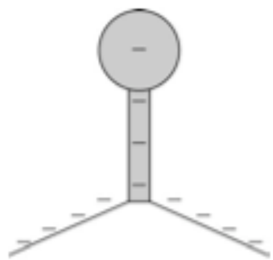
ECE1813 Fall 2015 Assignment #1 Solutions

Conceptual Questions

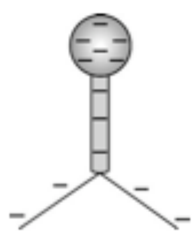
25.5. Upon touching the charged rod, the metal exchanges charge with the area of the rod touched by the sphere (we are assuming the rod is an insulator). Some of the local excess charge on the rod will spread over the conducting sphere, so that both the rod and the sphere will have an overall excess charge of the same type. Thus, the sphere and the rod will repel each other.

25.7. (a)



The negatively charged rod will repel the negative charges on the top of the electroscope, pushing more negative charge down onto the leaves. The leaves will separate more.

(b)



The positively charged rod will attract more negative charges to the top of the electroscope. As they depart from the leaves, the leaves will move closer together.

25.13. (a) The magnitude of the force on A quadruples (increases by a factor of 4), since the force between the charges is proportional to the product of the magnitudes of the charges. Therefore, $F'_A = 4F$.
(b) By Newton's third law, the force of A on B is equal in magnitude to the force of B on A; therefore the force on B also quadruples; $F'_B = F'_A = 4F$.

25.15. Since the force on a charge in an electric field has magnitude $F = qE$, the new force is

$$F' = (3q)\left(\frac{E}{2}\right) = \frac{3}{2}qE = \frac{3}{2}F.$$

25.16. Model: Charges A, B, and C are point charges.
Visualize: Please refer to Figure EX25.16. Charge A experiences an electric force $\vec{F}_{\text{B on A}}$ due to charge B and an electric force $\vec{F}_{\text{C on A}}$ due to charge C. The force $\vec{F}_{\text{B on A}}$ is directed to the right and the force $\vec{F}_{\text{C on A}}$ is directed to the left.
Solve: Coulomb's law yields:

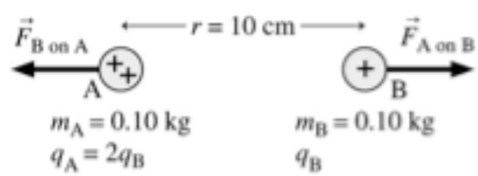
$$F_{\text{B on A}} = K\frac{|q_A||q_B|}{r^2} = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(1.0\times10^{-9}\text{ C})(1.0\times10^{-9}\text{ C})}{(1.0\times10^{-2}\text{ m})^2} = 9.0\times10^{-5}\text{ N}$$

$$F_{\text{C on A}} = K\frac{|q_C||q_A|}{r^2} = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(1.0\times10^{-9}\text{ C})(4.0\times10^{-9}\text{ C})}{(2.0\times10^{-2}\text{ m})^2} = 9.0\times10^{-5}\text{ N}$$

The net force on A is

$$\vec{F}_{\text{on A}} = \vec{F}_{\text{B on A}} + \vec{F}_{\text{C on A}} = (9.0\times10^{-5}\text{ N})\hat{i} + (9.0\times10^{-5}\text{ N})(-\hat{i}) = 0.0\text{ N}$$

25.32. Model: Objects A and B are point charges.
Visualize:



Solve: **(a)** It is given that $F_{\text{A on B}} = 0.45$ N. By Newton's third law, $F_{\text{B on A}} = F_{\text{A on B}} = 0.45$ N. Coulomb's law gives

$$F_{\text{B on A}} = F_{\text{A on B}} = 0.45\text{ N} = \frac{Kq_Aq_B}{r^2} = \frac{K(q_A)(\frac{1}{2}q_A)}{r^2}$$

$$q_A = \sqrt{\frac{2(0.45\text{ N})r^2}{K}} = \sqrt{\frac{2(0.45\text{ N})(10\times10^{-2}\text{ m})^2}{(9.0\times10^9\text{ N m}^2/\text{C}^2)}} = 1.0\times10^{-6}\text{ C}$$

(b) Newton's second law is $F_{\text{B on A}} = m_A a_A$. Hence,

$$a_A = \frac{F_{\text{B on A}}}{m_A} = \frac{F_{\text{A on B}}}{m_B} = \frac{0.45\text{ N}}{0.100\text{ kg}} = 4.5\text{ m/s}^2$$

25.40. Model: The charges are point charges.

Visualize: Please refer to Figure P25.40.

