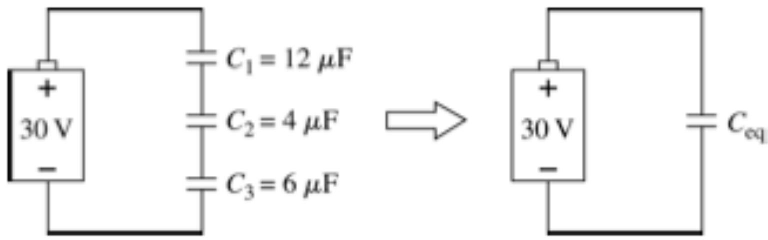
ECE1813 Fall 2015 Assignment #5 Solutions

Exercises and Problems from chapter 29

29.53. Model: Assume that the battery is ideal.
Visualize:



The pictorial representation shows the equivalent capacitance of the three capacitors.
Solve: Because C_1, C_2, and C_3 are in series,

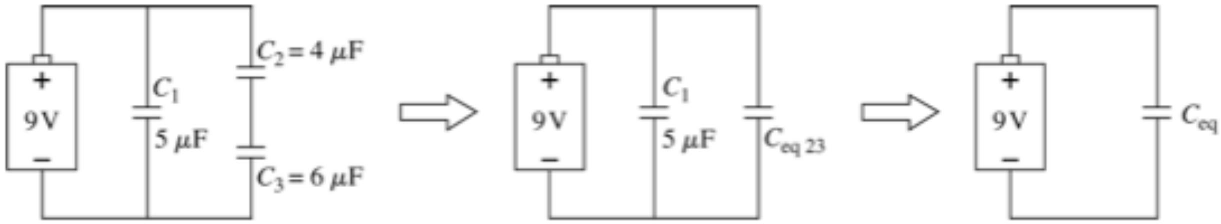
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{12\ \mu F} + \frac{1}{4\ \mu F} + \frac{1}{6\ \mu F} = \frac{1}{2}(\mu F)^{-1} \quad \Rightarrow \quad C_{eq} = 2\ \mu F$$

A potential difference of $\Delta V_C = \varepsilon = 30$ V across a capacitor of equivalent capacitance 2 μF produces a charge $Q = C_{eq}\Delta V_C = (2\mu F)(30\ V) = 60\mu C$. Because C_{eq} is a combination of three series capacitors, $Q_1 = Q_2 = Q_3 = 60\mu C$. We are now able to find the potential difference across each capacitor:

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{60\ \mu C}{12\ \mu F} = 5.0\ V \qquad \Delta V_2 = \frac{Q_2}{C_2} = \frac{60\ \mu C}{4\ \mu F} = 15\ V \qquad \Delta V_3 = \frac{Q_3}{C_3} = \frac{60\ \mu C}{6\ \mu F} = 10\ V.$$

Assess: $\Delta V_1 + \Delta V_2 + \Delta V_3 = 30\ V = \Delta V_{bat}$, as it should.

29.55. Model: Assume the battery is an ideal battery.
Visualize:



The pictorial representation shows how to find the equivalent capacitance of the three capacitors shown in the figure.
Solve: Because C_2 and C_3 are in series,

$$\frac{1}{C_{\text{eq }23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{4\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}} = \frac{10}{24}(\mu\text{F})^{-1} \quad \Rightarrow \quad C_{\text{eq }23} = \frac{12}{5}\mu\text{F} = 2.4\ \mu\text{F}$$

$C_{\text{eq }23}$ and C_1 are in parallel, so

$$C_{\text{eq}} = C_{\text{eq }23} + C_1 = 2.4\ \mu\text{F} + 5\ \mu\text{F} = 7.4\ \mu\text{F}$$

A potential difference of $\Delta V_{\text{C}} = 9$ V across a capacitor of equivalent capacitance 7.4 μF produces a charge

$$Q = C_{\text{eq}}\Delta V_{\text{C}} = \left(\frac{37}{5}\ \mu\text{F}\right)(9\text{ V}) = \frac{333}{5}\ \mu\text{C}$$

