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Two-Polarization Generalized Zero-Forcing Equalization for Channel Interference Suppression

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Abstract-**The purpose of the research is to obtain results toward effective methods for combating the deleterious effects of various impairments arising in digital data transmission over dually polarized communication channels, namely, metal telephone line, coaxial cable, radio channel and optical fiber.**

The improvements in spectral efficiency can cause more co-channel interference (CCI) and adjacent-channel interference (ACI). Moreover, transmission of M-state Quadrature Amplitude Modulated (QAM) signals via orthogonally (vertical and horizontal) polarized carriers is an effective method for reusing existing bandwidth with the advantages of more system capacity and higher bit rate. However, the obstacle in the way of realizing these advantages is the unavoidable presence of cross-polarization interference (CPI) between two polarized signals.

Channel equalization is a well-established theory when dealing with optimal communications system design. It is the technique employed in this research.

The utilization of wide transmitter and receiver bandwidth in order to suppress CCI, ACI and CPI over a dually polarized channel will be the first contribution of this research. The second contribution will be the study of the conditions to suppress all the interference by the generalized zero-forcing equalizer. These conditions are the transmitter bandwidth, receiver bandwidth, carrier spacing, and co-channel interference.

INTRODUCTION

The purpose of this research is to obtain results toward effective methods for combating the deleterious effects of various impairments arising in digital data transmission over dually polarized communication channels. The demand for services requires the use of the spectrum more efficiently. These improvements in spectral

efficiency can cause more co-channel interference (CCI), adjacent-channel interference (ACI) and cross-polarization interference (CPI). We will explore a design possibility and derive a mathematical solution using channel equalization and wide transmitter and receiver bandwidth. The conditions to suppress all the interference by the equalizer will also be studied.

