbol{\varepsilon}[n-L_p+2], \ldots, \boldsymbol{\varepsilon}[n]] \tag{24}$$

$$\boldsymbol{P}_\# = \begin{bmatrix} \boldsymbol{P}[L_p-1] \\ \boldsymbol{P}[L_p-2] \\ \vdots \\ \boldsymbol{P}[0] \end{bmatrix} = \left[\boldsymbol{p}_{\#1}^T, \boldsymbol{p}_{\#2}^T, \ldots, \boldsymbol{p}_{\#N}^T\right] \tag{25}$$

where the superscript 'T' denotes the transpose and $\boldsymbol{p}_{\#k}^T$ is an $(NL_p) \times 1$ column vector. The predicted error vector is then given by

$$\tilde{\boldsymbol{\varepsilon}}[n] = \boldsymbol{\varepsilon}_\#[n]\boldsymbol{P}_\#. \tag{26}$$

Since $\boldsymbol{P}$ has to be purely causal, the last $(N-k+1)$ components of $\boldsymbol{p}_{\#k}$ are constrained to be zero, i.e.

$$\boldsymbol{p}_{\#k} = [\boldsymbol{p}_k, \boldsymbol{o}_{N-k+1}] \tag{27}$$

where $\boldsymbol{p}_k$ is an $1 \times (NL_p - N + k - 1)$ row vector and $\boldsymbol{o}_m = [0, 0, \ldots, 0]$ is the $1 \times m$ zero vector. Note that for the k-th error $\tilde{\varepsilon}_k[n]$, the predictor uses as input information the errors $\varepsilon_q[n]$ for $q < k$ but not those for $q \geq k$. Let us order the users according to their mean-squared error (MSE) after the delay matrix:

$$e_1^2 \leq e_2^2 \leq \ldots \leq e_N^2 \tag{28}$$

where $e_k^2 = E[|\varepsilon_k[n]|^2]$ is the MSE of user k.

We need for the following calculations the spectrum and autocorrelation of the error $\boldsymbol{\varepsilon}$:

$$\boldsymbol{S}_\varepsilon(D) = E_M\left[\boldsymbol{\varepsilon}^H(D^{-*})\boldsymbol{\varepsilon}(D)\right] \tag{29}$$

$$\boldsymbol{R}_\varepsilon[m] = E\left[\boldsymbol{\varepsilon}^H[n]\boldsymbol{\varepsilon}[n+m]\right]. \tag{30}$$

Spectrum and autocorrelation are connected through the D-transform, i.e. $\boldsymbol{S}_\varepsilon(D) = \sum_{m=-\infty}^{\infty} \boldsymbol{R}_\varepsilon[m]D^m$. After substituting Eqns. (8), (10), (12) into (29), considering (4), (5), (6) and using (22), the error spectrum can be expressed as

$$\boldsymbol{S}_\varepsilon(D) = \boldsymbol{\Lambda}(D) = \boldsymbol{\Delta}^{-1}(D)\boldsymbol{L}(D)\boldsymbol{\Delta}(D). \tag{31}$$

The autocorrelation matrix may be determined by evaluating the inverse D-transform on the unit circle, i.e. for $D = e^{-j2\pi\check{f}}$ where $\check{f}$ is the normalized frequency:

$$\boldsymbol{R}_\varepsilon[m] = \int_0^1 \boldsymbol{\Lambda}(e^{-j2\pi\check{f}})e^{j2\pi\check{f}m}\, d\check{f}. \tag{32}$$

Let us partition the vector $\boldsymbol{\varepsilon}_\#[n]$ from Eqn. (24) into the following components:

$$\boldsymbol{\varepsilon}_\#[n] = [\boldsymbol{\varepsilon}_{\#k}[n], \varepsilon_k[n], \varepsilon_{k+1}[n], \ldots, \varepsilon_N[n]] \tag{33}$$

where $\boldsymbol{\varepsilon}_{\#k}[n]$ is a row vector and $\varepsilon_k[n]$ is the k-th component of $\boldsymbol{\varepsilon}[n]$. Furthermore, we define

$$\boldsymbol{R}_\# = E\left[\boldsymbol{\varepsilon}_\#^H[n]\boldsymbol{\varepsilon}_\#[n]\right] \tag{34}$$

$$\boldsymbol{R}_{\#k} = E\left[\boldsymbol{\varepsilon}_{\#k}^H[n]\boldsymbol{\varepsilon}_{\#k}[n]\right] \tag{35}$$

$$\boldsymbol{\varrho}_{\#k} = E\left[\varepsilon_k^*[n]\boldsymbol{\varepsilon}_{\#k}[n]\right]. \tag{36}$$