Because C_{eq} is a parallel combination of C_1 and $C_{\text{eq }23}$, these capacitors have $\Delta V_1 = \Delta V_{\text{eq }23} = \Delta V_{\text{C}} = 9$ V. Thus the charges on these two capacitors are

$$Q_1 = (5\ \mu\text{F})(9\text{ V}) = 45\ \mu\text{C} \qquad Q_{\text{eq }23} = (2.4\ \mu\text{F})(9\text{ V}) = 21.6\ \mu\text{C}$$

Because $Q_{\text{eq }23}$ is due to a series combination of C_2 and C_3, $Q_2 = Q_3 = 21.6\ \mu$C. This means

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{21.6\ \mu\text{C}}{4\ \mu\text{F}} = 5.4\text{ V} \qquad \Delta V_3 = \frac{Q_3}{C_3} = \frac{21.6\ \mu\text{C}}{6\ \mu\text{F}} = 3.6\text{ V}$$

Thus, to two significant figures, $Q_1 = 45\ \mu$C, $V_1 = 9$ V; $Q_2 = 22\ \mu$C, $V_2 = 5.4$ V; and $Q_3 = 22\ \mu$C, $V_3 = 3.6$ V.

29.59. Model: Assume the battery is ideal.
Visualize: Please refer to Figure P29.59. While the switch is in position A, the capacitors C_2 and C_3 are uncharged. When the switch is placed in position B, the charged capacitor C_1 is connected to C_2 and C_3. C_2 and C_3 are connected in series to form an equivalent capacitor $C_{\text{eq }23}$.
Solve: While the switch is in position A, a potential difference of $V_1 = 100$ V across C_1 charges it to

$$Q_1 = C_1 V_1 = (15\ \mu\text{F})(100\text{ V}) = 1500\ \mu\text{C}$$

When the switch is moved to position B, this initial charge Q_1 is redistributed. The charge Q_1' goes on C_1 and the charge $Q_{\text{eq }23}$ goes on $C_{\text{eq }23}$. The voltage across C_1 and $C_{\text{eq }23}$ is the same and $Q_1' + Q_{\text{eq }23} = Q_1 = 1500\ \mu$C. Combining these two conditions, we get

$$\frac{Q_1'}{C_1} = \frac{Q_{\text{eq }23}}{C_{\text{eq }23}} \quad \Rightarrow \quad \frac{1500\ \mu\text{C} - Q_{\text{eq }23}}{C_1} = \frac{Q_{\text{eq }23}}{C_{\text{eq }23}}$$

Since $C_{\text{eq }23} = \left(\frac{1}{20\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}}\right)^{-1} = 12\ \mu\text{F}$, we can rewrite this equation as

$$\frac{1500\ \mu\text{C} - Q_{\text{eq }23}}{15\ \mu\text{F}} = \frac{Q_{\text{eq }23}}{12\ \mu\text{F}} \quad \Rightarrow \quad Q_{\text{eq }23} = 0.67\text{ mC} \quad \Rightarrow \quad Q_1' = Q_1 - Q_{\text{eq }23} = 1.500\text{ mC} - 0.67\text{ mC} = 0.83\text{ mC}$$

Having found the charge $Q_{\text{eq }23}$, it is easy to see that $Q_2 = Q_3 = 0.67$ mC because $C_{\text{eq }23}$ is a series combination of C_2 and C_3. Thus,

$$\Delta V_1 = \frac{Q_1'}{C_1} = \frac{830\ \mu\text{C}}{15\ \mu\text{F}} = 55\text{ V} \qquad \Delta V_2 = \frac{Q_2}{C_2} = \frac{670\ \mu\text{C}}{20\ \mu\text{F}} = 34\text{ V} \qquad \Delta V_3 = \frac{Q_3}{C_3} = \frac{670\ \mu\text{C}}{30\ \mu\text{F}} = 22\text{ V}$$

Conceptual Questions from chapter 30

30.9. **(a)** $I_1 = I_2$. Due to conservation of current, the current everywhere in the wire is the same. The number of charges passing per unit time must be the same in wires 1 and 2.

(b) By definition, $J_1 = \dfrac{I}{A_1}$ and $J_2 = \dfrac{I}{A_2}$. Since $A_1 < A_2$, $J_1 > J_2$.