SYSTEM MODEL

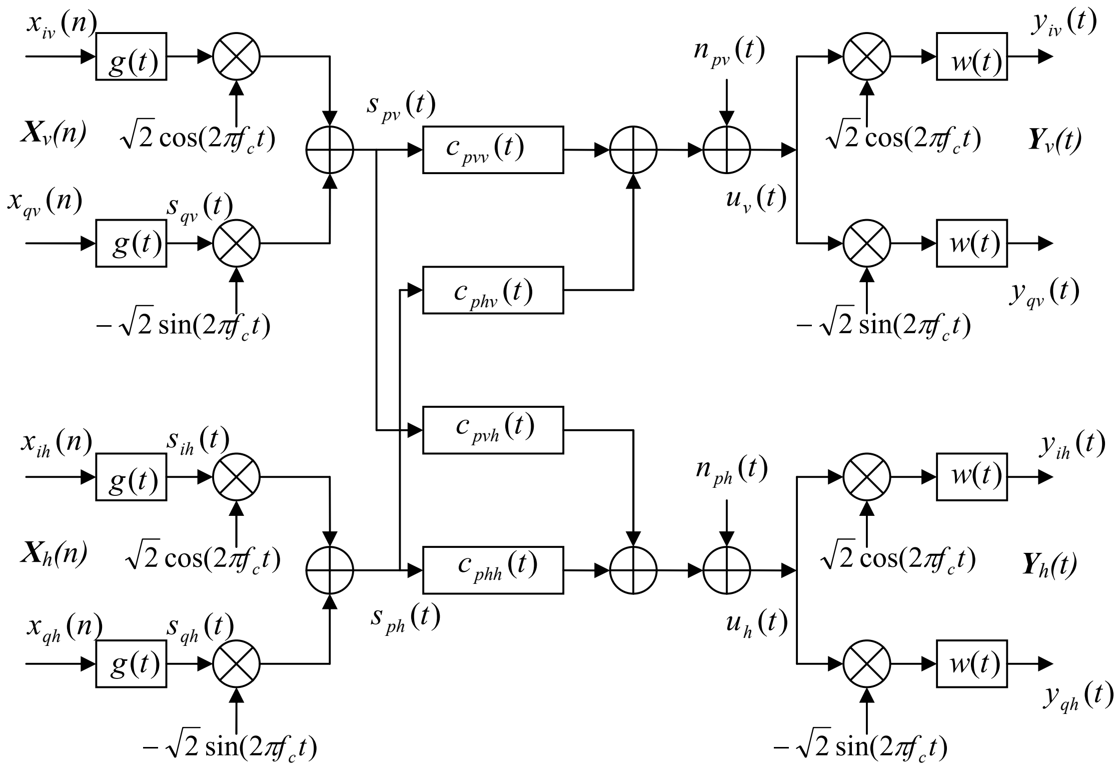
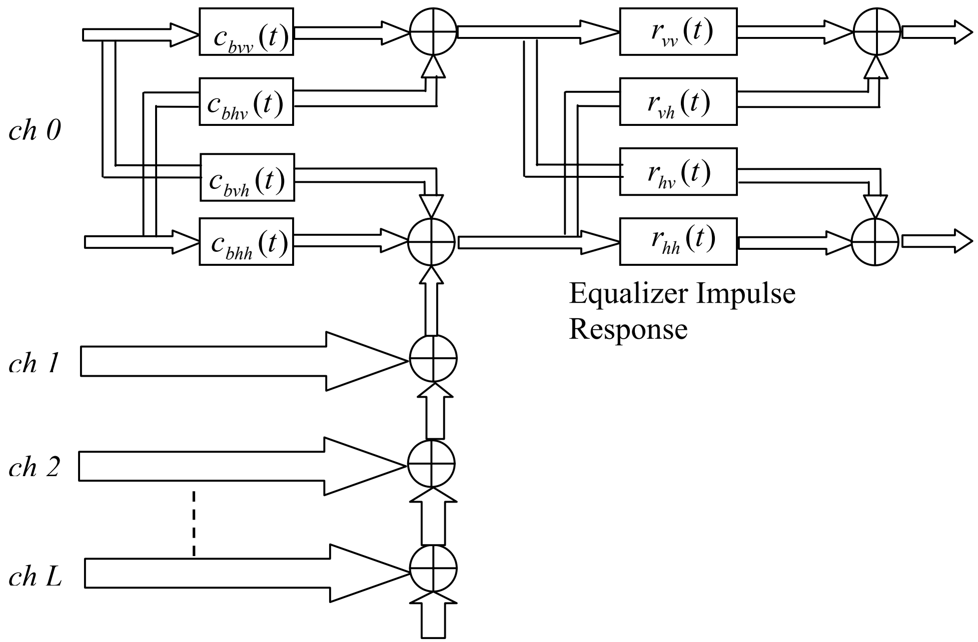


Figure 1-1 Passband System Model



Channel Impulse Response Matrix:

$$\boldsymbol{C}(f)=\begin{bmatrix} c_{bvv}(f) & c_{bvh}(f) \\ c_{bhv}(f) & c_{bhh}(f) \end{bmatrix}$$

Equalizer Impulse Response Matrix:

$$\boldsymbol{R}(f)=\begin{bmatrix} r_{vv}(f) & r_{vh}(f) \\ r_{hv}(f) & r_{hh}(f) \end{bmatrix}$$

Combined Channel Impulse Response:

$$\boldsymbol{H}(f)=\boldsymbol{C}(f)\boldsymbol{R}(f)$$

Figure 1-2 Baseband Model for Multi-User System

In order to extract the data of interest from the channel, the following conditions must be true:

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} \boldsymbol{H}_0\left(f+\frac{k}{T}\right) = \boldsymbol{I}$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} \boldsymbol{H}_l\left(f+\frac{k}{T}\right) = 0$$

We have

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} \left[\boldsymbol{C}_0\left(f+\frac{k}{T}\right)\boldsymbol{R}\left(f+\frac{k}{T}\right)\right] = \boldsymbol{I}$$

