An Introduction to Modeling a Fixed Wing UAV

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Convenient Vector Formulation

Used in AUV's and UAV's the following state space representation is very convent

The UAV's position is expresses relative to an inertial reference.

$$\eta_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \eta_2 = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \end{bmatrix}$$

Also the ν vector holds the linear and angular velocity

$$\nu_1 = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad \nu_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad \nu = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}$$

Formalizing the Kinematics

Applying the first R matrix it is possible to determine the change in location based on the current linear velocity and orientation.

$\dot{\eta}_1 = R_1(\eta_2)\nu_1$
where $R_1 =$

$$\begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\theta + \cos\psi\sin\theta\cos\phi \\ \sin\psi\cos\theta & \cos\psi\cos\phi & -\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

Implementing the second rotation matrix the change in orientation can be determined from the angular velocities

$\dot{\eta}_2 = R_2(\eta_2)\nu_2$
where

$$R_2(\eta_2) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\psi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\psi/\cos\theta \end{bmatrix}$$

Our UAV

EZ Hawk Powered Glider

Manufacture Specifications

- Wing span :1370mm (53.93 in)
- Length : 870mm (34.25 in)
- Weight : 680g (23.98 oz)

Measured data

- root chord :192mm
- tip chord : 170mm (no wing tip)
- tip length : 170mm
- $\ell_t = 520$mm

Our UAV

Calculated parameters

- The taper ratio $\lambda = c_t / c_o = .89$
- Approximate wing area : $S = c_{avg} b = 217200mm^2$
- The Aspect Ratio $AR = \frac{b^2}{s_w} = 8.6$

Airfoil data

- 13% thickness
- The airfoil appears to be a ClarkY from observation

Our UAV

EZ Hawk Powered Glider

Tail data

- Horizontal Tail moment lengthℓ_t $= 520$mm
- Vertical tail moment length $=510$mm
- Horizontal tail area$= 43050\text{mm}^2$
- Vertical tail area $=13600\text{mm}^2$
- tail airfoils symmetric

other data

- CG location $=$ 50mm
- approximate max thrust 550 grams

Dynamic Pressure

For convenience we will define the term dynamic pressure here

$$\bar{q} = 1/2\rho V^2$$

where ρ is the from the Standard atmosphere model mass density and is 1.2250 kg/m^3 at sea level
note: This is the pressure you will measure with the differential pressure sensor in flight.

Reynolds Number

the nature of the boundary layer viscous flow is determined by a single dimensionless parameter the Reynolds Number R_e

$$R_e = (\rho l V_T)/\mu$$

where μ is the from the viscosity of air and is very dependent on temperature but practically independent of pressure. Again using the standard atmosphere and 20°C $\mu = 1.7894x10^{-5} Ns/m^2$

Induced Drag from lift and a 3D wing

A three dimensional wing will not perform as well as a 2D airfoil the different come in the form of more drag

$$C_{D_i} = C_L^2/(\pi e AR)$$

this drag forms in the wing tip vortexes
e is the efficiency factor or how closely it approaches an elliptical wing (e=1 for an elliptical) for a tapered rectangular wing with elliptical wing tips 0.9 is a good estimate and rectangular wing with no tips is approximately 0.8

Create a thrust coefficient

Defined by normalizing the thrust the same way as the other coefficients

$$T_C = \frac{Thrust}{\bar{q} S_D}$$

where S_D the propeller disk area

Wind and Stability Axis definitions

Two more axis systems are defined for deaning with the the angle of attack α and side slipβ

These are the wind and stability axis systems and used to define the relative wind

Both the wind and stability axises systems are attached to the center of mass of the airplane

The wind axis and is rotated by angle β around z to rotate the aircraft into the oncoming wind.

The stability axis is attached to the aircraft center of mass but is rotated about the pitch axis by α

Calculating force and moment

Forces and moments are defined as dimensionless aerodynamics coefficients in the same manor as airfoils and are functions of two aerodynamic angles,Reynolds number and often mac number control surface deflection and thrust coefficient.

Drag $D = \bar{q}SC_D$

Lift $L = \bar{q}SC_L$

Crosswind force $C = \bar{q}SC_C$

Rolling moment $\ell_w = \bar{q}SC_\ell$

Pitching moment $m_w = \bar{q}SC_m$

Yawing moment $n_w = \bar{q}SC_n$

Building up aerodynamic coefficients

due to the dimensionless nature of the definitions a coefficient can be build up of as many peaces of information available

Example C_D
Drag can be made of of as little as $C_{D_{airfoil}} + C_{D_{induced}} + C_{friction}$ for the main wing of other parameters can be added such as wave drag, fuselage drag, interference drag, control surface drag, form drag for landing gear etc.

The flat earth, body axis 6-DOF equations

Force Equations

$$\dot{U} = RV - QW - g_D \sin\theta + (X_A +_T)/m$$
$$\dot{V} = -RU + PW + g_D \sin\phi \cos\theta + (Y_A + Y_T)/m$$
$$\dot{W} = QU - PV + g_D \cos\phi \cos\theta + (Z_A + Z_T)/m$$

where sub script A and T refer to the aerodynamic and thrust components respectively.

the Flat earth, body axis 6-DOF equations

Kinematic Equations

$$\dot{\phi} = P + \tan\theta(Q\sin\phi + R\cos\phi$$

$$\dot{\theta} = Q\cos\phi - R\sin\phi$$

$$\dot{\psi} = (Q\sin\phi + R\cos\phi)/\cos\theta$$

Moment equations

$$\Gamma\dot{P} = J_{xz}[J_x - J_y + J_z]PQ - J_z(J_z - J_y) + J_{xz}^2]QR + J_z\ell + J_{xz}n$$

$$J_y\dot{Q} = (J_x - J_y)PR - J_x + J_{xz}(P^2 - R^2) + m$$

$$\Gamma\dot{R} = [(J_x - J_y)J_x + J_{xz}^2]PQ - J_{xz}[J_x - J_y + J_z]QR + J_{xz}\ell + J_x n$$

$$\dot{\Gamma} = J_x J_z - J_{xz}^2$$

the Flat earth, body axis 6-DOF equations

Navigation Equations

$$\dot{P}N = U\cos\theta\cos\psi + V(-\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi) + W(\sin\theta\sin\psi + \cos\phi\sin\theta\cos\psi)$$

$$\dot{P}E = U\cos\theta\sin\psi + V(\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi) + W(-\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi)$$

$$\dot{h} = U\sin\theta - V\sin\theta\cos\theta - W\cos\phi\cos\theta$$