



Modeling Helicopter UAV

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Common formulation for helicopters

Common Assumptions for helicopters

State Formulation

Helicopter Autonomy

The State Space Model



Common Assumptions for Helicopters

1. The earth is flat, stationary and therefore an approximate inertial reference frame.
2. The atmosphere is at rest relative to the ground (zero wind)
3. The Rotor head RPM is constant
4. The model is only valid in near hover conditions
5. The model can not be derived like previous models and must be identified using experiments.
6. Change in mass during flight is negligible.
7. The Yaw rate gyro is still in place as a very fast rate control on Yaw and there for changes in torque because of control input are neglected



State Formulation

Used in AUV's and UAV's the following state space representation is very convenient

The UAV's position is expressed relative to an inertial reference.

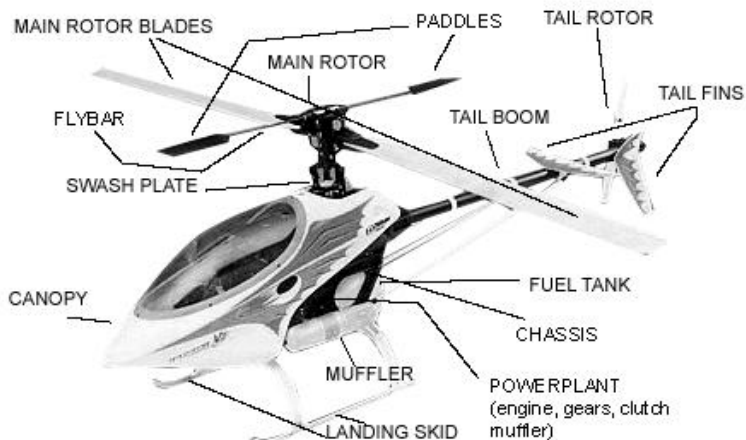
$$\eta_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \eta_2 = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \end{bmatrix}$$

Also the ν vector holds the linear and angular velocity

$$\nu_1 = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad \nu_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad \nu = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}$$



Tour of a helicopter



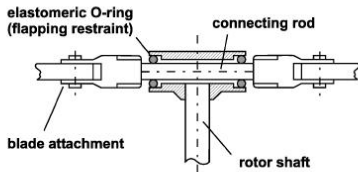


The Rotor head

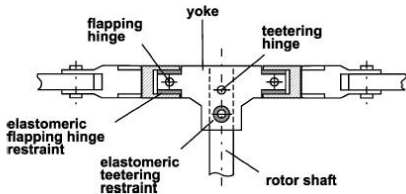
The Rotor head of helicopters will usually fall into two categories the Rigid or Hinged rotor head design full scale

helicopters must use hinged since the structural strength of the blades is too low to support its only moment about the rotor head.

X-Cell's rigid head

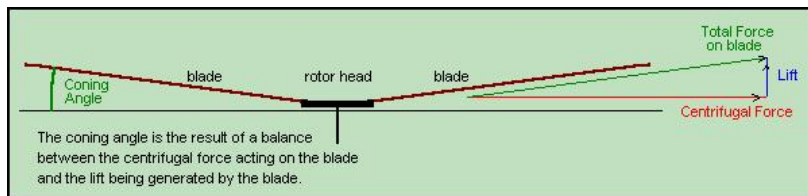


R-50's tri-hinge head





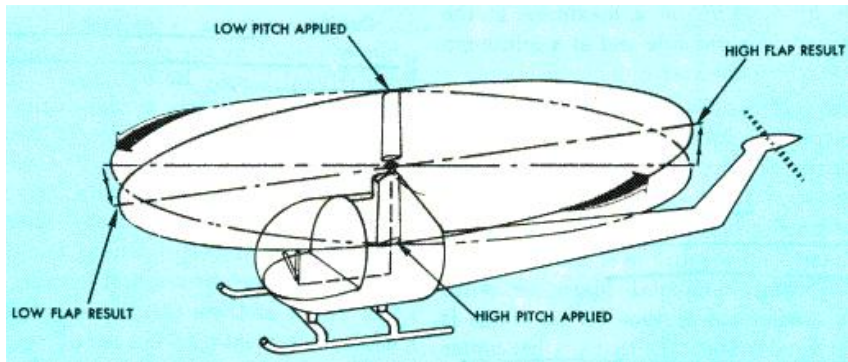
The Coning Angle



As a result of the hinged rotor head or the inherent flexibility of the blades the rotating disk will have an angle called the coning angle. This angle acts a little like the dihedral angle in fixed wing planes



