ECE1813 Fall 2015 Assignment #2 Solutions

Conceptual Questions

26.5. $F = eE = e\left(\dfrac{1}{4\pi E_0}\dfrac{2\lambda}{r}\right)$. If the charge density λ is doubled, then the distance r from the wire must also be doubled for the force to be the same. Thus $r = 2$ cm.

26.13. **(a)** Accelerates to the right.
(b) Remains in place.
(c) Accelerates to the left.

Exercises and Problems

26.4. Model: The electric field at the point is found by superposition of the fields due to the two charges located on the y-axis.
Visualize: The electric field due to the positive charge q_1 at the point is away from q_1. On the other hand, the electric field due to the negative charge q_2 at the point is toward q_2. These two electric fields are then added vertically to obtain the net electric field at the point.
Solve: The electric field from q_1 is

$$\vec{E}_1 = \left(\frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2}, \theta \text{ below } -x\text{-axis}\right) = \left(\frac{(9.0\times10^9 \text{ N m}^2/\text{C}^2)(3.0\times10^{-9}\text{ C})}{(0.050\text{ m})^2 + (0.050\text{ m})^2}\right)(-\cos\theta\hat{i} - \sin\theta\hat{j})$$

Because $\tan\theta = 5 \text{ cm}/5 \text{ cm}, \theta = 45°$. So,

$$\vec{E}_1 = (5400 \text{ N/C})\left(-\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right)$$

Similarly, the electric field from q_2 is

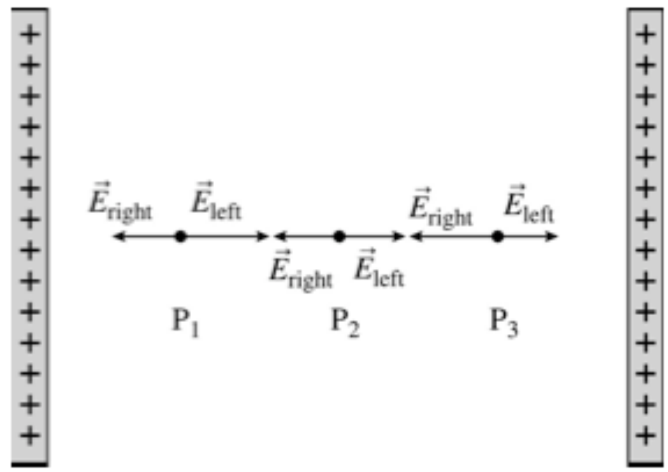
$$\vec{E}_2 = \left(\frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{r_2^2}, \theta \text{ below } +x\text{-axis}\right) = (5400 \text{ N/C})\left(+\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\Rightarrow \vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = 2(5400 \text{ N/C})\left(-\frac{1}{\sqrt{2}}\right)\hat{j} = -7637\,\hat{j} \text{ N/C} = -7.6\times10^3\ \hat{j} \text{ N/C}$$

Thus, the strength of the electric field is 7.6×10^3 N/C and its direction is vertically downward.
Assess: A quick visualization of the components of the two electric fields shows that the horizontal components cancel.

26.9. Model: The rods are thin. Assume that the charge lies along a *line*.

Visualize:



Because both the rods are positively charged, the electric field from each rod points away from the rod. Because the electric fields from the two rods are in opposite directions at P_1, P_2, and P_3, the net field strength at each point is the difference of the field strengths from the two rods.

Solve: Example 26.3 gives the electric field strength in the plane that bisects a charged rod:

$$E_{rod} = \frac{1}{4\pi\varepsilon_0} \frac{|Q|}{r\sqrt{r^2 + (L/2)^2}}$$

The electric field from the rod on the right at a distance of 1 cm from the rod is

$$E_{right} = (9.0\times10^9 \text{ N m}^2/\text{C}^2)\frac{10\times10^{-9}\text{ C}}{(0.01\text{ m})\sqrt{(0.01\text{ m})^2 + (0.05\text{ m})^2}} = 1.765\times10^5 \text{ N/C}$$