(c) Since $\sigma_1 = \sigma_2 = \sigma$, $E_1 = \dfrac{J_1}{\sigma}$ and $E_2 = \dfrac{J_2}{\sigma}$. So $E_1 > E_2$.

(d) We have $J = nev_d$. Since $J_1 > J_2$, $(v_d)_1 > (v_d)_2$.

30.11. $R_d > R_a = R_e > R_c > R_b$. Calculate $R = \dfrac{\rho L}{A}$:

$$R_a = \frac{\rho L}{\pi r^2} \qquad R_c = \frac{\rho(2L)}{(\pi)(2r)^2} = \frac{1}{2}R_a$$

$$R_b = \frac{\rho L}{(\pi)(2r)^2} = \frac{1}{4}R_a \qquad R_d = \frac{\rho(2L)}{\pi r^2} = 2R_a$$

$$R_e = \frac{\rho(4L)}{(\pi)(2r)^2} = R_a$$

Exercises and Problems from chapter 30

30.11. **Solve:** From Equation 30.13, the current in the wire is

$$I = JA = (7.50\times10^5\ \text{A/m}^2)(2.5\times10^{-6}\ \text{m}\times75\times10^{-6}\ \text{m}) = 0.141\ \text{mA}$$

The charge that flows in 15 min is the current times the time.

$$Q = I\Delta t = (0.141\ \text{A})(900\ \text{s}) = 127\ \text{C} \approx 130\ \text{C}$$

30.21. Solve: From Equations 30.17 and 30.18, the resistivity is

$$\rho = \frac{E}{J} = \frac{E}{I/A} = \frac{EA}{I} = \frac{E\pi r^2}{I} = \frac{(0.085 \text{ V/m})\pi(1.5\times10^{-3} \text{ m})^2}{12 \text{ A}} = 5.0\times10^{-8}\ \Omega \text{ m}$$

30.27. Solve: (a) Resistivity depends only on the type of material, not the geometry of the wire. Wires 1 and 2 are made of the same material, so $\rho_2 = \rho_1$ and thus $\rho_2/\rho_1 = 1.00$.

(b) The resistance of a wire of length L and radius r is given by Equation 30.22:

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$$

Because the two wires have the same resistivity,

$$\frac{R_2}{R_1} = \frac{\rho L_2/\pi r_2^2}{\rho L_1/\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2 \frac{L_2}{L_1} = \left(\frac{1}{2}\right)^2 \frac{2}{1} = \frac{1}{2} = 0.50$$

30.41. Solve: (a) The current associated with the moving film is the rate at which the charge on the film moves past a certain point. The tangential speed of the film is

$$v = \omega r = (90 \text{ rpm})(4.0 \text{ cm}) = 90\frac{\text{rev}}{\text{min}}\times\frac{1 \text{ min}}{60 \text{ s}}\times\frac{2\pi \text{ rad}}{1 \text{ rev}}\times1.0 \text{ cm} = 9.425 \text{ cm/s}$$

In 1.0 s the film moves a distance of 9.425 cm. This means the area of the film that moves to the right in 1.0 s is $(9.425 \text{ cm})(4.0 \text{ cm}) = 37.7 \text{ cm}^2$. The amount of charge that passes to the right in 1.0 s is

$$Q = (37.7 \text{ cm}^2)(-2.0\times10^{-9} \text{ C/cm}^2) = -75.4\times10^{-9} \text{ C}$$

Since $I = Q/\Delta t$, we have

$$I = \frac{\left|-(75.4\times10^{-9} \text{ C})\right|}{1 \text{ s}} = 75.4 \text{ nA}$$

The current is 75 nA.

(b) Having found the current in part (a), we can once again use $I = Q/\Delta t$ to obtain Δt:

$$\Delta t = \frac{Q}{I} = \frac{\left|-10\times10^{-6} \text{ C}\right|}{75.4\times10^{-9} \text{ A}} = 133 \text{ s} \approx 130 \text{ s}$$

30.46. Solve: The total charge in the battery is

$$Q = I\Delta t = (90 \text{ A})(3600 \text{ s}) = 3.2\times10^5 \text{ C}$$