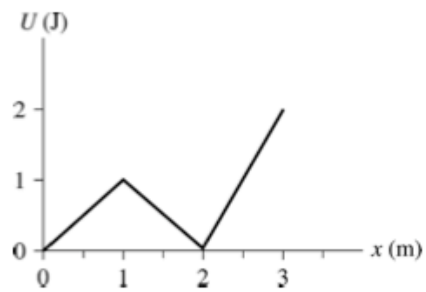
ECE1813 Fall 2015 Assignment # 3 Solutions

Conceptual Questions

**28.3.** **(a)** The electric potential energy of the charges is $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$. The potential energy varies as $\frac{1}{r}$, but because the two charges have opposite signs the potential energy actually increases as $r$ increases.
**(b)** Less than. Two opposite charges slow down as they move farther apart.

**28.6.** **(a)** Note $U = qV$.

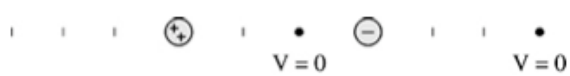



**(b)** At $x = 1$ m, the total energy $E = K_1 + U_1 = 1\text{ J} + 1\text{ J} = 2\text{ J}$. At the turning point $v = 0$ so $K_2 = 0$. Since $E = K_2 + U_2$, $2J = 0J + U_2 \Rightarrow U_2 = 2J$. This occurs at $x = 3$ m.

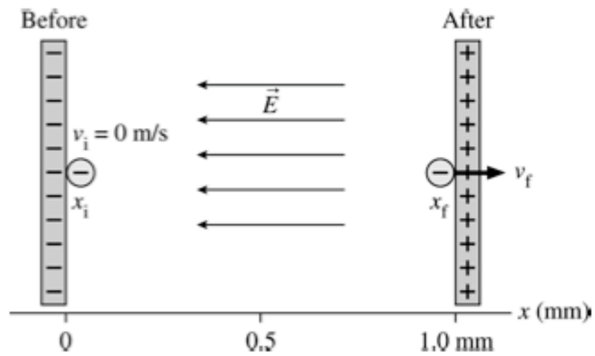
**28.12.** The potential due to the two charges at a position $x$ is

$$U(x) = \frac{1}{4\pi\varepsilon_0}\frac{2q}{|x|} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{|x-3|}$$

where the $2q$ charge is at $x = 0$ and the $-q$ charge is at $x = 3$. Setting $U(x) = 0$ and solving for $x$ yields the two locations.



**28.2. Model:** The mechanical energy of the electron is conserved. A parallel-plate capacitor has a uniform electric field.
**Visualize:**




The figure shows the before-and-after pictorial representation. The electron has an initial speed $v_i = 0$ m/s and a final speed $v_f$ after traveling a distance $d = 1.0$ mm.

**Solve:** The electron loses potential energy and gains kinetic energy as it moves toward the positive plate. The potential energy $U$ is defined as $U = U_0 + qEx$, where $x$ is the distance from the negative plate and $U_0$ is the potential energy at the negative plate (at $x = 0$ m). Thus, the change in the potential energy of the electron is

$$\Delta U_e = U_f - U_i = (U_0 + qEd) - (U_0 + 0\text{ J}) = qEd$$

The change in the kinetic energy of the electron is

$$\Delta K = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2$$

Now, the law of conservation of mechanical energy gives $\Delta K + \Delta U = 0$ J. This means

$$\tfrac{1}{2}mv_f^2 + qEd = 0\text{ J}$$

$$\Rightarrow v_f = \sqrt{\frac{-2qEd}{m}} = \sqrt{\frac{(-2)(-1.60\times10^{-19}\text{ C})(20{,}000\text{ N/C})(1.0\times10^{-3}\text{ m})}{9.11\times10^{-31}\text{ kg}}} = 2.7\times10^{6}\text{ m/s}$$

**Assess:** Note that $\Delta U_e = qEd$ is the change in the potential energy of the electron. It is negative because $q = -e$ for the electron. Thus, the potential energy becomes more negative as $d$ increases, that is, the potential energy of the electron decreases with an increase in $d$ (or $x$).

**28.7. Model:** The charges are point charges.

**Solve:** For a system of point charges, the potential energy is the sum of the potential energies due to all distinct pairs of charges:

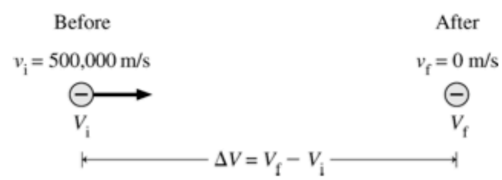
$$U_{\text{elec}} = \sum_{i,j} \frac{Kq_i q_j}{r_{ij}} = U_{12} + U_{13} + U_{23}$$

$$= (9.0\times10^{9}\text{ N m}^2/\text{C}^2)\left[\frac{(2.0\times10^{-9}\text{ C})(3.0\times10^{-9}\text{ C})}{0.030\text{ m}} + \frac{(2.0\times10^{-9}\text{ C})(3.0\times10^{-9}\text{ C})}{0.040\text{ m}} + \frac{(3.0\times10^{-9}\text{ C})(3.0\times10^{-9}\text{ C})}{\sqrt{(0.030\text{ m})^2 + (0.040\text{ m})^2}}\right]$$

$$= 1.80\times10^{-6}\text{ J} + 1.35\times10^{-6}\text{ J} + 1.62\times10^{-6}\text{ J} = 4.8\times10^{-6}\text{ J}$$

