The More you Know - The Information Gain Approach to Path Planning (Part 1)

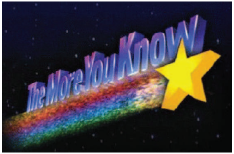
Liam Paull

Basic Motion Planning Concepts
  Some Important Terms
  Main Tasks
  Configuration Space

Brief Overview of Existing Path Planning Methods
  Bug Algorithms
  Potential Fields
  Roadmaps
  Cellular Decomposition

The More you Know - The Information Gain Approach to Path Planning (Part 1)

Liam Paull



Outline

1 Basic Motion Planning Concepts
- Some Important Terms
- Main Tasks
- Configuration Space

2 Brief Overview of Existing Path Planning Methods
- Bug Algorithms
- Potential Fields
- Roadmaps
- Cellular Decomposition

Outline

1 Basic Motion Planning Concepts
- Some Important Terms
- Main Tasks
- Configuration Space

2 Brief Overview of Existing Path Planning Methods
- Bug Algorithms
- Potential Fields
- Roadmaps
- Cellular Decomposition

Some Important Terms

Configuration space (Q_{free}): The set of all valid configuration that the robot can achieve
Workspace or **Environment**: The world that the robot exists in
Path: Parametrized curve through Q_{free}
Degrees of Freedom: The minimum number of independent variables required to represent the robot configuration. (e.g. How many for a car?)
Nonholonomic: A robot with less inputs than degrees of freedom (e.g. a car)
Optimality: An algorithm that optimizes (maximizes or minimizes) some objective

Some Important Terms - cont'd

Completeness: A plan is complete if it will always find a solution if one exists or determine that no solution exists in finite time

Resolution completeness: Complete subject to discretization

Offline planning: All knowledge of the environment is known and the plan is completed before execution begins

Online planning: The plan is incrementally constructed during execution

Sensor-based planning: Sensor information is processed online and used for planning

Some Important Terms - cont'd

Deliberative: Sense → Plan → Act cycle. An entire representation of the environment is built on each iteration
Reactive: Use sensory information to accomplish mission without representation of entire environment. Analogy: Brain vs. Spinal Cord

Some Important Terms - cont'd

Deliberative: Sense → Plan → Act cycle. An entire representation of the environment is built on each iteration
Reactive: Use sensory information to accomplish mission without representation of entire environment. Analogy: Brain vs. Spinal Cord
Coupled: Each agent is aware of others and their motions are planned together (optimal)
Decoupled: Each agent acts autonomously (scalable)

Outline

1 Basic Motion Planning Concepts
- Some Important Terms
- Main Tasks
- Configuration Space

2 Brief Overview of Existing Path Planning Methods
- Bug Algorithms
- Potential Fields
- Roadmaps
- Cellular Decomposition

Main Tasks of Motion Planning

Motion planning tasks usually fall into one or more of the following 4 areas:

1. Navigation - finding a collision-free path through an obstacle-laden environment

Main Tasks of Motion Planning

Motion planning tasks usually fall into one or more of the following 4 areas:

1. Navigation - finding a collision-free path through an obstacle-laden environment
2. Coverage - passing a sensor over every point in the environment

Main Tasks of Motion Planning

Motion planning tasks usually fall into one or more of the following 4 areas:

1. Navigation - finding a collision-free path through an obstacle-laden environment
2. Coverage - passing a sensor over every point in the environment
3. Localization - Using sensor data to determine the configuration of the robot within

Main Tasks of Motion Planning

Motion planning tasks usually fall into one or more of the following 4 areas:

1. Navigation - finding a collision-free path through an obstacle-laden environment
2. Coverage - passing a sensor over every point in the environment
3. Localization - Using sensor data to determine the configuration of the robot within the environment
4. Mapping - Using sensor to explore a previously unknown environment

Outline

1 Basic Motion Planning Concepts
- Some Important Terms
- Main Tasks
- Configuration Space

2 Brief Overview of Existing Path Planning Methods
- Bug Algorithms
- Potential Fields
- Roadmaps
- Cellular Decomposition

Configuration Space and Sensor Geometry

Q-Target (QT): Subset of Q such that target is in the field of view of the object sensor
Q-Obstacle (QB): Subset of Q that represents collision with an obstacle

Configuration Space and Sensor Geometry

Q-Target (QT): Subset of Q such that target is in the field of view of the object sensor
Q-Obstacle (QB): Subset of Q that represents collision with an obstacle

Example 1: Simple 2-D configuration space
Example 2: Robot Arm

Things get complicated fast!