Combining Equations (24), (30) and (34) we obtain

$$\boldsymbol{R}_\# = \begin{bmatrix} \boldsymbol{R}_\varepsilon[0] & \boldsymbol{R}_\varepsilon[1] & \ldots & \boldsymbol{R}_\varepsilon[L_p-1] \\ \boldsymbol{R}_\varepsilon[-1] & \boldsymbol{R}_\varepsilon[0] & \ldots & \boldsymbol{R}_\varepsilon[L_p-2] \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{R}_\varepsilon[1-L_p] & \boldsymbol{R}_\varepsilon[2-L_p] & \ldots & \boldsymbol{R}_\varepsilon[0] \end{bmatrix}. \tag{37}$$

According to Eqn. (33) we may write the vector $\boldsymbol{\varepsilon}_{\#k}[n]$ recursively as $\boldsymbol{\varepsilon}_{\#(k+1)}[n] = [\boldsymbol{\varepsilon}_{\#k}[n], \varepsilon_k[n]]$. Thus, the matrices in (34) and (35) can be written as

$$\boldsymbol{R}_\# = \begin{bmatrix} \boldsymbol{R}_{\#N} & \boldsymbol{\varrho}_{\#N}^H \\ \boldsymbol{\varrho}_{\#N} & e_N^2 \end{bmatrix} \tag{38}$$

$$\boldsymbol{R}_{\#(k+1)} = \begin{bmatrix} \boldsymbol{R}_{\#k} & \boldsymbol{\varrho}_{\#k}^H \\ \boldsymbol{\varrho}_{\#k} & e_k^2 \end{bmatrix}. \tag{39}$$

Substituting Equations (25), (27) and (33) into (26), the k-th component of the predicted error may be expressed as

$$\tilde{\varepsilon}_k[n] = \boldsymbol{\varepsilon}_{\#k}[n]\boldsymbol{p}_k^T. \tag{40}$$

We obtain the optimum predictor by applying the orthogonality principle:

$$E\left[\boldsymbol{\varepsilon}_{\#k}^H[n]\left(\tilde{\varepsilon}_k[n] - \varepsilon_k[n]\right)\right] = \boldsymbol{o}_{NL_p-N+k-1}^T. \tag{41}$$

Substituting (40) into (41), using the Definitions (35), (36), and solving for $\boldsymbol{p}_k^T$ yields

$$\boldsymbol{p}_k^T = \boldsymbol{R}_{\#k}^{-1}\boldsymbol{\varrho}_{\#k}^H. \tag{42}$$

Note that only one matrix inversion is required for the calculation of the predictor matrix $\boldsymbol{P}_\#$. To see that, let us decompose $\boldsymbol{R}_{\#(k+1)}^{-1}$ into

$$\boldsymbol{R}_{\#(k+1)}^{-1} = \begin{bmatrix} \boldsymbol{\Gamma}_k & \boldsymbol{\gamma}_k^H \\ \boldsymbol{\gamma}_k & g_k \end{bmatrix} \tag{43}$$

where $\boldsymbol{\Gamma}_k$ is a square matrix whose dimension is one less than that of $\boldsymbol{R}_{\#(k+1)}^{-1}$, $\boldsymbol{\gamma}_k$ is a row vector and g_k is a scalar. Since $\boldsymbol{R}_{\#k}$ is a submatrix of $\boldsymbol{R}_{\#(k+1)}$ according to Eqn. (39), the matrix $\boldsymbol{R}_{\#k}^{-1}$ can be computed efficiently with [6, pp. 445–46]

$$\boldsymbol{R}_{\#k}^{-1} = \boldsymbol{\Gamma}_k - g_k^{-1}\boldsymbol{\gamma}_k^H\boldsymbol{\gamma}_k. \tag{44}$$

Thus, only $\boldsymbol{R}_{\#N}$ needs to be inverted. The remaining matrices $\boldsymbol{R}_{\#q}$ $(q = 1, 2, \ldots, N-1)$ may be obtained recursively using Equation (44).

Let us define the normalized MSE after the delay matrix $\sigma_{\mathrm{lin},k} = 1/\mathcal{E}_a E\left[|\tilde{\alpha}_k[n] - \alpha_k[n]|^2\right]$ and the normalized MMSE of the MNP-DDFE before the decision element $\sigma_{\mathrm{ddfe},k} = 1/\mathcal{E}_a E\left[|\bar{\alpha}_k[n] - \alpha_k[n]|^2\right]$. Alternatively, these quantities may be expressed in terms of the error signals:

$$\sigma_{\mathrm{lin},k} = \mathcal{E}_a^{-1} E\left[|\varepsilon_k[n]|^2\right] \tag{45}$$

$$\sigma_{\mathrm{ddfe},k} = \mathcal{E}_a^{-1} E\left[|\epsilon_k[n]|^2\right]. \tag{46}$$