**Assess:** Note that $U_{12} = U_{21}$, $U_{13} = U_{31}$, and $U_{23} = U_{32}$.

**28.15. Model:** Energy is conserved. The potential energy is determined by the electric potential.
**Visualize:**




The figure shows a before-and-after pictorial representation of an electron moving through a potential difference.
**Solve: (a)** Because the electron is a negative charge and it slows down as it travels, it must be moving from a region of higher potential to a region of lower potential.
**(b)** Using the conservation of energy equation,

$$K_f + U_f = K_i + U_i \Rightarrow K_f + qV_f = K_i + qV_i$$
$$\Rightarrow V_f - V_i = \frac{1}{q}(K_i - K_f) = \frac{1}{(-e)}\left(\tfrac{1}{2}mv_i^2 - 0\text{ J}\right)$$
$$\Rightarrow \Delta V = -\frac{mv_i^2}{2e} = -\frac{(9.11\times10^{-31}\text{ kg})(500{,}000\text{ m/s})^2}{2(1.60\times10^{-19}\text{ C})} = -0.712\text{ V}$$

**Assess:** The negative sign with $\Delta V$ verifies that the electron moves from a higher potential region to a lower potential region.

**28.25. Model:** The net potential is the sum of the potentials due to each charge.
**Solve:** From the geometry in the figure,

$$\frac{1.5\text{ cm}}{r_1} = \frac{1.5\text{ cm}}{r_2} = \frac{1.5\text{ cm}}{r_3} = \cos 30° \Rightarrow r_1 = r_2 = r_3 = \frac{1.5\text{ cm}}{\cos 30°} = 1.732\text{ cm}$$

The potential at the dot is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$
$$= (9.0\times10^9\text{ N m}^2/\text{C}^2)\left[\frac{1.0\times10^{-9}\text{ C}}{0.01732\text{ m}} - \frac{2.0\times10^{-9}\text{ C}}{0.01732\text{ m}} - \frac{2.0\times10^{-9}\text{ C}}{0.01732\text{ m}}\right] = -1600\text{ V}$$

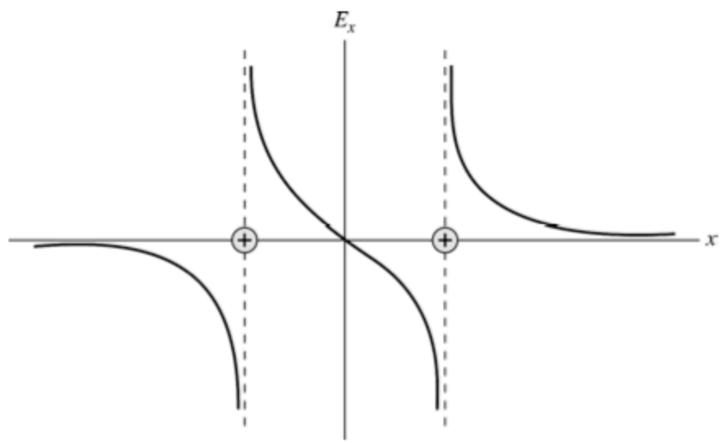
**Assess:** Potential is a scalar quantity, so we found the net potential by adding three scalar quantities.

**28.29. Model:** While the potential is the sum of the scalar potentials due to each charge, the electric field is the vector sum of the electric fields due to each charge.

**Solve: (a)** As $V$ is always positive, both charges must be positive. The positive signs for the two equal charges at $x = a$ and at $x = b$ are also consistent with the behavior of the potential in the range $a < x < b$.

**(b)** By the symmetry of the drawing about the middle, we infer that the magnitudes of the charges are the same. Thus $|q_a/q_b| = 1$.

**(c)**




The graph of $E_x$, the $x$-component of the electric field, as a function of $x$ is shown in the figure.

**28.42. Model:** Energy is conserved. The proton's potential energy inside the capacitor can be found from the capacitor's potential difference.

**Solve: (a)** The electric potential at the midpoint of the capacitor is 250 V. This is because the potential inside a parallel-plate capacitor is $V = Es$ where $s$ is the distance from the negative electron. The proton has charge $q = e$ and its potential energy at a point where the capacitor's potential is $V$ is $U = eV$. The proton will gain potential energy

$$\Delta U = e\Delta V = e(250\text{ V}) = 1.60\times10^{-19}\text{ C}(250\text{ V}) = 4.00\times10^{-17}\text{ J}$$

if it moves all the way to the positive plate. This increase in potential energy comes at the expense of kinetic energy which is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.67\times10^{-27}\text{ kg})(200{,}000\text{ m/s})^2 = 3.34\times10^{-17}\text{ J}$$

This available kinetic energy is not enough to provide for the increase in potential energy if the proton is to reach the positive plate. Thus the proton does not reach the plate because $K < \Delta U$.

**(b)** The energy-conservation equation $K_f + U_f = K_i + U_i$ is

$$\tfrac{1}{2}mv_f^2 + qV_f = \tfrac{1}{2}mv_i^2 + qV_i \Rightarrow \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 + q(V_i - V_f)$$

$$\Rightarrow v_f = \sqrt{v_i^2 + \frac{2q}{m}(V_i - V_f)} = \sqrt{(2.0\times10^5\text{ m/s})^2 + \frac{2(1.60\times10^{-19}\text{ C})(250\text{ V} - 0\text{ V})}{1.67\times10^{-27}\text{ kg}}} = 2.96\times10^5\text{ m/s}$$