ECE1813 Fall 2015 Assignment #7 Solutions

Exercises and Problems from chapter 31

31.45. Model: Assume that the connecting wires are ideal, but the battery is not. The battery has internal resistance. Also assume that the ammeter does not have any resistance.
Visualize: Please refer to Figure P31.45.

Solve: When the switch is open,

$$\varepsilon - Ir - I(5.0\ \Omega) = 0 \quad \Rightarrow \quad V = (1.636\ \text{A})(r + 5.0\ \Omega)$$

where we applied Kirchhoff's loop law starting from the lower-left corner. When the switch is closed, the current I comes out of the battery and splits at the junction. The current $I' = 1.565$ A flows through the $5.0\ \Omega$ resistor and the rest $(I - I')$ flows through the $10.0\ \Omega$ resistor. Because the potential differences across the two resistors are equal,

$$I'(5.0\ \Omega) = (I - I')(10.0\ \Omega) \quad \Rightarrow \quad (1.565\ \text{A})(5.0\ \Omega) = (I - 1.565\ \text{A})(10.0\ \Omega) \quad \Rightarrow \quad I = 2.348\ \text{A}$$

Applying Kirchhoff's loop law to the left loop of the closed circuit gives

$$\varepsilon - Ir - I'(5.0\ \Omega) = 0\ \text{V} \quad \Rightarrow \quad \varepsilon = (2.348\ \text{A})r + (1.565\ \text{A})(5.0\ \Omega) = (2.348\ \text{A})r + 7.825\ \text{V}$$

Combining this equation for ε with the equation obtained from the circuit when the switch was open gives

$$(2.348\ \text{A})r + 7.825\ \text{V} = (1.636\ \text{A})r + 8.18\ \text{V} \quad \Rightarrow \quad (0.712\ \text{A})r = 0.355\ \text{V} \quad \Rightarrow \quad r = 0.50\ \Omega$$

We also have $\varepsilon = (1.636\ \text{A})(0.50\ \Omega + 5.0\ \Omega) = 9.0\ \text{V}$.

31.49. Model: The batteries are ideal, the connecting wires are ideal, and the ammeter has a negligibly small resistance.
Visualize: Please refer to Figure P31.49.
Solve: Kirchhoff's junction law tells us that the current flowing through the $2.0\ \Omega$ resistance in the middle branch is $I_1 + I_2 = 3.0$ A. We can therefore determine I_1 by applying Kirchhoff's loop law to the left loop. Starting clockwise from the lower-left corner,

$$+9.0\ \text{V} - I_1(3.0\ \Omega) - (3.0\ \text{A})(2.0\ \Omega) = 0\ \text{V} \quad \Rightarrow \quad I_1 = 1.0\ \text{A}$$

$$I_2 = (3.0\ \text{A} - I_1) = (3.0\ \text{A} - 1.0\ \text{A}) = 2.0\ \text{A}$$

Finally, to determine the emf ε, we apply Kirchhoff's loop law to the right loop and start counterclockwise from the lower-right corner of the loop:

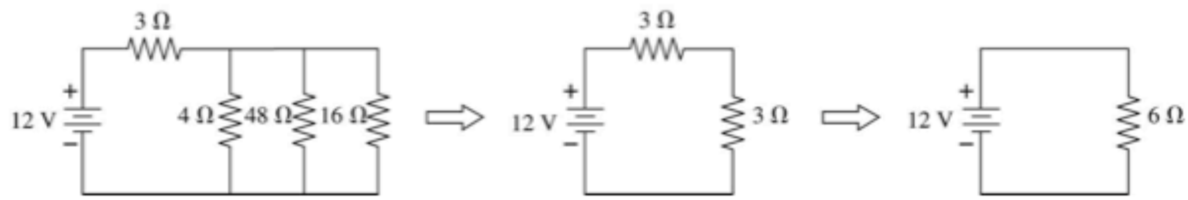
$$+\varepsilon - I_2(4.5\ \Omega) - (3.0\ \text{A})(2.0\text{W}) = 0\ \text{V} \quad \Rightarrow \quad \varepsilon - (2.0\ \text{A})(4.5\ \Omega) - 6.0\ \text{V} = 0\ \text{V} \quad \Rightarrow \quad \varepsilon = 15\ \text{V}$$

31.52. Solve: (a) The cost per month of the 1000 W refrigerator is

$$(1000\ \text{W})\left(\frac{1\ \text{kW}}{1000\ \text{W}}\right)\left(\frac{30\ \text{day}}{1\ \text{month}}\right)\left(\frac{24\ \text{h}}{1\ \text{day}}\right)(0.20)\left(\frac{\$0.10}{1\ \text{kW h}}\right)(1\ \text{month}) = \$14.40$$

(b) The cost per month of a refrigerator with a 800 W compressor is \$11.52. The difference in the running cost of the two refrigerators is \$2.88 per month. So, the number of months before you recover the additional cost of \$100 (of the energy-efficient refrigerator) is $\$100/\$2.88 = 34.7$ months.