Since $\sigma_{\mathrm{lin},k}$ is the MMSE of the linear MMSE E/C, we obtain [1, 3]

$$\sigma_{\mathrm{lin},k} = \frac{1}{\mathcal{E}_a}\int_0^1 L_{k,k}(e^{-j2\pi\check{f}})\, d\check{f} \tag{47}$$

where $L_{k,k}(D)$ is the k-th diagonal element of the matrix $\boldsymbol{L}(D)$ from Eqn. (22). Expanding Eqn. (46) and using (14), (35), (36), (40), (42), the normalized MMSE of the MNP-DDFE can be written as

$$\sigma_{\mathrm{ddfe},k} = \sigma_{\mathrm{lin},k} - \mathcal{E}_a^{-1} \boldsymbol{\varrho}_{\#k} \boldsymbol{R}_{\#k}^{-1} \boldsymbol{\varrho}_{\#k}^{H}. \tag{48}$$

As a result of the typical indoor channel behavior and the receiver front end including a matched filter matrix, the main cochannel interference of a particular symbol comes from those symbols of the other data streams which are sent at the same time or immediately before and after it. Assume for example individual delays of $\Delta_1 = 0$ and $\Delta_k = \Delta_{k-1} + \delta_0$ ($k = 2, 3, \ldots, N$), where δ_0 is an integer greater than zero. In this case, the signal of the N-th user is delayed by $\delta_0(N-1)$ symbols relative to that of user 1. Since the significant interference stems mainly from symbols sent at about the same time, the predictor length L_p should be on the order of $\delta_0(N-1)$ symbols. This may result in matrices $\boldsymbol{R}_{\#k}$ with large dimensions and a high computationally load to invert them, especially if there are many users in the system. One solution to this problem is to insert for every user a different delay matrix $\boldsymbol{\Delta}_{P,k} = \mathbf{D}\mathrm{iag}\langle D^{\max\{\delta_0(k-i-1),0\}}\rangle$ ($i = 1, \ldots, N$) in front of the predictor $\boldsymbol{p}_k$. This ensures that all error signals ε_q for $q < k$ are delayed by only δ_0 symbols relative to $\boldsymbol{\varepsilon}_k$ when they enter the predictor. L_p may then be chosen to a value on the order of δ_0, which can reduce the dimension of $\boldsymbol{R}_{\#N}$ significantly.

The receiver complexity might be additionally reduced by feeding into the predictor only the error signals ε_q with $q \leq k$. This reduces the performance in many cases only slightly because the contribution of the ε_v with $v > k$ to $\tilde{\varepsilon}_k$ is usually small, especially if the delays in $\boldsymbol{\Delta}$ are large.

These reduced complexity versions of the MNP-DDFE are not analyzed in this paper because they require a somewhat lengthy notation. However, the analysis of these structures is straightforward if the methods in this section are applied analogously.

5 Numerical Results

We compare in this section the performance of the MNP-DDFE, the MNP-DFE and the linear E/C. The channel matrix $\boldsymbol{S}_x$ has been calculated based on the general multiuser system in [1] by following the procedure described therein. For the individual channels, we have used the same indoor channel impulse response (CIR) measurements as in [1].

The results have been obtained for a system with a symbol period of $T = 50$ ns, $A = 4$ receive antennas at the base station and a double-sided system bandwidth of $K = 4$ times the Nyquist bandwidth $1/T$. Therefore, the total degree of diversity is $AK = 16$, allowing no more than 16 users in the system if a zero-forcing equalizer/combiner is used [7]. For our computations, the number of system users has been varied between 1 and 30. The reverse link of the system has been simulated by randomly selecting 30 out of 2044 CIR sets and assigning each to one of the 30 users. The users have been divided into several groups of N portables for which the theoretical MMSE's (47) and (48) have been calculated. This procedure has been repeated 100 times for each value of N with different CIR sets.

The linear MMSE E/C consists of the forward matrix filter $\boldsymbol{L}$, Eqn. (22), only. The MNP-DFE is a special case of the structure described in Section 3 with no delay matrix, or, equivalently, with $\boldsymbol{\Delta}(D) = \boldsymbol{I}_N$. The numerical results have been obtained for the reduced complexity version of the MNP-DDFE with additional delay matrices in front of the predictor and no feedback of error signals with a larger $\sigma_{\mathrm{lin},k}$. The individual delays of the MNP-DDFE have been chosen to $\Delta_1 = 0$ and $\Delta_k = \Delta_{k-1} + 3$ for $k = 2, 3, \ldots, N$. The length of the feedback filters has been set to $L_p = 7$ for both the MNP-DFE and the MNP-DDFE.