The Flapping Angle

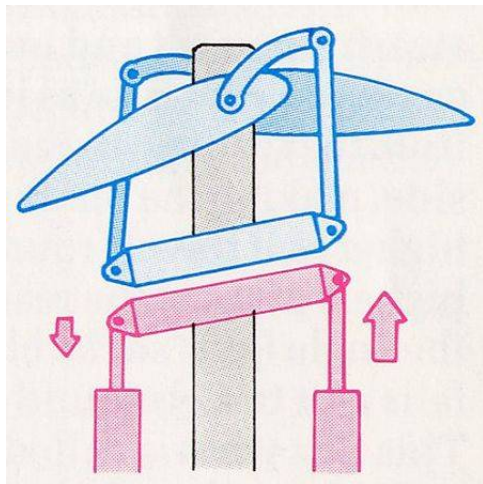


The other two significant angles are the longitudinal and lateral flapping angles. They are the result of control input from the swash plate and actuators. The angle tilts the rotor disk and changes the thrust vector relative to the center of gravity allowing translational motion.

The Swash Plate

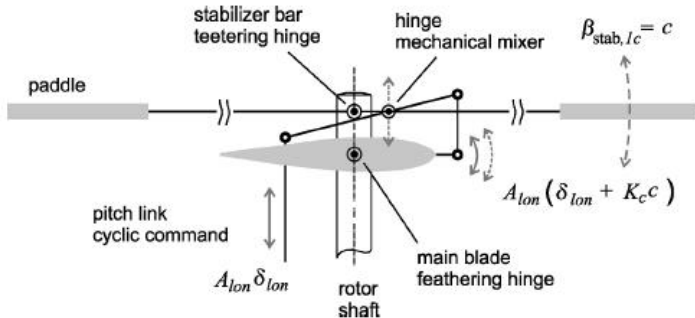
The swash plate is what transfers control input from the non rotating body to the rotating rotor head and flybar.

the lower part is actuated by the actuators and is free to move in height roll and pitch but not yaw as that would effect the phase angle where the rotor head is actuated.





The Flybar



The flybar is used to dampen the control inputs for model control since model scale helicopters have smaller rotor inertial in the man rotor disk this type of aerodynamic gearing allows of unassisted human control.



The Full CCPM Rotor Head and Swash Plate





The Parameterized State-space model

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{p} \\ \dot{q} \\ \dot{\phi} \\ \dot{\theta} \\ \tau_f \dot{a} \\ \tau_f \dot{b} \\ \dot{w} \\ \dot{r} \\ \dot{r}_{fb} \\ \tau_s \dot{c} \\ \tau_s \dot{d} \end{bmatrix} = \begin{bmatrix} X_u & 0 & 0 & 0 & 0 & -g & X_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & g & 0 & 0 & Y_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline L_u & L_v & 0 & 0 & 0 & 0 & 0 & L_b & L_w & 0 & 0 & 0 & 0 & 0 & 0 \\ M_u & M_v & 0 & 0 & 0 & 0 & M_a & 0 & M_w & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -\tau_f & 0 & 0 & -1 & A_b & 0 & 0 & 0 & 0 & A_c & 0 & 0 \\ 0 & 0 & -\tau_f & 0 & 0 & 0 & B_a & -1 & 0 & 0 & 0 & 0 & 0 & B_d & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & Z_a & Z_b & Z_w & Z_r & 0 & 0 & 0 & 0 & 0 \\ 0 & N_v & N_p & 0 & 0 & 0 & 0 & 0 & N_w & N_r & N_{rfb} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_r & K_{rfb} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -\tau_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -\tau_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ p \\ q \\ \phi \\ \theta \\ a \\ b \\ w \\ r \\ r_{fb} \\ c \\ d \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & Y_{ped} & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{col} \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline A_{lat} & A_{lon} & 0 & 0 \\ B_{lat} & B_{lon} & 0 & 0 \\ \hline 0 & 0 & 0 & Z_{col} \\ 0 & 0 & N_{ped} & N_{col} \\ 0 & 0 & 0 & 0 \\ \hline 0 & C_{lon} & 0 & 0 \\ D_{lat} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{lat} \\ \delta_{lon} \\ \delta_{ped} \\ \delta_{col} \end{bmatrix}$$

Identified model from the MIT paper used for the assignment