Outline

1 Basic Motion Planning Concepts
- Some Important Terms
- Main Tasks
- Configuration Space

2 Brief Overview of Existing Path Planning Methods
- Bug Algorithms
- Potential Fields
- Roadmaps
- Cellular Decomposition

Bug Algorithms

Exhibit two behaviors: Move in a straight line, and follow a boundary

- Bug1

Bug Algorithms

Exhibit two behaviors: Move in a straight line, and follow a boundary

- Bug1
- Bug2

Bug Algorithms

Exhibit two behaviors: Move in a straight line, and follow a boundary

- Bug1
- Bug2
- Tangent Bug

Outline

1. Basic Motion Planning Concepts
   - Some Important Terms
   - Main Tasks
   - Configuration Space
2. Brief Overview of Existing Path Planning Methods
   - Bug Algorithms
   - Potential Fields
   - Roadmaps
   - Cellular Decomposition

Potential Fields

Potential Function: Differentiable function $U : R^m \rightarrow R$ that represents the energy such that ∇U represents force

Potential Fields

Potential Function: Differentiable function $U : R^m \rightarrow R$ that represents the energy such that ∇U represents force

Vector Field: Assigns a vector to each point on a manifold

Potential Fields

Potential Function: Differentiable function $U : R^m \rightarrow R$ that represents the energy such that ∇U represents force

Vector Field: Assigns a vector to each point on a manifold

Gradient Descent: $\dot{c}(t) = -\nabla U(c(t))$

Potential Fields

Potential Function: Differentiable function $U : R^m \rightarrow R$ that represents the energy such that ∇U represents force
Vector Field: Assigns a vector to each point on a manifold
Gradient Descent: $\dot{c}(t) = -\nabla U(c(t))$
Critical Point: Point where $\nabla U(q) = 0$ (local min, local max, or saddle point)

Potential Fields

Potential Function: Differentiable function $U : R^m \rightarrow R$ that represents the energy such that ∇U represents force
Vector Field: Assigns a vector to each point on a manifold
Gradient Descent: $\dot{c}(t) = -\nabla U(c(t))$
Critical Point: Point where $\nabla U(q) = 0$ (local min, local max, or saddle point)

Analogy: Electric field where robot is positively charged, obstacles are positively charged, and goal is negatively charged

Potential Fields - Potential Function Formulation

$$U(q) = U_{att}(q) + U_{rep}(q) \quad (1)$$

$$\nabla U(q) = \nabla U_{att}(q) + \nabla U_{rep}(q) \quad (2)$$

Potential Fields - Potential Function Formulation

$$U(q) = U_{att}(q) + U_{rep}(q) \tag{1}$$

$$\nabla U(q) = \nabla U_{att}(q) + \nabla U_{rep}(q) \tag{2}$$

$$U_{att}(q) = \frac{1}{2}\xi d^2(q, q_{goal}) \tag{3}$$

$$\nabla U_{att}(q) = \xi(q - q_{goal}) \tag{4}$$

Where ξ is a scalar multiplier

Potential Fields - Potential Function Formulation cont'd

$d_i(q)$ is the minimum distance to obstacle i
c_i is the closest point to q on obstacle i
Q_i^* is the domain of influence for obstacle i
η is a scalar multiplier

$$U_{rep_i}(q) = \begin{cases} \frac{1}{2}\eta(\frac{1}{d_i(q)} - \frac{1}{Q_i^*})^2 & , d_i(q) \leq Q_i^* \\ 0 & , d_i(q) > Q_i^* \end{cases} \tag{5}$$

Potential Fields - Potential Function Formulation cont'd

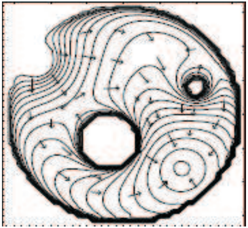
$d_i(q)$ is the minimum distance to obstacle i
c_i is the closest point to q on obstacle i
Q_i^* is the domain of influence for obstacle i
η is a scalar multiplier

$$U_{rep_i}(q) = \begin{cases} \frac{1}{2}\eta(\frac{1}{d_i(q)} - \frac{1}{Q_i^*})^2 & , d_i(q) \leq Q_i^* \\ 0 & , d_i(q) > Q_i^* \end{cases} \tag{5}$$

$$\nabla U_{rep_i} = \begin{cases} \eta(\frac{1}{Q_i^*} - \frac{1}{d_i(q)})\frac{q-c_i}{d_i^3(q)} & , d_i q \leq Q_i^* \\ 0 & , d_i(q) > Q_i^* \end{cases} \tag{6}$$

$$\nabla U_{rep} = \sum_{i=1}^{n} \nabla U_{rep_i} \tag{7}$$

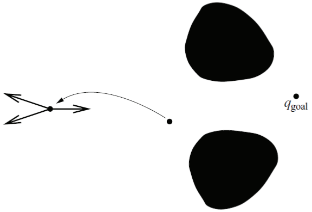
Potential Fields



Are there any problems with this approach?