Figure 2 shows the normalized MMSE for the best and worst of the 30 users, averaged over all 100 trials. The received SNR of the individual users varies by up to ± 5 dB around a mean of 20 dB. This situation describes thus a system with no or less stringent power control. The graph shows that the DFE structures perform significantly better than the linear MMSE E/C in both the worst and best user case. While the MNP-DDFE yields the lowest average MMSE for the worst system user, the MNP-DFE performs superior if the best user is chosen as the criterion. Note that the worst user curve of the MNP-DDFE stays almost constant at approximately -15 dB until it approaches the best user curve of the linear MMSE E/C. For large N, these two curves are very close. While strongly improving on the performance of initially bad users, the MNP-DDFE reduces the MMSE of good users only marginally. For $N \geq 16$ the most users which perform worse after $\boldsymbol{L}$ benefit from the predictor such that their final MMSE $\sigma_{\mathrm{ddfe},k}$ is smaller than that of the initially best user 1. The MMSE difference between the best and worst user is approximately 10 dB in the single user case. This difference remains almost constant for the linear MMSE E/C when the number of users increases. The MMSE gap between the

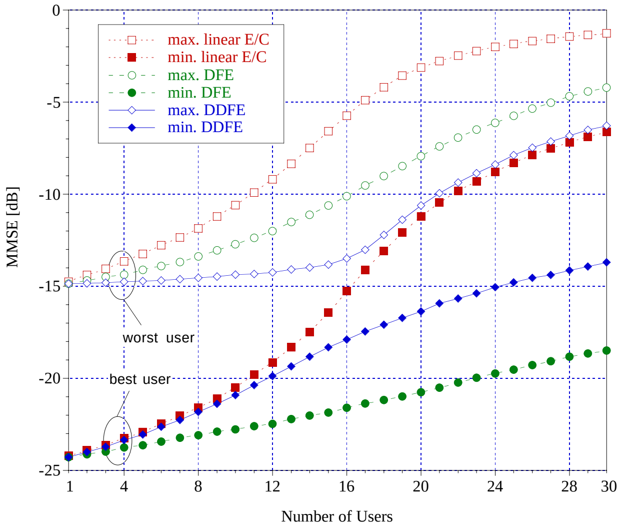

Figure 2: MMSE for linear E/C, MNP-DFE and MNP-DDFE (SNR's per user varying by up to 10 dB)

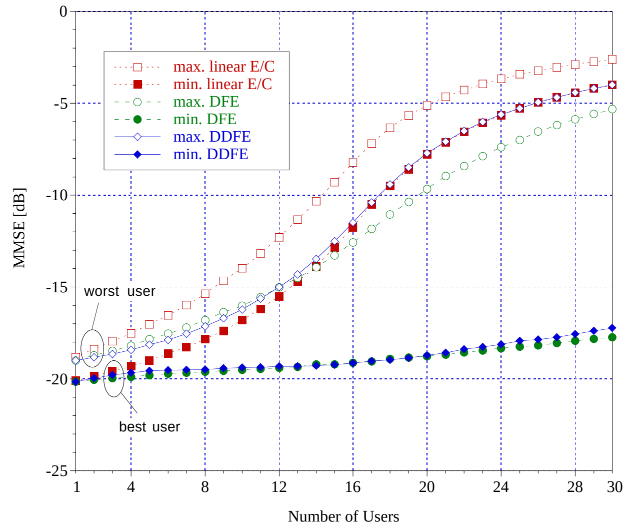

Figure 3: MMSE for linear E/C, MNP-DFE and MNP-DDFE (SNR's per user varying by up to 1 dB)

best and worst user widens slightly for the MNP-DFE. The MNP-DDFE, however, reduces this difference to about 4 dB at $N = 16$. This shows again that bad users benefit more from the feedback error reduction than good users. In contrast, since there are no delays in the MNP-DFE structure, all users improve on average equally.

Figure 3 shows results for the case that the individual SNR's of the users vary by at most ± 0.5 dB around a mean of 20 dB. Except for this, all system parameters and CIR sets have been the same as before. Again, we can see that the DFE receivers performs better than the linear E/C. While the relative improvement of the MNP-DFE compared to the linear equalizer in the worst user case is almost identical to that shown in Figure 2, the MNP-DDFE's worst user does not nearly perform as well as before. It has only a marginally lower MMSE than the MNP-DFE for a small number of users. For large N, its performance is even worse. The best user curves for the MNP-DDFE and MNP-DFE are almost identical. This suggests that there is no benefit from introducing large individual delays after the forward filter matrix $\boldsymbol{L}$ if all users are received with approximately the same power.

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