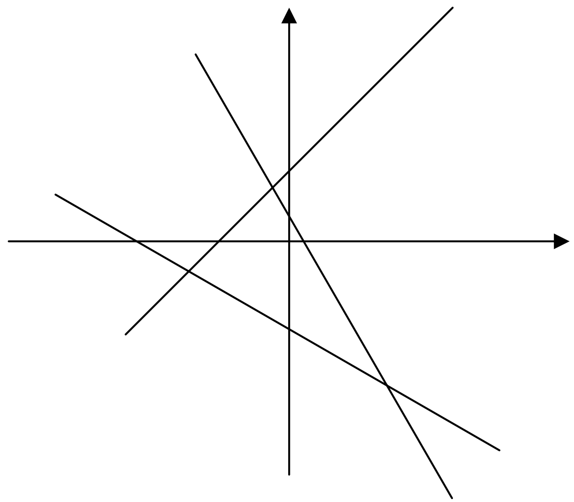
$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} \left[\boldsymbol{C}_1\left(f+\frac{k}{T}\right)\boldsymbol{R}\left(f+\frac{k}{T}\right)\right] = \boldsymbol{0}$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} \left[\boldsymbol{C}_2\left(f+\frac{k}{T}\right)\boldsymbol{R}\left(f+\frac{k}{T}\right)\right] = \boldsymbol{0}$$

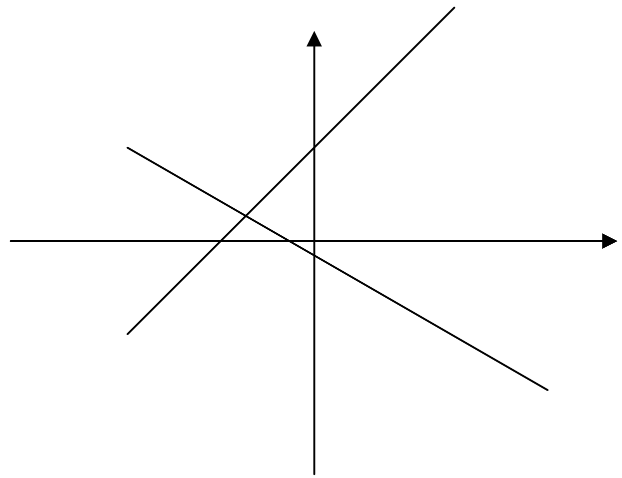
....

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} \left[\boldsymbol{C}_l\left(f+\frac{k}{T}\right)\boldsymbol{R}\left(f+\frac{k}{T}\right)\right] = \boldsymbol{0}$$

MATH STARTS HERE



Case One: The number of equations is greater than the number of unknowns, there is no solution



Case Two: The number of equations is less or equal to the number of unknowns, there will be at least one solution

Suppose the receiver is bandlimited, $\frac{-1}{2T} < f < \frac{1}{2T}$ and f_1 is in this range:

$$\frac{1}{T}\left[H_{0vv}(f_1-\frac{1}{T})+H_{ovv}(f_1)+H_{ovv}(f_1+\frac{1}{T})\right]=1$$

Since

$$H_{0vv}(f)=c_{0bvv}(f)r_{vv}(f)+c_{0bvh}(f)r_{hv}(f)$$

we have

$$\frac{1}{T}\begin{bmatrix} c_{0bvv}(f_1-\frac{1}{T})r_{vv}(f_1-\frac{1}{T})+c_{0bvh}(f_1-\frac{1}{T})r_{hv}(f_1) \\ +c_{0bvv}(f_1)r_{vv}(f_1)+c_{obvh}(f_1)r_{hv}(f_1) \\ +c_{0bvv}(f_1+\frac{1}{T})r_{vv}(f_1+\frac{1}{T})+c_{obvh}(f_1+\frac{1}{T})r_{hv}(f_1+\frac{1}{T}) \end{bmatrix}=1$$

3 more equations for H_o

The number of unknowns=12 (3bands, 4 filters)

Assume there are 2 more ACI channels, ch_1 and ch_2. For ch_1 and ch_2, apply the same analysis.