The electric field from the rod on the right at distances 2 cm and 3 cm from the rod are 0.835×10^5 N/C and 0.514×10^5 N/C. The electric fields produced by the rod on the left at the same distances are the same. Point P_1 is 1.0 cm from the rod on the left and is 3.0 cm from the rod on the right. Because the electric fields at P_1 have opposite directions, the net electric field strengths are

$$\begin{aligned} &\text{At 1.0 cm} \quad E = 1.765\times10^5 \text{ N/C} - 0.514\times10^5 \text{ N/C} = 1.3\times10^5 \text{ N/C} \\ &\text{At 2.0 cm} \quad E = 0.835\times10^5 \text{ N/C} - 0.835\times10^5 \text{ N/C} = 0 \text{ N/C} \\ &\text{At 3.0 cm} \quad E = 1.765\times10^5 \text{ N/C} - 0.514\times10^5 \text{ N/C} = 1.3\times10^5 \text{ N/C} \end{aligned}$$

26.18. Model: The electric field is uniform in a region of space between closely spaced capacitor plates.

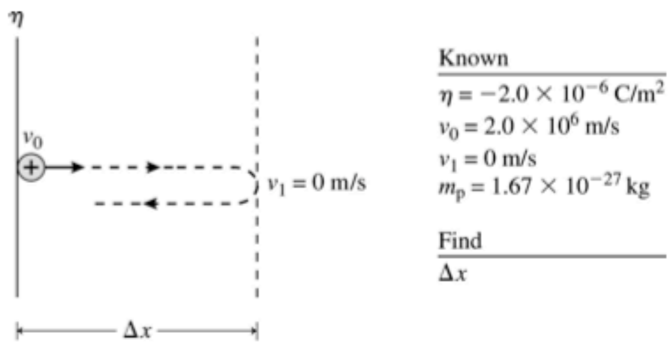
Solve: The electric field inside a capacitor is $E = Q/\varepsilon_0 A$. Thus, the charge needed to produce a field of strength E is

$$Q = \varepsilon_0 AE = (8.85\times10^{-12} \text{ C}^2/\text{N m}^2)\pi(0.030\text{ m})^2(1.0\times10^6 \text{ N/C}) = 25 \text{ nC}$$

Thus, one plate has a charge of 25 nC and the other has a charge of –25 nC.

Assess: Note that the capacitor as a whole has no net charge.

26.23. Model: The infinite negatively charged plane produces a uniform electric field that is directed toward the plane.
Visualize:



Solve: From the kinematic equation of motion $v_1^2 = 0 = v_0^2 + 2a\Delta x$ and $F = qE = ma$,

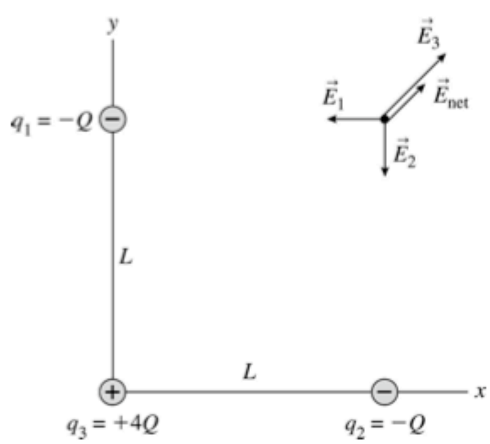
$$a = \frac{qE}{m} = \frac{-v_0^2}{2\Delta x} \Rightarrow \Delta x = \frac{-mv_0^2}{2qE}$$

Furthermore, the electric field of a plane of charge with surface charge density η is $E = \eta/2\varepsilon_0$. Thus,

$$\Delta x = \frac{-mv_0^2\varepsilon_0}{q\eta} = \frac{-(1.67\times10^{-27}\text{ kg})(2.0\times10^{6}\text{ m/s})^2(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)}{(1.60\times10^{-19}\text{ C})(-2.0\times10^{-6}\text{ C/m}^2)} = 0.18\text{ m}$$

26.31. Model: The electric field is that of three point charges $q_1 = -Q$, $q_2 = -Q$, and $q_3 = +4Q$.

Visualize: Assume the charges are in the x-y plane. The net electric field at point P is $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. The procedure will be to find the magnitudes of the electric fields, to write them in component form, and to add the components.



