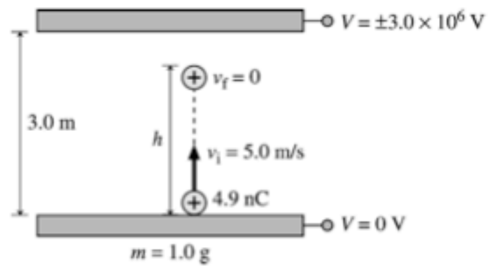
ECE1813 Fall 2015   Assignment #2   Solutions

Exercises and Problems from chapter 28

28.45. Model: Mechanical energy is conserved.
Visualize:



Solve: (a) Both gravitational and electric potential energy are involved. The electric potential between the plates is $V = Es$, where s is measured from the more negative plate. The electric field between the plates is

$$E = \frac{\Delta V}{d} = \frac{3.0 \times 10^6 \text{ V}}{3.0 \text{ m}} = 1.00 \times 10^6 \text{ V/m}$$

Conservation of energy is used to find the height.

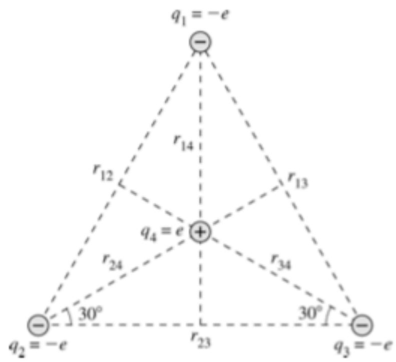
$$K_i + U_{gi} + U_{ei} = K_f + U_{gf} + U_{ef}$$

$$\frac{1}{2} m v_i^2 + 0 \text{ J} + q(0 \text{ V}) = 0 \text{ J} + mgh + q(Eh)$$

$$\Rightarrow h = \frac{\frac{1}{2} m v_i^2}{mg + qE} = \frac{\frac{1}{2}(1.0 \times 10^{-3} \text{ kg})(5.0 \text{ m/s})^2}{(1.0 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2) + (4.0 \times 10^{-9} \text{ C})(1.00 \times 10^6 \text{ V/m})} = 0.85 \text{ m}$$

28.49. Model: The electrons and the proton are point charges.

Visualize:



Solve: We are given that $r_{12} = r_{23} = r_{13} = 1.0\times10^{-9}$ m. From the geometry of the figure,

$$\frac{\frac{1}{2}r_{23}}{r_{24}} = \cos 30° \Rightarrow r_{24} = \frac{r_{23}}{2\cos 30°} = 0.5774\times10^{-9}\ \text{m} = r_{14} = r_{34}$$

The contributions to the total potential energy are

$$U_{12} = U_{13} = U_{23} = \frac{(9.0\times10^9\ \text{N m}^2/\text{C}^2)(-1.60\times10^{-19}\ \text{C})(-1.60\times10^{-19}\ \text{C})}{1.0\times10^{-9}\ \text{m}} = 2.304\times10^{-19}\ \text{J}$$

$$U_{14} = U_{24} = U_{34} = \frac{(9.0\times10^9\ \text{N m}^2/\text{C}^2)(-1.60\times10^{-19}\ \text{C})(1.60\times10^{-19}\ \text{C})}{0.5774\times10^{-9}\ \text{m}} = -3.990\times10^{-19}\ \text{J}$$

Summing all of the contributions,

$$U_{\text{elec}} = U_{12} + U_{13} + U_{23} + U_{14} + U_{24} + U_{34}$$
$$= 3(2.304\times10^{-19}\ \text{J}) + 3(-3.990\times10^{-19}\ \text{J}) = -5.1\times10^{-19}\ \text{J}$$

Assess: Note that $U_{12} = U_{21}$, $U_{13} = U_{31}$, and $U_{23} = U_{32}$, $U_{14} = U_{41}$, $U_{24} = U_{42}$, and $U_{34} = U_{43}$.

Conceptual Questions from chapter 29

29.6. (a) The electric field vector points in the direction of decreasing potential. Therefore $V_a > V_b$.

(b) $|\Delta V_{ab}| < |\Delta V_{cd}| < |\Delta V_{ef}|$. For a uniform electric field $|\Delta V| = |-E\Delta s| = |E\Delta s|$. The order is determined by the fact that $\Delta s_{ab} < \Delta s_{cd} < \Delta s_{ef}$.

(c) Surface 1 is an equipotential surface because it is perpendicular to the electric field vectors. Surface 2 is not perpendicular to the electric field so it is not an equipotential surface.

29.8. (a) $V_1 = V_2$. Both spheres and the wire become one conductor and so are all at the same potential.

(b) Since $V_1 = V_2$, $\frac{1}{4\pi\epsilon_0}\frac{Q_1}{r_1} = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r_2}$, thus $\frac{Q_1}{r_1} = \frac{Q_2}{r_2}$. Since $r_1 > r_2$, $Q_1 > Q_2$.

(c) Recall $E_1 = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{r_1^2} = \frac{V_1}{r_1}$ and $E_2 = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r_2^2} = \frac{V_2}{r_2}$. Since $r_1 > r_2$, $E_1 < E_2$.