ch_1:

$$\frac{1}{T}\begin{bmatrix} c_{1bvv}(f_1-\frac{1}{T})r_{vv}(f_1-\frac{1}{T})+c_{1bvh}(f_1-\frac{1}{T})r_{hv}(f_1) \\ +c_{1bvv}(f_1)r_{vv}(f_1)+c_{1bvh}(f_1)r_{hv}(f_1) \\ +c_{1bvv}(f_1+\frac{1}{T})r_{vv}(f_1+\frac{1}{T})+c_{1bvh}(f_1+\frac{1}{T})r_{hv}(f_1+\frac{1}{T}) \end{bmatrix}=1 \quad \textbf{(1 of 4)}$$

ch_2:

$$\frac{1}{T}\begin{bmatrix} c_{2bvv}(f_1-\frac{1}{T})r_{vv}(f_1-\frac{1}{T})+c_{2bvh}(f_1-\frac{1}{T})r_{hv}(f_1) \\ +c_{2bvv}(f_1)r_{vv}(f_1)+c_{2bvh}(f_1)r_{hv}(f_1) \\ +c_{2bvv}(f_1+\frac{1}{T})r_{vv}(f_1+\frac{1}{T})+c_{2bvh}(f_1+\frac{1}{T})r_{hv}(f_1+\frac{1}{T}) \end{bmatrix}=1 \quad \textbf{(1 of 4)}$$

The total number of equations: $4(ch_0)$ $+4(ch_1)$ $+4(ch_2)$ $=12$
The total number of equations equals the total number of unknowns. The solution exists (It is a unique solution).

THE RESULTS

N_{ar}: number of ACI signals

$$N_{ar} = \begin{cases} 2\,\boldsymbol{int}(\frac{B_t + B_{ri}}{c_t}), c_t \neq 0, c_t < 2B_t \\ 0, c_t \neq 0, c_t \geq 2B_t \end{cases}$$

N_E: number of equations

$$N_E = 4(1 + N_{ar})$$

B_{ri} takes the values as the following:

$$B_{ri} = \left\{ \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \ldots, \frac{\boldsymbol{int}(2B_r)}{2} \right\}$$