Solve: Placing the 1.0 nC charge at the origin and calling it q_1, the –6.0 nC is q_3, the q_2 charge is in the first quadrant, and the q_4 charge is in the second quadrant. The net electric force on q_1 is the vector sum of the electric forces from the three charges q_2, q_3, and q_4. We have

$$\vec{F}_{2\text{ on }1} = \left(\frac{K|q_1||q_2|}{r^2}, \text{away from } q_2\right)$$
$$= \left(\frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(1.0\times10^{-9}\text{ C})(2.0\times10^{-9}\text{ C})}{(5.0\times10^{-2}\text{ m})^2}, \text{away from } q_2\right)$$
$$= (0.72\times10^{-5}\text{ N, away from } q_2) = (0.720\times10^{-5}\text{ N})[-\cos(45°)\hat{\imath} - \sin(45°)\hat{\jmath}]$$

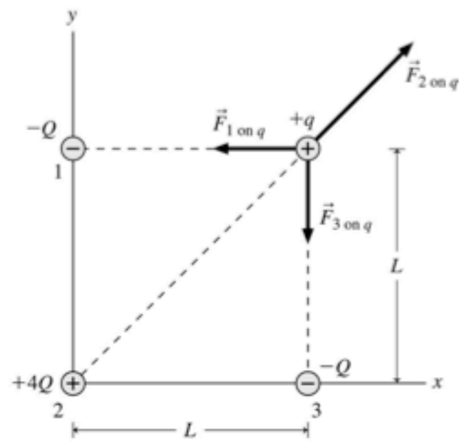
$$\vec{F}_{3\text{ on }1} = \left(\frac{K|q_1||q_3|}{r^2}, \text{toward } q_3\right)$$
$$= \left(\frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(1.0\times10^{-9}\text{ C})(6.0\times10^{-9}\text{ C})}{(5.0\times10^{-2}\text{ m})^2}, \text{toward } q_3\right)$$
$$= (2.16\times10^{-5}\text{ N, away from } q_3) = 2.16\times10^{-5}\,\hat{\jmath}\text{ N}$$
$$\vec{F}_{4\text{ on }1} = \left(\frac{K|q_1||q_4|}{r^2}, \text{away from } q_4\right) = (0.720\times10^{-5}\text{ N})[\cos(45°)\hat{\imath} - \sin(45°)\hat{\jmath}]$$

Summing these forces vectorially gives

$$\vec{F}_{\text{on }1} = \vec{F}_{2\text{ on }1} + \vec{F}_{3\text{ on }1} + \vec{F}_{4\text{ on }1} = [(2.16\times10^{-5}\text{ N}) - 2(0.720\times10^{-5}\text{ N})\sin(45°)]\hat{\jmath} = 1.1\times10^{-5}\,\hat{\jmath}\text{ N}$$

Assess: By symmetry, we see that the horizontal forces must cancel, and that the vertical force must be upward, which agrees with the calculation.

25.47. Model: The charges are point charges.
Solve:



We will denote the charges $-Q$, $4Q$ and $-Q$ by 1, 2, and 3, respectively.

$$\vec{F}_{1\text{ on }q} = \left(\frac{K|-Q||q|}{L^2}, \text{ toward } -Q\right) = \frac{KQq}{L^2}(-\hat{i})$$

$$\vec{F}_{2\text{ on }q} = \left(\frac{K|4Q||q|}{(\sqrt{2}L)^2}, \text{ away from } 4Q\right) = \frac{4KQq}{2L^2}[+\cos(45°)\hat{i} + \sin(45°)\hat{j}] \Rightarrow F_{2\text{ on }q} = \frac{2KQq}{L^2} = 2F_{1\text{ on }q}$$