Exercises and Problems from chapter 29

29.4. Model: The potential difference is the negative of the area under the E_x vs. x curve.
Visualize: Please refer to Figure EX29.4.
Solve: The potential difference between the origin and $x = 3.0$ m is

$$\Delta V = V(x = 3.0\text{ m}) - V(x = 0.0\text{ m}) = -\left[\tfrac{1}{2}(-100\text{ V})(1.0\text{ m} - 0\text{ m}) + \tfrac{1}{2}(200\text{ V})(3.0\text{ m} - 1.0\text{ m})\right]$$
$$= -150\text{ V}$$

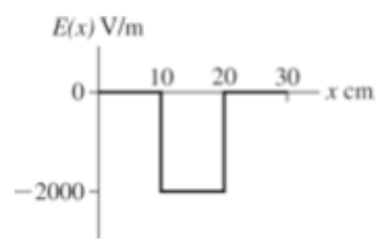
Thus $V(3.0\text{ m}) = V(0\text{ m}) - 150\text{ V} = -50\text{ V} - 150\text{ V} = -200\text{ V} = -0.20\text{ kV}$
Assess: The potential decreases from the origin to $x = 3.0$ m.

29.11. Model: The electric field is the negative of the slope of the graph of the potential function.
Visualize: Please refer to Figure EX29.11.
Solve: There are three regions of different slope. For $0\text{ cm} \le x < 10\text{ cm}$ and $20\text{ cm} \le x < 30\text{ cm}$,

$$\frac{\Delta V}{\Delta x} = 0\text{ V/m} \quad \Rightarrow \quad E_x = 0\text{ V/m}$$

For $10\text{ cm} \le x < 20\text{ cm}$,

$$\frac{\Delta V}{\Delta x} = \frac{100\text{ V} - (-100\text{ V})}{0.20\text{ m} - 0.10\text{ m}} = 2000\text{ V/m} \quad \Rightarrow \quad E_x = -2000\text{ V/m}$$



Assess: Because $E_s = -dV/ds$, the electric field is zero where the potential is not changing.

29.20. Solve: According to Equation 29.23,

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \left(\frac{1}{6\ \mu\text{F}} + \frac{1}{10\ \mu\text{F}} + \frac{1}{16\ \mu\text{F}}\right)^{-1} = 3.0 \times 10^{-6}\text{ F} = 3.0\ \mu\text{F}$$

29.21. Solve: According to Equation 29.21,

$$C_{eq} = C_1 + C_2 + C_3 = 6\ \mu\text{F} + 10\ \mu\text{F} + 16\ \mu\text{F} = 32\ \mu\text{F}$$

29.38. Model: The electric field is the negative of the slope of the graph of the potential function.
Visualize: Please see Figure P29.38.
Solve: **(a)** At $x = -2$ cm, the slope of the potential curve is

$$dV/dx = \frac{-10\text{ V}}{2\text{ m}} = -5\text{ V/m}$$

so the electric field is $E = 5$ V/m.
(b) At $x = -0$ cm, the slope of the potential curve is

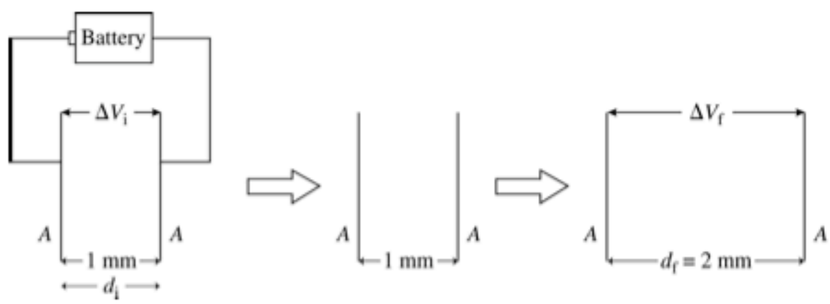
$$dV/dx = \frac{20\text{ V}}{2\text{ m}} = 10\text{ V/m}$$

so the electric field is $E = -10$ V/m.
(c) At $x = 2$ cm, the slope of the potential curve is

$$dV/dx = \frac{-10\text{ V}}{2\text{ m}} = -5\text{ V/m}$$

so the electric field is $E = 5$ V/m.

29.48. Model: The battery is assumed to be ideal.
Visualize:



The pictorial representation shows the capacitor plates connected to a battery, the battery removed from the plates, and the plates moved apart with insulating handles.
Solve: **(a)** The battery charges the plates through the wires. Once the wires are disconnected, the charge is trapped on the plates and *will not change* as their spacing is increased. The initial capacitance of the plates is

$$C_i = \frac{\epsilon_0 A}{d_i} = \frac{(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)(0.020\text{ m}\times0.020\text{ m})}{0.0010\text{ m}} = 3.54\times10^{-12}\text{ F}$$

Consequently, the initial voltage $\Delta V_i = 9.0$ V charges the plates to

$$Q = \pm C\Delta V = \pm(3.54\times10^{-12}\text{ F})(9.0\text{ V}) = \pm3.2\times10^{-11}\text{ C}$$

(b) The charge is still $Q = \pm3.2\times10^{-11}$ C after the spacing is increased, but the final capacitance is

$$C_f = \frac{\epsilon_0 A}{d_f} = \frac{\epsilon_0 A}{2d_i} = \frac{C_i}{2}$$

As a result, the final potential difference is

$$\Delta V_f = \frac{Q}{C_f} = \frac{Q}{C_i/2} = 2\frac{Q}{C_i} = 2\Delta V_i = 2\times9.0\text{ V} = 18\text{ V}$$