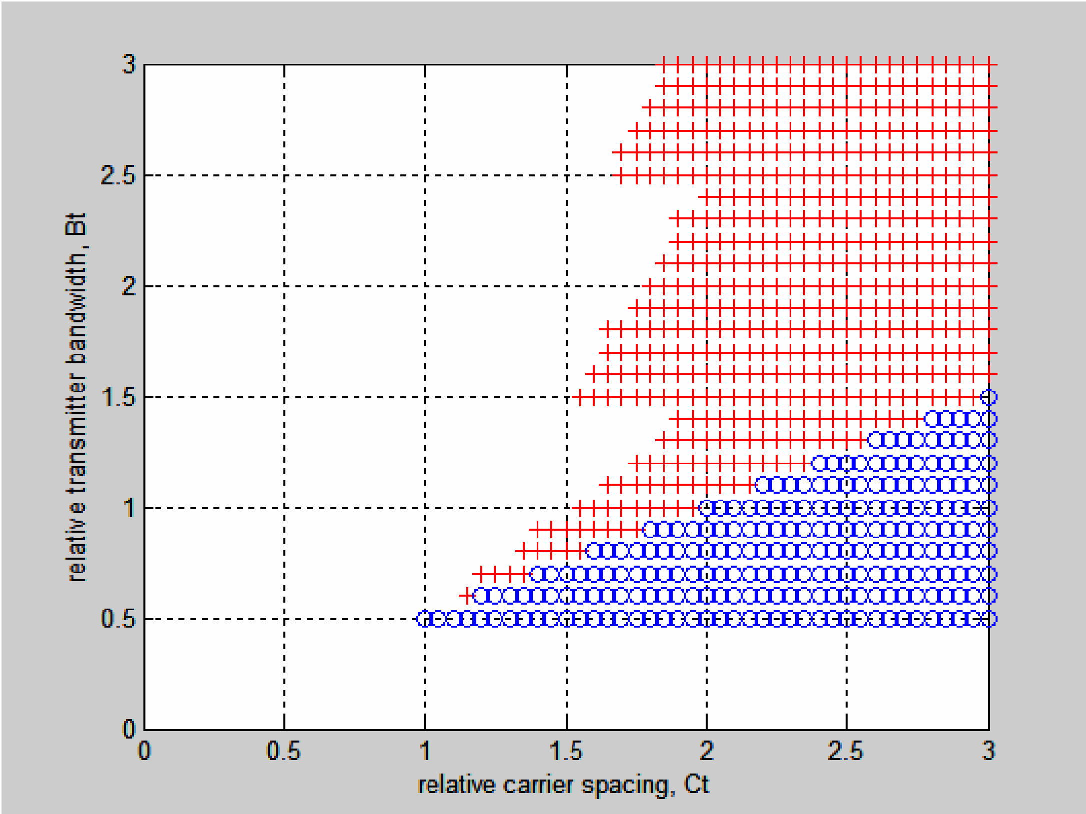
N_u: number of unknowns

$$N_u = \begin{cases} 0, 2B_t < 1 \\ 8B_{ri}, 2B_t \geq 1, c_t < 2B_t \\ 4[\boldsymbol{int}(2\,\boldsymbol{min}(B_{ri}; B_t)) + 1], 2B_t \geq 1, c_t \leq 2B_t \end{cases}$$

B_t: relative transmitter bandwidth

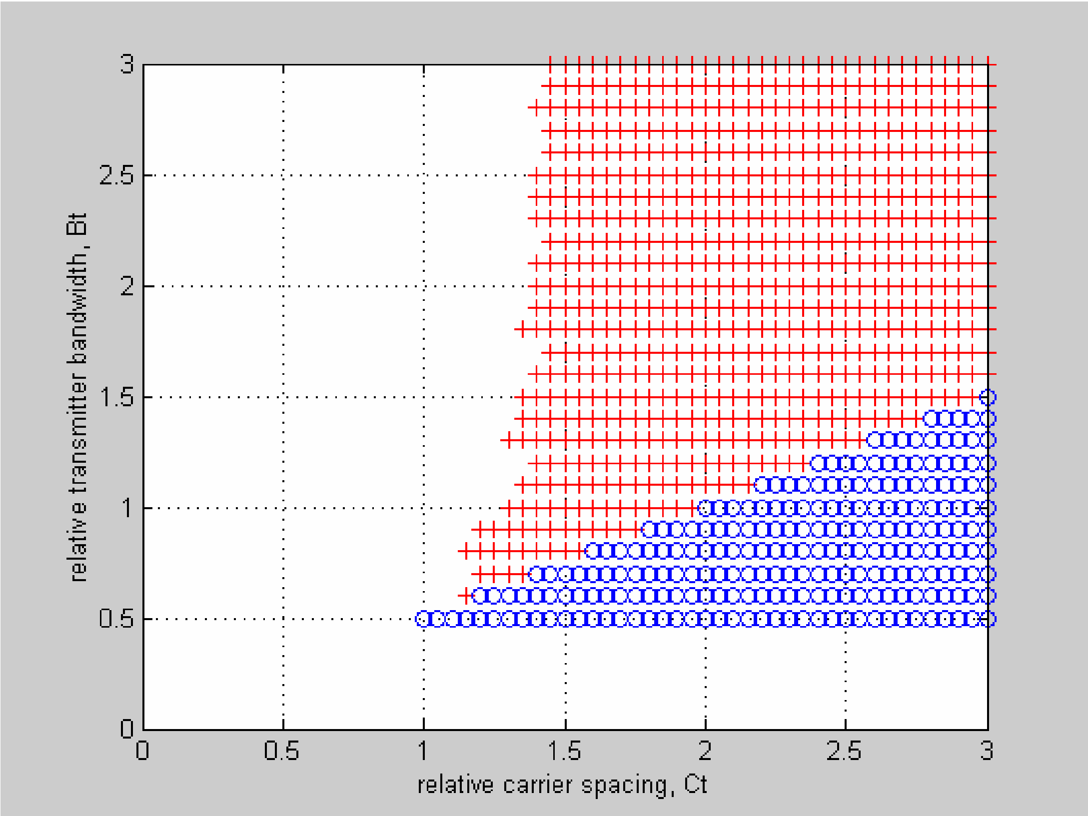
c_t: relative carrier spacing

When $B_r = B_t$, the region where the Generalized Zero-Forcing Equalizer can suppress ACI, CCI and CPI is shown below:



The region where ACI, CCI and CPI can be suppressed is shown by '+' and 'o'

When $B_r = 2B_t$, the region where the Generalized Zero-Forcing Equalizer can suppress ACI, CCI and CPI is shown below:



The region where ACI, CCI and CPI can be suppressed is shown by '+' and 'o'