ECE1813 Fall 2015 Assignment #6 Solutions

Exercises and Problems from chapter 30

30.59. Solve: From Equation 30.17 and Table 30.2, the electric field is

$$E = \frac{J}{\sigma} = \frac{I}{\sigma A} = \frac{5.0\text{ A}}{(1.0\times10^{7}\ \Omega^{-1}\text{m}^{-1})\pi(1.0\times10^{-3}\text{ m})^{2}} = 0.16\text{ V/m}$$

30.61. Model: Assume the resistors are ohmic.
Solve: The potential difference across the resistor on the left is ε, so the current in it is $I = \varepsilon/R$. The potential difference across the resistor on the right is 2ε. For the current to be the same in that resistor, the resistance must be twice as much, so that $I = 2\varepsilon/(2R)$. Therefore, the resistance of the resistor on the right is $2R$.
Assess: The two resistors are neither in parallel nor in series.

30.65. Model: The wire is uniform. The electric field in the wire is the negative of the slope of the V vs. s curve.
Solve: The electric field in the wire is

$$E = -\frac{\Delta V}{\Delta s} = -\frac{(3.0\text{ V} - 0.0\text{ V})}{(0.30\text{ m} - 0.0\text{ m})} = -10\text{ V/m}$$

The negative sign indicates the electric field direction is along the negative s direction, and is not needed to find current density. From Table 30.2, the conductivity of tungsten is $\sigma = 1.8\times10^{7}\ \Omega^{-1}\text{m}^{-1}$. Thus the current density in the wire is

$$J = \sigma E = (1.8\times10^{7}\ \Omega^{-1}\text{m}^{-1})(10\text{ V/m}) = 1.8\times10^{8}\text{ A/m}^{2}$$

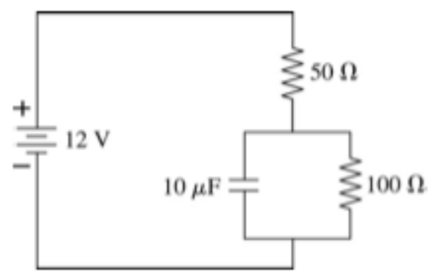
31.4. R_1 dissipates more power, since $P = I^2R$ and the same current flows through each resistor.

31.5. R_2 dissipates more power, since $P = (\Delta V)^2/R$ and both resistors have the same potential drop ΔV because they are connected in parallel.

Conceptual Questions from chapter 31

Exercises and Problems from chapter 31

31.2. Solve: In Figure EX31.2, the positive terminal of the battery is connected to the 50 Ω resistor, whose other end is connected to the 100 Ω resistor and the capacitor, which are in parallel. Thus, we have a resistor connected in series with a parallel combination of a resistor and a capacitor.

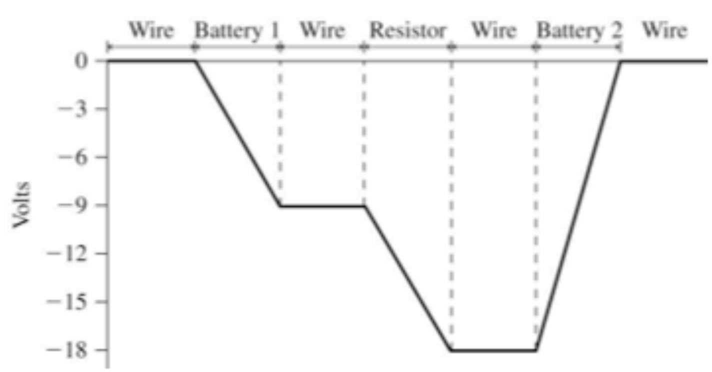


31.5. Model: The batteries and the connecting wires are ideal.
Visualize: Please refer to Figure EX31.5.
Solve: (a) Choose the current I to be in the clockwise direction. If I ends up being a positive number, then the current really does flow in this direction. If I is negative, the current really flows counterclockwise. There are no junctions, so I is the same for all elements in the circuit. With the 9 V battery labeled 1 and the 6 V battery labeled 2, Kirchhoff's loop law gives

$$\sum \Delta V_i = \Delta V_{\text{bat 1}} + \Delta V_{\text{R}} + \Delta V_{\text{bat 2}} = +\mathcal{E}_1 - IR - \mathcal{E}_2 = 0$$

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R} = \frac{9 \text{ V} - 18 \text{ V}}{10 \ \Omega} = -0.9 \text{ A}$$

Note the signs: Potential is gained in battery 1, but potential is lost both in the resistor and in battery 2. Because I is negative, we can say that $I = 0.9$ A and flows from right to left through the resistor.
(b) In the graph below, we start at the lower-left corner of the circuit and travel clockwise around the circuit (i.e., against the current). We start by losing 9 V going through battery 1, then loss $\Delta V_{\text{R}} = -IR = 9$ V going through the resistor. We then gain 18 V going through battery 2. The final potential is the same as the initial potential, as required.