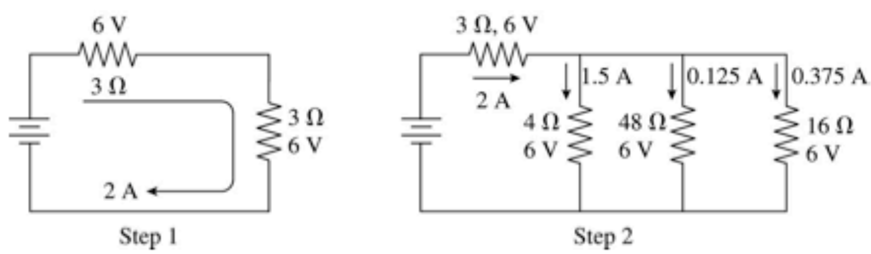
31.59. Model: The battery and the connecting wires are ideal.
Visualize:



The figure shows how to simplify the circuit in Figure P31.59 using the laws of series and parallel resistances. Having reduced the circuit to a single equivalent resistance, we will reverse the procedure and "build up" the circuit using the loop law and the junction law to find the current and potential difference across each resistor.
Solve: From the last circuit in the diagram,

$$I = \frac{\varepsilon}{6\ \Omega} = \frac{12\ \text{V}}{6\ \Omega} = 2\ \text{A}$$

Thus, the current through the battery is 2 A. As we rebuild the circuit, we note that series resistors *must* have the same current I and that parallel resistors *must* have the same potential difference ΔV.



In Step 1, the $6\ \Omega$ resistor is returned to a $3\ \Omega$ and $3\ \Omega$ resistor in series. Both resistors must have the same 2 A current as the $6\ \Omega$ resistance. We then use Ohm's law to find

$$\Delta V_3 = (2\ \text{A})(3\ \Omega) = 6\ \text{V}$$

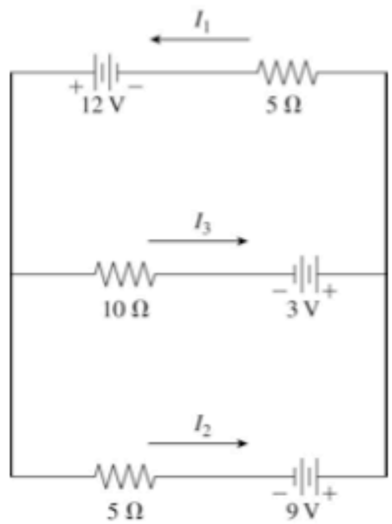
As a check, $6\ \text{V} + 6\ \text{V} = 12\ \text{V},$ which was ΔV of the $6\ \Omega$ resistor. In Step 2, one of the two $3\ \Omega$ resistances is returned to the $4\ \Omega$, $48\ \Omega$, and $16\ \Omega$ resistors in parallel. The three resistors must have the same $\Delta V = 6\ \text{V}$. From Ohm's law,

$$I_4 = \frac{6\ \text{V}}{4\ \Omega} = 1.5\ \text{A} \qquad I_{48} = \frac{6\ \text{V}}{48\ \Omega} = \frac{1}{8}\ \text{A} \qquad I_{16} = \frac{6\ \text{V}}{16\ \Omega} = \frac{3}{8}\ \text{A}$$

Resistor	Potential difference (V)	Current (A)
3 Ω	6	2
4 Ω	6	1.5
48 Ω	6	1/8
16 Ω	6	3/8

Assess: Note that the currents flowing through the three parallel resistors sum to the 2 A flowing through the battery, as required.

31.63. Model: The wires and batteries are ideal.
Visualize:



Solve: Assign currents I_1, I_2, and I_3 as shown in the figure above. If I_3 turns out to be negative, we'll know it really flows right to left. Apply Kirchhoff's loop rule counterclockwise to the top loop from the top-right corner:

$$-I_1(5\ \Omega)+12\ \text{V}-I_3(10\ \Omega)+3\ \text{V}=0$$

Apply the loop rule counterclockwise to the bottom loop starting at the lower-left corner:

$$-I_2(5\ \Omega)+9\ \text{V}-3\ \text{V}+I_3(10\ \Omega)=0$$

Note that, because we went against the current direction through the $10\ \Omega$ resistor, the potential increased across this resistor. Apply the junction rule to the right middle junction:

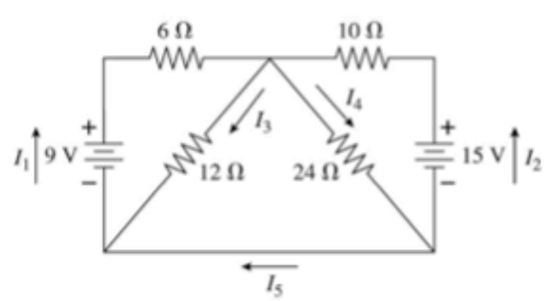
$$I_1=I_2+I_3.$$

These three equations can be solved for the current I_3:

$$(-I_1+I_2)(5\ \Omega)+9\ \text{V}-2I_3(10\ \Omega)=0 \;\Rightarrow\; -I_3(5\ \Omega)+9\ \text{V}-2I_3(10\ \Omega)=0 \;\Rightarrow\; I_3=\frac{9}{25}\ \text{A}$$

The result is $I_3=9/25\ \text{A}=0.12\ \text{A}$ flowing from left to right (as shown in the figure above).