Potential Fields - Some Issues

Local minima not corresponding to the goal



Possible Solutions:

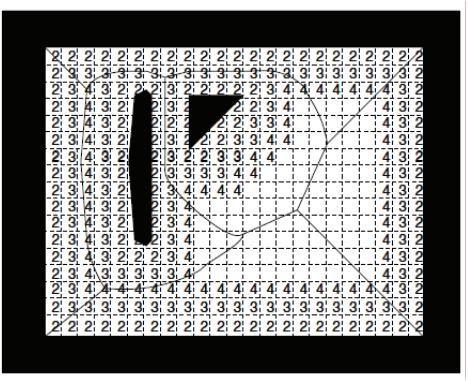
- Add search based planner on top (Examples: Wave-Front Planner))
- Ensure there are no local minima: Navigation Potential Functions

Potential Fields - Some Issues cont'd

How do we calculate $d_i(q)$?

Potential Fields - Some Issues cont'd

How do we calculate $d_i(q)$?
One solution: brushfire algorithm



Outline

1 Basic Motion Planning Concepts
- Some Important Terms
- Main Tasks
- Configuration Space

2 Brief Overview of Existing Path Planning Methods
- Bug Algorithms
- Potential Fields
- Roadmaps
- Cellular Decomposition

Roadmaps

A **roadmap** consists of nodes, which correspond to specific locations, and edges, which define the path between locations. Reduces motion planning to 3 step procedure:

1. Find collision-free path to the roadmap
2. Use the roadmap to get close to the goal
3. Find collision-free path from roadmap to the goal

Roadmaps

A **roadmap** consists of nodes, which correspond to specific locations, and edges, which define the path between locations. Reduces motion planning to 3 step procedure:

1. Find collision-free path to the roadmap
2. Use the roadmap to get close to the goal
3. Find collision-free path from roadmap to the goal

DEFINITION: A roadmap (RM) is a union of 1-D curves where $\forall q_{start}, q_{goal} \in Q_{free}$:

- There exists a collision-free path from q_{start} to $q_{start'} \in RM$ (accessibility)
- There exists a collision-free path from $q_{goal'} \in RM$ to q_{goal} (departability)
- There exists a path in RM between $q_{start'}$ and $q_{goal'}$

Roadmaps - Visibility Map

In a **visibility map** nodes share an edge if they are within line of sight of each other and all points in Q_{free} are in the line of sight of at least one node

Roadmaps - Visibility Map

In a **visibility map** nodes share an edge if they are within line of sight of each other and all points in Q_{free} are in the line of sight of at least one node

Example: Visibility Graph - q_{start}, q_{goal} and all obstacle vertices are nodes

Roadmaps - Visibility Map

In a **visibility map** nodes share an edge if they are within line of sight of each other and all points in Q_{free} are in the line of sight of at least one node

Example: Visibility Graph - q_{start}, q_{goal} and all obstacle vertices are nodes

Supporting Lines are tangent to 2 obstacles with both of them on the same side of the line
Separating Lines are tangent to 2 obstacles with each on opposite sides of the line

The best solution will be made of supporting and separating lines

Roadmaps - Deformation Retracts

A **retraction** is a function $f : X \to A$ with $A \subset X$ and $f(a) = a, \forall a, a \in A$

Roadmaps - Deformation Retracts

A **retraction** is a function $f : X \to A$ with $A \subset X$ and $f(a) = a, \forall a, a \in A$
A **homotopy** is a function $H : U \times [0, 1] \to V$ such that $H(x, 0) = a(x)$ and $H(x, 1) = b(x)$
E.g $H(x, t) = (1 - t)a(x) + tb(x)$

Roadmaps - Deformation Retracts

A **retraction** is a function $f : X \rightarrow A$ with $A \subset X$ and $f(a) = a, \forall a, a \in A$
A **homotopy** is a function $H : U \times [0,1] \rightarrow V$ such that $H(x,0) = a(x)$ and $H(x,1) = b(x)$
E.g $H(x,t) = (1-t)a(x) + tb(x)$

DEFINITION: Given that $f : X \rightarrow A$ is a retraction, a **deformation retraction**, $H : X \times [0,1] \rightarrow X$ is a homotopy between f and the identity function.