31.12. Solve: The cost of running the waterbed 25% of the time for a year is

$$(0.25)(450 \text{ W})(365 \text{ days})\left(\frac{24 \text{ hr}}{\text{day}}\right)\left(\frac{\text{kW}}{1000 \text{ W}}\right)\left(\frac{\$0.12}{\text{kW hr}}\right) = \$118$$

To two significant figures, the cost is $120.

31.17. Model: Assume ideal connecting wires but not an ideal battery.
Visualize: The circuit for an ideal battery is the same as the circuit in Figure EX31.17, except that the $1\ \Omega$ resistor is not present.
Solve: In the case of an ideal battery, we have a battery with $\mathcal{E} = 15\text{ V}$ connected to two series resistors of $10\ \Omega$ and $20\ \Omega$ resistance. Because the equivalent resistance is $R_{eq} = 10\ \Omega + 20\ \Omega = 30\ \Omega$ and the potential difference across R_{eq} is 15 V, the current in the circuit is

$$I = \frac{\Delta V}{R_{eq}} = \frac{\mathcal{E}}{R_{eq}} = \frac{15\text{ V}}{30\ \Omega} = 0.50\text{ A}$$

The potential difference across the $20\ \Omega$ resistor is

$$\Delta V_{20} = IR = (0.50\text{ A})(20\ \Omega) = 10\text{ V}$$

In the case of a real battery, we have a battery with $\mathcal{E} = 15\text{ V}$ connected to three series resistors: $10\ \Omega$, $20\ \Omega$, and an internal resistance of $1\ \Omega$. Now the equivalent resistance is

$$R'_{eq} = 10\ \Omega + 20\ \Omega + 1\ \Omega = 31\ \Omega$$

The potential difference across R_{eq} is the same as before $\mathcal{E} = 15\text{ V}$. Thus,

$$I' = \frac{\Delta V'}{R'_{eq}} = \frac{\mathcal{E}}{R'_{eq}} = \frac{15\text{ V}}{31\ \Omega} = 0.4839\text{ A}$$

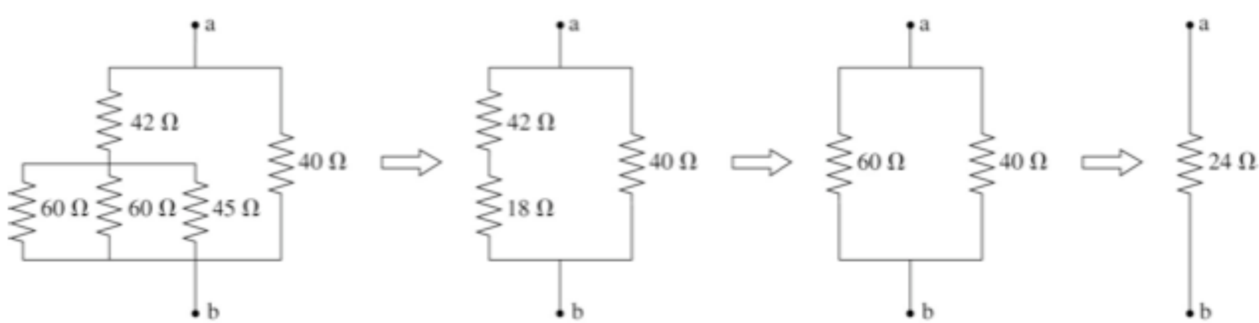
Therefore, the potential difference across the $20\ \Omega$ resistor is

$$\Delta V'_{20} = I'R = (0.4839\text{ A})(20\ \Omega) = 9.68\text{ V}$$

That is, the potential difference across the $20\ \Omega$ resistor is reduced from 10 V to 9.68 V due to the internal resistance of $1\ \Omega$ of the battery. The percentage change in the potential difference is

$$\left(\frac{10.0\text{ V} - 9.68\text{ V}}{10.0\text{ V}}\right) \times 100\% = 3.2\%$$

31.22. Model: The connecting wires are ideal with zero resistance.
Solve:



For the first step, the two 60 W resistors and the $45\ \Omega$ resistor are in parallel. Their equivalent resistance is

$$\frac{1}{R_{eq\,1}} = \frac{2}{60\ \Omega} + \frac{1}{45\ \Omega} = \frac{5}{90}\ \Omega \quad \Rightarrow \quad R_{eq\,1} = 18\ \Omega$$

For the second step, resistors $42\ \Omega$ and $R_{eq\,1} = 18\ \Omega$ are in series. Therefore,

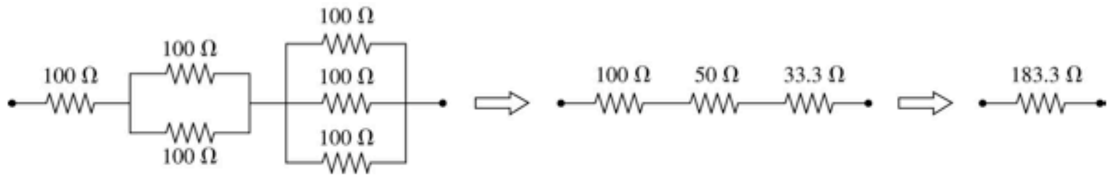
$$R_{eq\,2} = R_{eq\,1} + 42\ \Omega = 18\ \Omega + 42\ \Omega = 60\ \Omega$$

For the third step, the resistors $40\ \Omega$ and $R_{eq\,2} = 60\ \Omega$ are in parallel. So,

$$\frac{1}{R_{eq\,3}} = \frac{1}{60\ \Omega} + \frac{1}{40\ \Omega} \quad \Rightarrow \quad R_{eq\,3} = 24\ \Omega$$

The equivalent resistance of the circuit is $24\ \Omega$.