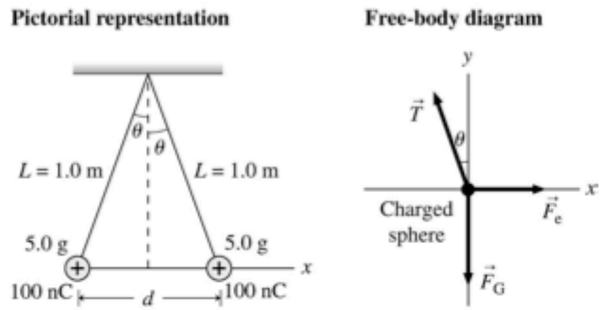
$$\vec{F}_{3\text{ on }q} = \left(\frac{K|-Q||q|}{L^2}, \text{ toward } -Q\right) = \frac{KQq}{L^2}(-\hat{j})$$

The net electric force on the charge $+q$ is the vector sum of the electric forces from the other three charges. The net force is

$$\vec{F}_{\text{net}} = \frac{KQq}{L^2}(-\hat{i}) + \frac{2KQq}{L^2}\left(+\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right) + \frac{KQq}{L^2}(-\hat{j}) = -\frac{KQq}{L^2}\hat{i}(1-\sqrt{2}) - \frac{KQq}{L^2}\hat{j}(1-\sqrt{2})$$

$$F_{\text{net}} = \sqrt{\left[\frac{KQq}{L^2}(1-\sqrt{2})\right]^2 + \left[\frac{KQq}{L^2}(1-\sqrt{2})\right]^2} = (2-\sqrt{2})\frac{KQq}{L^2}$$

25.58. Model: The charged spheres behave as point charges.
Visualize:



Each sphere is in static equilibrium and the string makes an angle θ with the vertical. The three forces acting on each sphere are the electric force, the gravitational force on the sphere, and the tension force.

Solve: In static equilibrium, Newton's first law gives $\vec{F}_{\text{net}} = \vec{T} + \vec{F}_G + \vec{F}_e = \vec{0}$. In component form,

$$(F_{\text{net}})_x = T_x + (F_G)_x + (F_e)_x = 0 \text{ N} \qquad (F_{\text{net}})_y = T_y + (F_G) + (F_e)_y = 0 \text{ N}$$

$$-T\sin\theta + 0 \text{ N} + \frac{Kq^2}{d^2} = 0 \text{ N} \qquad T\cos\theta - mg + 0 \text{ N} = 0 \text{ N}$$

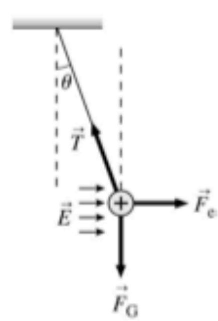
$$T\sin\theta = \frac{Kq^2}{d^2} = \frac{Kq^2}{(2L\sin\theta)^2} \qquad T\cos\theta = +mg$$

Dividing the two equations gives

$$\sin^2\theta\tan\theta = \frac{Kq^2}{4L^2mg} = \frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(100\times10^{-9} \text{ C})^2}{4(1.0 \text{ m})^2(5.0\times10^{-3} \text{ kg})(9.8 \text{ N/kg})} = 4.59\times10^{-4}$$

For small-angles, $\tan\theta \approx \sin\theta$. With this approximation we obtain $\sin\theta = 0.07714$ rad, , so $\theta = 4.4°$.

25.67. Model: The charged ball attached to the string is the point charge.
Visualize:



The charged ball is in static equilibrium in the electric field when the string makes an angle θ with the vertical. The three forces acting on the charge are the electric force due to the electric field, the gravitational force on the ball, and the tension force.

Solve: In static equilibrium, Newton's second law for the charged ball gives $\vec{F}_{\text{net}} = \vec{T} + \vec{F}_G + \vec{F}_3 = \vec{0}$. In component form,

$$(F_{\text{net}})_x = T_x + 0 \text{ N} + qE = 0 \text{ N} \qquad (F_{\text{net}})_y = T_y - mg + 0 \text{ N} = 0 \text{ N}$$

These two equations become $T\sin\theta = qE$ and $T\cos\theta = mg$. Dividing the equations gives

$$\tan\theta = \frac{qE}{mg} = \frac{(25\times10^{-9} \text{ C})(200{,}000 \text{ N/C})}{(2.0\times10^{-3} \text{ kg})(9.8 \text{ N/kg})} = 0.255 \quad\Rightarrow\quad \theta = 14°$$