Roadmaps - Deformation Retracts

A **retraction** is a function $f : X \to A$ with $A \subset X$ and $f(a) = a, \forall a, a \in A$
A **homotopy** is a function $H : U \times [0,1] \to V$ such that $H(x,0) = a(x)$ and $H(x,1) = b(x)$
E.g $H(x,t) = (1-t)a(x) + tb(x)$

DEFINITION: Given that $f : X \to A$ is a retraction, a **deformation retraction**, $H : X \times [0,1] \to X$ is a homotopy between f and the identity function.
More precisely:

$$H(x,0) = x \tag{8}$$
$$H(x,1) \in A \tag{9}$$
$$H(a,t) = a, \forall a, a \in A, t \in [0,1] \tag{10}$$

Roadmaps - Deformation Retracts

The **Generalized Voronoi Diagram** GVD is the set of points in R^2 whose distance to the 2 closest obstacles is the same

It can be shown that the (GVD) is a deformation retract and therefore a valid RM

Roadmaps - Deformation Retracts

The **Generalized Voronoi Diagram** GVD is the set of points in R^2 whose distance to the 2 closest obstacles is the same

It can be shown that the (GVD) is a deformation retract and therefore a valid *RM*

The concept can be extended to higher dimensions (The Generalized Voronoi Graph (GVG) generated by the intersection of planes generated by the GVD) and to non-Euclidean configuration spaces (Rod-Hierarchical GVG)

Roadmaps - Silhouette Method

Procedure:

- Pick a dimension at random q_1 and evaluate $\pi_2 : R^{m-1} \rightarrow R$ along slices

Roadmaps - Silhouette Method

Procedure:

- Pick a dimension at random q_1 and evaluate $\pi_2 : R^{m-1} \rightarrow R$ along slices
- Extrema of π_2 become part of the silhouette

Roadmaps - Silhouette Method

Procedure:

- Pick a dimension at random q_1 and evaluate $\pi_2 : R^{m-1} \rightarrow R$ along slices
- Extrema of π_2 become part of the silhouette
- Identify *critical points* as points where the number of extrema changes. The slices containing the critical points are *critical slices*

Roadmaps - Silhouette Method

Procedure:

- Pick a dimension at random q_1 and evaluate $\pi_2 : R^{m-1} \rightarrow R$ along slices
- Extrema of π_2 become part of the silhouette
- Identify *critical points* as points where the number of extrema changes. The slices containing the critical points are *critical slices*
- Recursively scan critical slices, reducing the dimension of the slice by one each time, until all slices are 1-D and all critical points have been identified

Roadmaps - Silhouette Method

Procedure:

- Pick a dimension at random q_1 and evaluate $\pi_2 : R^{m-1} \rightarrow R$ along slices
- Extrema of π_2 become part of the silhouette
- Identify *critical points* as points where the number of extrema changes. The slices containing the critical points are *critical slices*
- Recursively scan critical slices, reducing the dimension of the slice by one each time, until all slices are 1-D and all critical points have been identified
- *RM* consists of the silhouette and the final 1-D critical slices

Outline

1. Basic Motion Planning Concepts
   - Some Important Terms
   - Main Tasks
   - Configuration Space
2. Brief Overview of Existing Path Planning Methods
   - Bug Algorithms
   - Potential Fields
   - Roadmaps
   - Cellular Decomposition

Cellular Decomposition

DEFINITION: Cellular decomposition breaks down the workspace into cells

A connectivity graph can then be built where the cells represents the nodes, and any two cells that share a boundary are connected in the graph

Most useful for achieving **coverage** because the workspace is broken into cells and then each cell can be covered using a simple method (e.g. back and forth motions)

Exact Cellular Decomposition

- Exact Trapezoidal Decomposition

Exact Cellular Decomposition

- Exact Trapezoidal Decomposition
- Morse Decomposition

Exact Cellular Decomposition

- Exact Trapezoidal Decomposition
- Morse Decomposition
    - Reeb Graph

Exact Cellular Decomposition

- Exact Trapezoidal Decomposition
- Morse Decomposition
  - Reeb Graph
- Exact Hexagon Decomposition

Approximate Cellular Decomposition

Union of cells does not equal of the target environment, therefore coverage cannot be guaranteed, but can be used for obstacle avoidance and target searching using information gain

Example: Approximate rectangloid decomposition