31.67. Model: The wires and batteries are ideal.
Visualize:



Solve: The circuit is redrawn above for clarity and the currents are shown. We must find I_5.
Repeatedly apply Kirchhoff's rules to the loops. The loop rule applied clockwise about the three triangles yields

$$\text{Left: } 9\ \text{V}-I_1(6\ \Omega)-I_3(12\ \Omega)=0 \;\Rightarrow\; I_1=1.5\ \text{A}-2I_3$$

$$\text{Center: } -I_4(24\ \Omega)+I_3(12\ \Omega)=0 \;\Rightarrow\; I_4=I_3/2$$

$$\text{Right: } 15\ \text{V}-I_2(10\ \Omega)-I_4(24\ \Omega)=0 \;\Rightarrow\; I_2=1.5\ \text{A}-2.4I_4$$

The junction rule applied at the bottom corners gives equations into which the results above may be substituted:

$$I_1=I_3+I_5 \;\Rightarrow\; 1.5\ \text{A}-2I_3=I_3+I_5 \;\Rightarrow\; I_5=1.5\ \text{A}-3I_3$$

$$I_4=I_2+I_5 \;\Rightarrow\; I_4=1.5\ \text{A}-2.4I_4+I_5 \;\Rightarrow\; I_5=3.4I_4-1.5\ \text{A}$$

Using $I_4 = I_3/2$ and solving for I_3 gives

$$1.5 \text{ A} - 3I_3 = 3.4(I_3/2) - 1.5 \text{ A} \quad \Rightarrow \quad I_3 = \frac{30}{47} \text{ A}$$

$$I_5 = 1.5 \text{ A} - 3\left(\frac{30}{47} \text{ A}\right) = -\frac{201}{94} \text{ A} = -0.41 \text{ A}$$

Since the result is negative, 0.40 A flows from left to right through the bottom wire.

31.31. Model: The capacitor discharges through a resistor. Assume ideal wires.
Visualize: The switch in the circuit in Figure EX31.31 is in position a. When the switch is in position b the circuit consists of a capacitor and a resistor.
Solve: **(a)** The switch has been in position a for a long time, which means the capacitor is fully charged to a charge

$$Q_0 = C\Delta V = C\varepsilon = (4 \text{ }\mu\text{F})(9 \text{ V}) = 36 \text{ }\mu\text{C}$$

Immediately after the switch is moved to the b position, the charge on the capacitor is $Q_0 = 36 \text{ }\mu\text{C}$. The current through the resistor is

$$I_0 = \frac{\Delta V_\text{R}}{R} = \frac{9 \text{ V}}{25 \text{ }\Omega} = 0.36 \text{ A}$$

Note that, as soon as the switch is closed, the potential difference across the capacitor ΔV_C appears across the 25 Ω resistor.
(b) The charge Q_0 decays as $Q = Q_0 e^{-t/\tau}$, where

$$\tau = RC = (25 \text{ }\Omega)(4 \text{ }\mu\text{F}) = 100 \text{ }\mu\text{s}$$

Thus, at $t = 50 \text{ }\mu\text{s}$, the charge is

$$Q = (36 \text{ }\mu\text{C})e^{-50 \text{ }\mu\text{s}/100 \text{ }\mu\text{s}} = (36 \text{ }\mu\text{C})e^{-0.5} = 22 \text{ }\mu\text{C}$$

and the resistor current is

$$I = I_0 e^{-t/\tau} = (0.36 \text{ A})e^{-50 \text{ }\mu\text{s}/100 \text{ }\mu\text{s}} = 0.22 \text{ A}$$

(c) Likewise, at $t = 200 \text{ }\mu\text{s}$, the charge is $Q = 4.9 \text{ }\mu\text{C}$ and the current is $I = 49$ mA.

31.73. Model: The battery and the connecting wires are ideal.
Visualize: Please refer to Figure P31.73.
Solve: (a) A very long time after the switch has closed, the potential difference ΔV_C across the capacitor is ε. This is because the capacitor charges until $\Delta V_C = \varepsilon$ while the charging current approaches zero.

(b) The full charge of the capacitor is $Q_{max} = C(\Delta V_C)_{max} = C\varepsilon$.

(c) In this circuit, $I = +dQ/dt$ because the capacitor is charging; that is, because the charge on the capacitor is increasing.

(d) From Equation 31.36, capacitor charge at time t is $Q = Q_{max}(1 - e^{-t/\tau})$. Therefore,

$$I = \frac{dQ}{dt} = C\varepsilon \frac{d}{dt}(1 - e^{-t/\tau}) = C\varepsilon\left(\frac{1}{\tau}\right)e^{-t/\tau} = C\varepsilon\left(\frac{1}{RC}\right)e^{-t/\tau} = \frac{\varepsilon}{R}e^{-t/\tau}$$

A graph of I as a function of t is shown below.