31.23. Model: The connecting wires are ideal with zero resistance.
Solve:



For the first step, the two resistors in the middle of the circuit are in parallel, so their equivalent resistance is

$$\frac{1}{R_{\text{eq 1}}} = \frac{1}{100\ \Omega} + \frac{1}{100\ \Omega} \quad \Rightarrow \quad R_{\text{eq 1}} = 50\ \Omega$$

The three $100\ \Omega$ resistors at the end are in parallel. Their equivalent resistance is

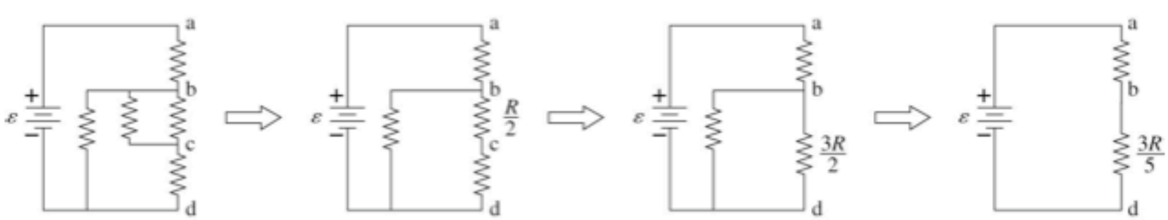
$$\frac{1}{R_{\text{eq 2}}} = \frac{1}{100\ \Omega} + \frac{1}{100\ \Omega} + \frac{1}{100\ \Omega} \quad \Rightarrow \quad R_{\text{eq 2}} = 33.3\ \Omega$$

For the second step, the three resistors are in series, so their equivalent resistance is

$$100\ \Omega + 50\ \Omega + 33.3\ \Omega = 183\ \Omega$$

The equivalent resistance of the circuit is $183\ \Omega$.

31.35. Model: Assume ideal wires.
Visualize: The following equivalent circuits will be useful, where we have labeled three points in the circuits a, b, and c. The unlabeled resistors all have resistance R.



Solve: Because the bulbs are identical, they all have the same resistance, which we shall call R. The bulb brightness is proportional to the power $P = V^2/R$, so we shall calculate the voltage difference across each bulb. First, we find the potential at points a, b, and c with respect to point d (i.e., the negative terminal of the battery), which we shall assign as the zero of the potential. The potential at point a is ε, so $V_a = \varepsilon$. Considering the last equivalent circuit above, we see that the current flowing around the circuit is

$$I = \frac{V_a}{R + 3R/5} = \frac{5\varepsilon}{8R}$$

Therefore, the potential at point b is

$$V_b = V_a - IR = \varepsilon - \frac{5\varepsilon}{8} = \frac{3\varepsilon}{8}$$

Considering the second-to-last circuit, we see that the current flowing from point b to point d must be

$$I_R = \frac{V_b}{3R/2} = \frac{2}{3R}\frac{3\varepsilon}{8} = \frac{\varepsilon}{4R}$$

so, looking at the second circuit, we can find the potential at point c:

$$V_c = V_b - I_R\left(\frac{R}{2}\right) = \frac{3\varepsilon}{8} - \left(\frac{\varepsilon}{4R}\right)\left(\frac{R}{2}\right) = \frac{\varepsilon}{4}$$

In the following table, we put the bulbs and the potential difference across them:

Bulb	Potential Difference	Power
P	$\Delta V = V_{\mathrm{b}} - V_{\mathrm{a}} = -\frac{5\varepsilon}{8}$	$\frac{25}{64}\frac{\varepsilon^2}{R}$
Q	$\Delta V = V_{\mathrm{d}} - V_{\mathrm{b}} = -\frac{3\varepsilon}{8}$	$\frac{9}{64}\frac{\varepsilon^2}{R}$
R	$\Delta V = V_{\mathrm{c}} - V_{\mathrm{b}} = -\frac{\varepsilon}{8}$	$\frac{1}{64}\frac{\varepsilon^2}{R}$
S	$\Delta V = V_{\mathrm{c}} - V_{\mathrm{b}} = -\frac{\varepsilon}{8}$	$\frac{1}{64}\frac{\varepsilon^2}{R}$
T	$\Delta V = V_{\mathrm{d}} - V_{\mathrm{c}} = -\frac{\varepsilon}{4}$	$\frac{4}{64}\frac{\varepsilon^2}{R}$

Thus, ordering the bulbs from brightest to dimmest gives

$$\mathrm{P} > \mathrm{Q} > \mathrm{T} > \mathrm{R} = \mathrm{S}$$

so the response is D.