Solve: The electric field produced by q_1 points toward q_1 and is given by

$$\vec{E}_1 = -\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{L^2}\right)\hat{i}$$

The electric field produced by q_2 points toward q_2 and is given by

$$\vec{E}_2 = -\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{L^2}\right)\hat{j}$$

The electric field produced by q_3 is

$$E_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{4Q}{L^2+L^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{L^2}\right)$$

$\vec{E}_3$ points away from q_3 and makes an angle $\phi = \tan^{-1}(L/L) = 45°$ with the x-axis. So

$$\vec{E}_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{L^2}\right)(\cos\phi\hat{i} + \sin\phi\hat{j}) = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{L^2}\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

Adding these three vectors gives

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{L^2}\left[\left(\frac{2}{\sqrt{2}}-1\right)\hat{i} + \left(\frac{2}{\sqrt{2}}-1\right)\hat{j}\right] = \frac{1}{4\pi\varepsilon_0}\frac{Q}{L^2}(\sqrt{2}-1)(\hat{i}+\hat{j})$$

26.46. Model: Assume that the plastic sheets are planes of charge.

Solve: At point 1 the electric fields due to the left sheet and the right sheet are

$$\vec{E}_{\text{left}} = \left(\frac{\eta_0}{2\varepsilon_0}, \text{ toward right}\right) = \frac{\eta_0}{2\varepsilon_0}\hat{i} \qquad \vec{E}_{\text{right}} = \left(\frac{3\eta_0}{2\varepsilon_0}, \text{ toward left}\right) = -\frac{3\eta_0}{2\varepsilon_0}\hat{i}$$

$$\Rightarrow \vec{E}_{\text{net}} = \vec{E}_{\text{left}} + \vec{E}_{\text{right}} = -\frac{\eta_0}{\varepsilon_0}\hat{i}$$

At point 2, $\vec{E}_{\text{left}} = -(\eta_0/2\varepsilon_0)\hat{i}$, $\vec{E}_{\text{right}} = -(3\eta_0/2\varepsilon_0)\hat{i}$, and $\vec{E}_{\text{net}} = -(2\eta_0/\varepsilon_0)\hat{i}$. At point 3, $\vec{E}_{\text{net}} = +(\eta_0/\varepsilon_0)\hat{i}$.

26.50. Model: Assume that the electric field inside the capacitor is constant, so constant-acceleration kinematic equations apply.

Solve: (a) The force on the electron inside the capacitor is

$$\vec{F} = m\vec{a} = q\vec{E} \Rightarrow \vec{a} = \frac{q\vec{E}}{m}$$

Because $\vec{E}$ is directed upward (from the positive plate to the negative plate) and $q = -1.60\times10^{-19}$ C, the acceleration of the electron is downward. We can therefore write the above equation as simply $a_y = qE/m$. To determine E, we must first find a_y. From kinematics,

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2 \Rightarrow 0.040\text{ m} = 0\text{ m} + v_0\cos 45°(t_1 - t_0) + 0\text{ m}$$

$$\Rightarrow (t_1 - t_0) = \frac{(0.040\text{ m})}{(5.0\times10^6\text{ m/s})\cos 45°} = 1.1314\times10^{-8}\text{ s}$$

Using the kinematic equations for the motion in the y-direction,

$$v_{1y} = v_{0y} + a_y\left(\frac{t_1 - t_0}{2}\right) \Rightarrow 0\text{ m/s} = v_0\sin 45° + \left(\frac{qE}{m}\right)\left(\frac{t_1 - t_0}{2}\right)$$

$$\Rightarrow E = -\frac{2\text{ m }v_0\sin 45°}{q(t_1 - t_0)} = -\frac{2(9.1\times10^{-31}\text{ kg})(5.0\times10^6\text{ m/s})\sin 45°}{(-1.60\times10^{-19}\text{ C})(1.1314\times10^{-8}\text{ s})} = 3550\text{ N/C} = 3.6\times10^3\text{ N/C}$$

(b) To determine the separation between the two plates, we note that $y_0 = 0$ m and $v_{0y} = (5.0\times10^6\text{ m/s})\sin 45°$, but at $y = y_1$, the electron's highest point, $v_{1y} = 0$ m/s. From kinematics,

$$v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0) \Rightarrow 0\text{ m}^2/\text{s}^2 = v_0^2\sin^2 45° + 2a_y(y_1 - y_0)$$

$$\Rightarrow (y_1 - y_0) = -\frac{v_0^2\sin^2 45°}{2a_y} = -\frac{v_0^2}{4a_y}$$

From part (a),

$$a_y = \frac{qE}{m} = \frac{(-1.60\times10^{-19}\text{ C})(3550\text{ N/C})}{9.1\times10^{-31}\text{ kg}} = -6.242\times10^{14}\text{ m/s}^2$$

$$\Rightarrow y_1 - y_0 = -\frac{(5.0\times10^6\text{ m/s})^2}{4(-6.242\times10^{14}\text{ m/s}^2)} = 0.010\text{ m} = 1.0\text{ cm}$$

This is the height of the electron's trajectory, so the minimum spacing is 1.0 cm.